Capacity Estimation for Lithium-ion Batteries Using Adaptive Filter on Affine Space

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TR2015-071 May 2015

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Asian Control Conference 2015
Capacity Estimation for Lithium-ion Batteries Using Adaptive Filter on Affine Space

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Abstract—In this paper, we propose methods to estimate the full charge capacity (FCC) of a battery based on adaptive filters. The FCC is estimated as a ratio of the accumulated charge current to the state of charge (SoC) of the battery, which is estimated by an extended Kalman filter. We consider bias errors on the estimated SoC caused by the error of typical value of FCC, which is assumed in the SoC estimation. We also consider the current sensor offset, which causes unboundedness of variables in the FCC estimation. We compose the adaptive filters on an affine space to avoid the unboundedness, which is undesirable for an implementation in embedded systems.

I. INTRODUCTION

Lithium-ion batteries (LiBs) have been widely used in electric appliances and electric cars; the application of LiBs becomes wider because of its high energy density, high power density and long life [1]. While it is important to assess the battery lifetime to use LiBs for larger scale applications, such as peak shaving, photovoltaic power generation and so on, the battery lifetime is however difficult to predict because of limited measurability relative to the complexity of the internal processes.

For designing battery systems, the battery lifetime is commonly assessed by its full charge capacity (FCC), although it should be evaluated by its energy storage capacity. Because the energy storage capacity depends on the internal resistance of the battery, which substantially varies depending on the temperature [2]. In addition, FCC is one of the most basic battery model parameter for the state of charge (SoC) estimation, where SoC is obtained as a ratio of the accumulated charge current to the FCC in Coulomb counting and several model-based methods [3]–[6].

The FCC has an initial variation in each battery cell, and decreases due to the degradation. But it takes a long time for an actual measurement of the FCC according to its definition: the electric quantity to charge the battery full from the empty. Worse yet, the battery system has to be suspended during the measurement. A solution of this problem is applying a signal processing technology. An online FCC estimation method based on an adaptive filter is proposed in [7], and another method based on Kalman filter is proposed in [8]. Although additive Gaussian noises on the terminal voltage and the current measurements are taken into consideration in these methods, a significant degradation in the estimation accuracy is caused by an offset on the current measurements, which are inevitable in widely used Hall effect sensors [9].

We consider a simultaneous estimation of the FCC and the current sensor offset by an adaptive filter. Let \( q_{cc,k} \) be the Coulomb counting calculated recursively by the following algorithm:

\[
q_{cc,k} = q_{cc,k-1} + t_s I_{k-1}, \quad q_{cc,0} = 0 \tag{1}
\]

where \( t_s \) is the sampling period, \( I_{k} \) is the measured current, the FCC is able to be estimated based on the following relationship:

\[
q_{cc,k} = F_{cc} s_k + t_k I_{off} - q_0. \tag{2}
\]

where \( t_k := k t_s \), \( I_{off} \) is the current sensor offset, \( F_{cc} \) is the FCC, \( s_k \) is the SoC and \( q_0 \) is the initial electric quantity charged in the battery. It is able to implement a recursive least squares (RLS) filter or a recursive total least squares (RTLS) filter to estimate the \( I_{off} \), \( F_{cc} \) and \( q_0 \), if an accurate estimation of \( s_k \) is obtained [7].

The first difficulty in the FCC estimation based on the adaptive filters is that the model-based methods for SoC estimation depend on the FCC. If a typical value of the FCC is used in these methods, bias errors of \( s_k \) are caused by the difference between the typical value and the true value of the FCC. The second difficulty is caused by the term \( t_k I_{off} \) in (2), which increases unlimitedly with time. It is obviously undesirable for an implementation in embedded systems.

In this paper, we propose methods to estimate the FCC of a battery and the current sensor offset based on an RLS filter and an RTLS filter proposed in [10], in which the bias errors of the estimated SoC is compensated by a first order evaluation of \( s_k \) obtained [7].

II. MODEL BASED SOC ESTIMATION

A. Lithium-ion battery

An LiB consists mainly of a positive electrode, a negative electrode, current collectors and a separator; all components are soaked with electrolyte solution (Fig. 1). In a typical design, the positive electrode is made of a porous material composed of metal oxide particles such as LiCoO\(_2\), LiMn\(_2\)O\(_4\) and so on. The negative electrode is also made of a porous
Fig. 1. A typical structure of lithium-ion battery cells. Lithium-ions pass through the separator, while electrons conduct via the electric circuit.

\[ \text{Collector} \quad \begin{array}{c|c|c} \text{Positive} & \text{Separator} & \text{Negative} \\ \text{electrode} & & \text{electrode} \\ \text{Collector} \end{array} \]

Fig. 2. An equivalent circuit expression of a simplified lithium-ion battery model. The voltage source \( E \) depends on the state of charge of the battery.

The voltage source \( E \) is referred to as an open circuit voltage (OCV) which is a function of the SoC \([15]\). Let \( \varrho_{b,k} \) and \( \varrho_{l,k} \) be the electric quantities charged in the battery and the capacitor respectively, and let \( \tau_d := R_d C_d \), the terminal voltage of the battery \( V_k \) is described as follows:

\[ x_{k+1} = F_k x_k + G I_k, \quad V = h(x_k) + R_0 I_k, \] (6)

where \( x_k := [\varrho_{b,k} \quad \varrho_{l,k}]^T \), \( h(x_k) := E(\varrho_{b,k}/F_{cc}) \) and \( F_k := \left[ e^{-\frac{t_k}{\tau_d}} \quad 1 \right] \), \( G := \left[ \tau_d(1-e^{-\frac{t_k}{\tau_d}}) \right] \).

C. Extended Kalman filter for SoC estimation

Let \( \sigma_f^2 \) and \( \sigma_v^2 \) be the variances of the noises on the current and the voltage measurement respectively. A prediction step of an extended Kalman filter (EKF) for SoC estimation is written as follows:

\[ \hat{x}_{k|k} = F_k \hat{x}_{k|k-1} + G I_k, \] (7)

\[ P_{k|k} = F_k P_{k|k-1} F_k^T + Q, \] (8)

where \( \hat{x}_{|k} \) is an estimation of \( x \) at \( k \), and \( Q \) is a symmetric positive definite matrix depending on \( \sigma_v \). An update step of the EKF is written as follows:

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(V_k - z_k), \] (9)

\[ P_{k|k} = (I - K_k H_k) P_{k|k-1}, \] (10)

where

\[ z_k := h(\hat{x}_{k|k-1}) + R_0 I_k, \quad H_k := \frac{\partial h}{\partial \hat{x}_k}(\hat{x}_{k|k-1}), \]

\[ S_k := \sigma_v^2 + H_k P_{k|k-1} H_k^T, \quad K_k := P_{k|k-1} H_k^T S_k^{-1}. \]

Then the estimation of the SoC is given by \( \hat{s}_k := \hat{\varrho}_{b,k}/F_{cc} \), where \( [\varrho_{b,k}|k] := \frac{\tau_d}{\tau_s} \hat{x}_{k|k} \).

III. FCC AND CURRENT SENSOR OFFSET ESTIMATION

A. Bias error compensation

The SoC of the battery is estimated from a time series of the measured current and the terminal voltage. As we described in section II-C, the estimation algorithm depends on the FCC of the battery. Therefore the dependency of the estimation is expressed by:

\[ \hat{s}_k = F_k((I_k, V_k), \ldots, (I_0, V_0)|F_{cc}), \] (11)

where \( F_k \) is a map from a set of measured values to an estimated value of SoC. If a typical value of the FCC used in the SoC estimation is slightly different from the true value of the FCC, and/or the current sensor offset, the estimated value of SoC is biased as expressed by:

\[ \hat{s}_k = F_k((I_k + I_{off}, V_k), \ldots, (I_0 + I_{off}, V_0)|\hat{F}_{cc}). \] (12)

Let the current sensor offset and difference between \( F_{cc} \) and \( \hat{F}_{cc} \) parameterize by \( I_{off} \approx I_{typ} p_1 \) and \( \hat{F}_{cc} \approx F_{cc}(1 + p_2) \), the biased estimation \( \hat{s}_k \) is approximated as:

\[ \hat{s}_k \approx \hat{s}_k + \frac{\partial F_k}{\partial p_1} p_1 + \frac{\partial F_k}{\partial p_2} p_2 \] (13)
by a Taylor series expansion (see [16] for matrix derivatives), where \( I_{\text{typ}} \) is a constant introduced to ensure \( p_1 \ll 1 \). Then we get the following relation:

\[
q_{cc,k} = \frac{\hat{F}_{cc}}{1 + p_2} \left( \hat{s}_k - \frac{\partial F_k}{\partial p_1} p_1 - \frac{\partial F_k}{\partial p_2} p_2 \right) + I_{\text{typ}} p_1 t_k - q_0
\]

by substituting (13) into (2). The equation (14) is rewritten as:

\[
q_{cc,k} - \hat{F}_{cc} \hat{s}_k = \left( I_{\text{typ}} t_k - \hat{F}_{cc} \frac{\partial F_k}{\partial p_1} \right) p_1 - \hat{F}_{cc} \left( \hat{s}_k + \frac{\partial F_0}{\partial p_2} \right) p_2 - q_0
\]

by omitting higher order terms of \( p_1 \) and \( p_2 \).

**B. Adaptive filter on affine space**

For simplicity, we rewrite (15) as:

\[
y_k = p^T u_k - q_0,
\]

where

\[
y_k := q_{cc,k} - \hat{F}_{cc} \hat{s}_k, \quad u_k := \left[ I_{\text{typ}} t_k - \hat{F}_{cc} \frac{\partial F_k}{\partial p_1} \right] - \hat{F}_{cc} \left( \hat{s}_k + \frac{\partial F_0}{\partial p_2} \right)
\]

and \( p := [p_1 \ p_2]^T \). Obviously the parameter \( p \) is able to estimate by an RLS filter or an RTLS filter. We simply refer to these methods as RLS and RTLS respectively in the following sections.

The pair of the term \( Q_{cc,k} \) and the term \( I_{\text{typ}} t_k \) is the main obstacle due to those unboundedness. A simple solution is a differential approach, that is an RLS filter or an RTLS filter based on the following relation:

\[
\Delta y_k = p^T \Delta u_k,
\]

where \( \Delta y_k := y_k - y_{k-1}, \Delta u_k := u_k - u_{k-1} \). We refer to these methods as DRLS and DRTLS.

Our approach is considering a time-dependent local coordinate system of the vector space to which the pair \((y_k, u_k)\) is belonging. The local coordinate of the pair is able to be bounded, if the origin of the local coordinate system is set close to the pair.

Let \( \mathcal{V} \) be a vector space to which the pair \((y_k, u_k)\) is belonging, we regard \( \mathcal{V} \) as an affine space. Let \( T_k \) be a tangent space of \( \mathcal{V} \) whose origin is set on \((y_k, u_k)\) and whose bases are same to those of the original vector space. Then the local coordinate of \((y_l, u_l) \in \mathcal{V}\) is written as \((y_l - y_k, u_l - u_k) := (\phi^{-1}_k(y_l), \psi^{-1}_k(u_l))\) in \( T_k \) for all \( l \).

Now we consider a weighted mean of \( y_k \) and \( u_k \) defined as follows:

\[
\bar{y}_k := \frac{1}{S_k} \sum_{l=0}^{k} \lambda^{k-l} y_l, \quad \bar{u}_k := \frac{1}{S_k} \sum_{l=0}^{k} \lambda^{k-l} u_l
\]

where \( S_k := \sum_{l=0}^{k} \lambda^{k-l} \) and \( \lambda \) is a forgetting factor in \((0, 1)\). Then the following relationship holds between the deviations of \( y_l \) and \( u_l \) from the weighted mean: \( y_l - \bar{y}_k = p^T(u_l - u_k) \).

Fig. 3. Geometric relationship among the tangent spaces \( T_k \), the samples \((y_k, u_k)\) and the weighted mean \((\bar{y}_k, \bar{u}_k)\). The ellipses express the distributions of the samples. The equations (20) and (21) are the direct calculation from \((\zeta_k, \xi_k)\) to \((\zeta_{k+1}, \xi_{k+1})\). Remark the different between the two distributions is extremely emphasized.

The relationship also holds on the tangent space \( T_k \), because the map \( \phi_k \) and \( \psi_k \) conserves the inner product of \( \mathcal{V} \). Therefore

\[
\zeta_k = p^T \xi_k, \quad \zeta_{k+1} = p^T \xi_{k+1}
\]

where \((\zeta_k, \xi_k) := (\phi^{-1}_k(y_k), \psi^{-1}_k(u_k))\) (see Fig. 3).

The local coordinate of the weighted means \( \zeta_k, \xi_k \) are calculated recursively by follows:

\[
\bar{\zeta}_k = \phi^{-1}_k(\lambda S_{k-1} \bar{\zeta}_{k-1} + y_k)/S_k = \frac{\lambda S_{k-1}}{S_k} (\bar{\zeta}_{k-1} - \Delta y_k),
\]

\[
\bar{\xi}_k = \psi^{-1}_k(\lambda S_{k-1} \bar{\xi}_{k-1} + u_k)/S_k = \frac{\lambda S_{k-1}}{S_k} (\bar{\xi}_{k-1} - \Delta u_k).
\]

The variables \( \Delta y_k \) and \( \Delta u_k \) are bounded if \( y_k \) and \( u_k \) are smooth time series, and calculated without dealing with the term \( q_{cc,k} \) and \( t_k \), by using following equations: \( t_k - t_{k-1} = t_s, q_{cc,k} - q_{cc,k-1} = t_s I_{\text{typ}} \). The term \( S_k \) is also bounded obviously, and calculated recursively by \( S_k = S_{k-1} + \lambda^k \) and \( \lambda^k = \lambda \cdot \lambda^{k-1} \). Therefore \( \zeta_k \) and \( \xi_k \) are bounded for each \( k \), because \( \lambda S_k/S_{k-1} < 1 \).

An adaptive filter based on (17) or (19) is more desirable for implementation in embedded systems than that based on (16), because all variables in the recursive calculation are bounded.

**C. Rayleigh quotient-based fast RTLS filter**

Let \( \delta_k \) be the error of \( \zeta_k \) and \( \epsilon_k \) be the error of \( \xi_k \), an RTLS filter minimizes the following objective function:

\[
J_k(p, \hat{\zeta}_k, \hat{\xi}_k) := \sum_{l=0}^{k} \lambda^{k-l} (\delta_l^2 + \epsilon_l^T W^{-1} \epsilon_l),
\]

such that \( \zeta_k - \delta_k = p^T (\xi_k - \epsilon_k) \), where \( \hat{\zeta}_k := \{\zeta_0, \ldots, \zeta_k\} \), \( \hat{\xi}_k := \{\xi_0, \ldots, \xi_k\} \), \( \zeta_k := \zeta_k - \delta_k, \xi_k := \xi_k - \epsilon_k \) and \( W \) is a symmetric positive definite weighting matrix. Let \( p_k \) be an
estimator of \( \mathbf{p} \) at \( t_k \), the minimum point of \((\hat{z}_k, \hat{z}_k)\) for each \( \hat{p}_k \) is given by:

\[
\hat{n}_k := A_k (A_k^T \Lambda^{-1} A_k)^{-1} A_k^T \Lambda^{-1} \eta_k,
\]

where

\[
\hat{n}_k := \frac{\hat{z}_k}{\xi_k}, \quad \eta_k := \left[ \begin{array}{c} \zeta_k \\ \xi_k \end{array} \right], \quad A_k := \left[ \begin{array}{c} \hat{p}_k^T \\ I \end{array} \right]
\]

and \( \Lambda := \text{diag}(1, W) \).

A nonzero vector \( \mathbf{a}_k \) such that \( A_k^T \mathbf{a}_k = 0 \) exists uniquely except for scalar multiplies, because the kernel of \( A_k \) is an 1-dimensional subspace. Then minimizing objective function \( J_k \) is equivalent to minimizing a Rayleigh quotient defined as follows:

\[
Q_k := \frac{\mathbf{a}_k^T \hat{R}_k \mathbf{a}_k}{\mathbf{a}_k^T \Lambda \mathbf{a}_k}, \quad \text{where} \quad \hat{R}_k := \sum_{i=0}^{k} \lambda_k^{i-1} \eta_i \eta_i^T.
\]  

The vector \( \mathbf{a}_k \) must be parameterized as \( \mathbf{a}_k = [-1, \mathbf{p}_k^T] \) to satisfy the orthogonality condition. Now we consider the following incremental update equation:

\[
\hat{p}_k = \hat{p}_{k-1} + \theta_k \mathbf{w}_k,
\]

where \( \mathbf{w}_k \) is a direction of a line search. The direction \( \mathbf{w}_k \) is desirably a time series of vectors which efficiently spans the image of \( A_k \) in a short time range. Although authors employ \( \xi_k \) as \( \mathbf{w}_k \) in [10], we employ a random unit vector uniformly distributed on the unit circle (actually it is not necessarily normalized), because \( \xi_k \) is strongly time-correlated and inefficient to span the image of \( A_k \) in our case.

Substituting (25) into (24), the Rayleigh quotient \( Q_k \) forms a rational function of \( \theta_k \). The numerator \( N_k \) and the denominator \( D_k \) of \( Q_k \) is written as

\[
N_k = N_{2,k} \theta_k^2 + 2N_{1,k} \theta_k + N_{0,k}, \\
D_k = D_{2,k} \theta_k^2 + 2D_{1,k} \theta_k + D_{0,k},
\]

where

\[
N_{2,k} := \mathbf{w}_k^T \hat{R}_k \mathbf{w}_k, \\
N_{1,k} := \mathbf{w}_k^T \hat{R}_k \hat{p}_{k-1} - c_k^T \mathbf{w}_k, \\
N_{0,k} := \hat{p}_{k-1}^T \hat{R}_k \hat{p}_{k-1} - 2c_k^T \hat{p}_{k-1} + d \\
D_{2,k} := \mathbf{w}_k^T \mathbf{w}_k \mathbf{w}_k^T, \\
D_{1,k} := \mathbf{w}_k^T \hat{R}_k \mathbf{w}_k, \\
D_{0,k} := \hat{p}_{k-1}^T \mathbf{w}_k \mathbf{w}_k^T + 1
\]

and

\[
\left[ \begin{array}{c} d_k \\ c_k \\ R_k \end{array} \right] := \hat{R}_k := \sum_{i=0}^{k} \lambda_k^{i-1} \left[ \begin{array}{c} \zeta_i^2 \\ \xi_i \hat{z}_i \xi_i \xi_i^T \end{array} \right].
\]

The extreme points of \( Q_k \) are given by the solutions of following equation

\[
\frac{\partial N_k}{\partial \theta_k} D_k - N_k \frac{\partial D_k}{\partial \theta_k} = \alpha_k \theta_k^2 + \beta_k \theta_k + \gamma_k
\]

where

\[
\theta_k := \frac{\beta_k \theta_k^2}{D_k^2}.
\]

IV. NUMERICAL EXAMPLE

In this section, we illustrate the performance of our algorithm by a numerical simulation. In our simulation, we employed the simplified battery model shown in Fig. 2 with the battery model parameters in Table II, and sampling period \( t_s = 100 \text{ ms} \). We assumed the dependency of the voltage source \( E \) on the state of charge \( s \) is described as \( E = E_0 + E_1 s \), where \( E_0 \) and \( E_1 \) are constants.

First, we calculated the terminal voltage of the battery using the input current shown in Fig. 4 (the true SoC is also shown in Fig. 5 for visibility). We slightly varied the FCC of the battery from the typical value as \( F_{cc} = 0.9F_{cc} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 )</td>
<td>10 mΩ</td>
<td>( F_{cc} )</td>
<td>1.0 Ah</td>
</tr>
<tr>
<td>( R_d )</td>
<td>1.0 mΩ</td>
<td>( E_0 )</td>
<td>2.6 V</td>
</tr>
<tr>
<td>( C_d )</td>
<td>5.0 kF</td>
<td>( E_1 )</td>
<td>1.6 V</td>
</tr>
</tbody>
</table>

Fig. 4. Input current for the simulation. The time axis is zoomed in during the first 50 minutes.
Next, we estimated the SoC of the battery and the derivative thereof from the current and the voltage using a Kalman filter based on the simplified battery model, where we offset the current by 0.3 A, and added a Gaussian noise of variance $\sigma_i^2 = 10^{-4}$ and $\sigma_v^2 = 10^{-5}$ to the current and the voltage respectively.

After that, we estimated the FCC and the current sensor offset from the estimated SoC, the derivative thereof and the measured current (or Coulomb counting in some methods) by various adaptive filters shown in Table III. The differential filters are based on (17), while the basic and affine algorithms are based on (16) and (19) respectively.

We employed $\lambda = e^{-\tau/\tau_c}$ where $\tau = 20$ min, $I_{typ} = 100$ A and $W = I$. Then we tuned the initial $R_k, c_k, d_k$ as far as possible, so that the estimated values converge while $t_k \leq 100$ min.

The results of the estimation are shown in Fig. 6–11. All filters well estimate the current sensor offset, and well estimate the FCC except RTLS. The estimation errors, means and standard deviations thereof are shown in Fig. 12–15. ARLS and ARTLS achieve equivalent accuracy to that of RLS, while the results of DRLS and DRTLS are biased in $F_{cc}$ and have larger deviations in $I_{off}$.

### V. Conclusion

In this paper, we propose a method to estimate the FCC of a lithium-ion battery by an adaptive filter from the current and the SoC of the battery. The SoC is estimated by a Kalman filter based on a simplified battery model from the current and the terminal voltage of the battery. The bias error of the estimated SoC caused by the current sensor offset and the difference between a true value of the FCC and a typical value thereof is assumed in the Kalman filter is compensated in the meaning of first order approximation.

In the FCC estimation by commonly used RLS filter or RTLS filter where the current sensor offset is taken into consideration, the variables used in the estimation algorithms are unbounded. In our approach, the RLS filter or RTLS filter is composed on an affine space to avoid the unboundedness, instead of differentiating the input signals, which is another
our approach outperforms the differentiation approach by a numerical example.

REFERENCES