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Interference Investigation for Cognitive Spectrum Sharing Networks with Reactive DF Relay Selection

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Abstract—Cognitive radio (CR) with spectrum-sharing has been envisioned as emerging technology for the next generation of mobile and wireless networks by allowing the unlicensed customers simultaneously utilize the licensed radio frequency spectrums. However, the CR has faced some practical challenges due to its deduced system performance as compared to non spectrum-sharing counterpart. In this paper, we therefore consider the potential of incorporating the cooperative communications into CR by introducing the concept of reactive multiple decode-and-forward (DF) relays. In particular, we derive new results for exact and asymptotic expressions for the performance of cognitive relay networks with $K$-th best relay selection. Our novel results have exhibited the significance of using relay networks to enhance the system performance of CR.

I. INTRODUCTION

With the ever increasing demand of mobile multimedia services, wireless systems have encountered several practical constraints, e.g., bandwidth availability, multi-path signal degradation, interference management. Among these affects, shortage of radio frequency spectrum is the most critical issue. To cope with this problem, cognitive radio (CR) has been proposed as an efficient sharing scheme among licensed and unlicensed users. By allowing the unlicensed user to concurrently occupy the radio spectrum, the utilization of frequency spectrum is remarkably enhanced, which is a promising solution to the lack of radio spectrum [1]. However, by limiting the transmit power at the unlicensed user, regulated by the primary network, the performance at the secondary receiver is drastically reduced, especially when the channels experience heavy pathloss and severe shadowing effects.

Recently, opportunistic relaying has been realized as a supreme means to enhance communication coverage [2]. As such, the extension of using relay in CR has attracted great attention in the research community [3]–[8]. Specifically, the performance of proactive relay networks has been considered in terms of outage probability, error probability, and ergodic capacity [4]. In [5], the ergodic capacity of reactive multiple decode-and-forward (DF) relays has been investigated. It is important to note that the works [4], [5] have selected the best relays among all of the active nodes in the networks. However, such scheduling may be not applicable in the dense heterogenous networks due to the load balance and imperfect channel state information. Very recently, by considering the $N$-th best relay, the outage probability of reactive DF relays has been derived in [6]. However, this work is limited to the case of identical fading channels and only exact outage probability. Therefore, we take a step further to reveal the importance of $N$-th best reactive DF relay in CR by consider a more general fading model where all links are assumed independent but non-identically distributed (i.n.i.d.). In addition, we provide additional insights by deriving the asymptotic outage probability, where both diversity and coding gains are also obtained. Our contributions are summarized as follows:

- We consider the joint impact of peak interference power constraint of licensed user and maximal transmit power of unlicensed user on the performance of CR networks with reactive multiple DF relays and $N$-th best relay selections.
- We derived exact outage probability and ergodic capacity for the considered networks. Our derivation is valid for general channels where all links are i.n.i.d. fading.
- To provide more insights into the system performance, we also derived the asymptotic outage probability where both diversity and coding gains are obtained. It has been shown that the full diversity can be realized when the peak interference power is proportional to the maximal transmit power.

II. SYSTEM AND CHANNEL MODELS

The cognitive network consists of a secondary source ($S$), a secondary destination ($D$), $M$ secondary relays ($R$) and a primary user ($PU$). Let $d_{i1}$, $d_{i2}$, $d_{i3}$, and $d_{i4}$ denote distances of the $S \rightarrow R_i$, $R_i \rightarrow D$, $S \rightarrow PU$, and $R_i \rightarrow PU$ links, respectively, where $i \in \{1,2,\ldots,M\}$. We also denote $h_{i1}$, $h_{i2}$, $h_{i3}$, and $h_{i4}$ as channel coefficients of the $S \rightarrow R_i$, $R_i \rightarrow D$, $S \rightarrow PU$, and $R_i \rightarrow PU$ links, respectively. We assume that all of the channels follow a Rayleigh fading distribution. Hence, the channel gains $\gamma_{i1} = |h_{i1}|^2$, $\gamma_{i2} = |h_{i2}|^2$, $\gamma_{i3} = |h_{i3}|^2$, and $\gamma_{i4} = |h_{i4}|^2$ follow exponential distributions. To take path-loss into account, we model the parameters of $\gamma_{i1}$, $\gamma_{i2}$, $\gamma_{i3}$, and $\gamma_{i4}$ as in [9]: $\lambda_{i1} = (d_{i1})^\beta$, $\lambda_{i2} = (d_{i2})^\beta$, $\lambda_{i3} = (d_{i3})^\beta$, and $\lambda_{i4} = (d_{i4})^\beta$, where $\beta$ denotes the path-loss exponent.

In cognitive underlay networks, the source and relay must adapt their transmit power so that interference caused at PU is lower than a maximum interference level, denoted by $I_{th}$. In addition, it is also assumed that their transmit power must be lower than a maximum threshold, denoted by $P_{th}$. We assume that all of the nodes are equipped with a single antenna and
operate on half-duplex mode. Similar to the model proposed in [7], the maximum transmit power of the source \( S \) and relay \( R_i \) are, respectively, given as \( P_S = \min \{ P_{th}, I_{th}/\gamma_3 \} \) and \( P_{R_i} = \min \{ P_{th}, I_{th}/\gamma_4 i \} \). Therefore, the instantaneous signal-to-noise (SNR) of the SNR of the signal from the source to the relays and destination. Also, we define normalized quantities \( \gamma_P = P_{th}/N_0 \) and \( \gamma_I = I_{th}/N_0 \). Without loss of generality, we assume that the ratio between \( \gamma_P \) and \( \gamma_I \) is constant, i.e.,

\[
\bar{\gamma}_P = \frac{\gamma_P}{\gamma_I} = \mu.
\]  

The operation of the proposed protocol is realized by TDMA technique. In the first time slot, the source \( S \) broadcasts its signal to the destination. Then, the relays try to decode the source’s signal from the received signal. Let us denote \( Q_1 \) and \( Q_2 \) as the set of the relays which decode the source successfully and unsuccessfully, respectively. We can assume that \( Q_1 = \{ R_{j_1}, R_{j_2}, ..., R_{j_N} \} \) and \( Q_2 = \{ R_{j_{N+1}}, R_{j_{N+2}}, ..., R_{j_{M}} \} \), where \( N \) is the cardinality of \( Q_1 \), \( N \in \{ 0, 1, 2, ..., M \} \), and \( j_1, j_2, ..., j_M \in \{ 1, 2, ..., M \} \). The considered cognitive radio networks, we consider two cases as follows:

- (C1) : The \( K \)th-best relay is chosen among \( N \geq K \) successful relays to forward the source’s signal to the destination at the second time slot. The partial relay selection is realized by the following strategy

\[
R_{j_c} : \Psi_{2j_c} = K \text{th} \max_{t \in 1, 2, ..., N} (\Psi_{2j_t})
\]  

where \( R_{j_c} \) denotes the chosen relay.

- (C2) : The system cannot choose a relay to forward the source’s signal to the destination since \( N < K \). Hence, in this case, the signal is dropped.

### III. PERFORMANCE EVALUATION

#### A. Derivation of the CDF and PDF of the Instantaneous SNR by \( K \)th-Best Relay Selection

We observe that among \( N \) successful relays of the set \( Q_1 \), there are \( (K-1) \) relays whose \( \Psi_{2j_t} \) is larger than \( \Psi_{2j_c} \) and \( (N-K) \) relays whose \( \Psi_{2j_t} \) is smaller than \( \Psi_{2j_c} \), where \( t \in \{ 1, 2, ..., N \} / \{ c \} \). We, respectively, denote these sets as \( W_1 = \{ R_{z_1}, R_{z_2}, ..., R_{z_{K-1}} \} \) and \( W_2 = \{ R_{z_{K+1}}, R_{z_{K+2}}, ..., R_{z_N} \} \), where \( W_1 \subset Q_1, W_2 \subset Q_1 \), and \( W_1 \cap W_2 = Q_1/\{ R_{j_c} \} \). Thus, from [10, Eq. (8)], we can write the CDF and PDF of \( \Psi_{2i}, i \in \{ 1, 2, ..., M \} \), as follows

\[
F_{\Psi_{2i}}(x) = 1 - \exp(-\frac{x}{\gamma_P}) + \frac{\lambda_2i\bar{x}}{\lambda_{2i} + P_{\gamma_P}} \exp\left(-\frac{\lambda_{2i}\bar{x}}{\gamma_P} - \lambda_{4i}\mu\right)
\]

and

\[
f_{\Psi_{2i}}(x) = \frac{\lambda_2i\bar{x}}{\gamma_P} \exp\left(-\frac{\lambda_{2i}\bar{x}}{\gamma_P}\right) - \frac{\lambda_{2i}^2\bar{x}}{\lambda_{2i} + \bar{\gamma}_P}\exp\left(-\frac{\lambda_{2i}\bar{x}}{\gamma_P} - \lambda_{4i}\mu\right)
\]

Let us denote \( Y_1 = \min(\Psi_{2z_1}, ..., \Psi_{2z_{K-1}}) \) and \( Y_2 = \max(\Psi_{2z_{K+1}}, ..., \Psi_{2z_{M}}) \). Now we can express the CDFs of the RVs \( Y_1 \) and \( Y_2 \) as

\[
F_{Y_1}(x) = \prod_{v=1}^{K-1} (1 - \exp(-\frac{\lambda_{2z_v}x}{\gamma_P}))
\]

and

\[
F_{Y_2}(x) = \prod_{v=K+1}^{M} (1 - \exp(-\frac{\lambda_{2z_v}x}{\gamma_P}))
\]

In addition, we can formulate the CDF of \( \Psi_{2j_c} \) as follows

\[
F_{\Psi_{2j_c}}(x) = \sum_{c=1}^{N} \sum_{W_1, W_2} \Pr(\Psi_{2j_c} < x, Y_1 \leq \Psi_{2j_c} \leq Y_2)
\]

which is equivalent to the following expression

\[
F_{\Psi_{2j_c}}(x) = \sum_{c=1}^{N} \sum_{W_1, W_2} \int_0^x \left( \frac{\lambda_{2j_c}x}{\gamma_P} \exp\left(-\frac{\lambda_{2j_c}y}{\gamma_P}\right) \right.
\]

\[
\left. + \left( \frac{\lambda_{2j_c}\lambda_{4j_c}\bar{\gamma}_P}{(\lambda_{2j_c}y + \lambda_{4j_c}\bar{\gamma}_P)^2} - \frac{(\lambda_{2j_c})^2y}{\gamma_P(\lambda_{2j_c}y + \lambda_{4j_c}\bar{\gamma}_P)} \right) \exp\left(-\frac{\lambda_{2j_c}y}{\gamma_P} - \lambda_{4j_c}\mu\right) \right) 
\]

\[
(1 - F_{Y_1}(y)) F_{Y_2}(y) dy.
\]

Based on (7), the corresponding PDF is given by

\[
f_{\Psi_{2j_c}}(x) = \partial F_{\Psi_{2j_c}}(x) / \partial x
\]

which is evaluated as

\[
f_{\Psi_{2j_c}}(x) = \sum_{c=1}^{N} \sum_{W_1, W_2} \left[ \frac{\lambda_{2j_c}x}{\gamma_P} \exp\left(-\frac{\lambda_{2j_c}x}{\gamma_P}\right) \right.
\]

\[
\left. + \left( \frac{\lambda_{2j_c}\lambda_{4j_c}\bar{\gamma}_P}{(\lambda_{2j_c}x + \lambda_{4j_c}\bar{\gamma}_P)^2} - \frac{(\lambda_{2j_c})^2x}{\gamma_P(\lambda_{2j_c}x + \lambda_{4j_c}\bar{\gamma}_P)} \right) \exp\left(-\frac{\lambda_{2j_c}x}{\gamma_P} - \lambda_{4j_c}\mu\right) \right]
\]

\[
(1 - F_{Y_1}(x)) F_{Y_2}(x).
\]
B. Exact Outage Probability of Non-homogeneous Networks

We assume that the relay $R_i$ (the destination $D$) can decode the signal successfully if the instantaneous SNR of the $S \rightarrow R_i$ $(R_i \rightarrow D)$ link exceeds a threshold $\gamma_{th}$. Therefore, the outage probability of the proposed protocol can be calculated as follows

$$P_{out} = \Pr(N < K) + \sum_{v=1}^{M} \Pr(\Psi_{2v} < \gamma_{th}, N \geq K) \quad (11)$$

where $P_{out}^1$ represents case $C_1$ where the system cannot choose any relays to forward the source’s signal to the destination, while $P_{out}^2$ represents case $C_2$ where the system can choose the $K$th-best relay for the cooperation but the transmission between the selected relay and the destination is in outage. Considering the outage probability $P_{out}^1$, we can formulate it as

$$P_{out}^1 = \sum_{\bar{q_1}, \bar{q_2} \in \bar{q}} \Pr(\Psi_{1\bar{q}_1} \geq \gamma_{th}, \ldots, \Psi_{1N} \geq \gamma_{th}, \Psi_{1N+1} < \gamma_{th}, \ldots, \Psi_{1M} < \gamma_{th}) \quad (12)$$

which can be rewritten by (13) at the top of next page where,

$$V_1 = \frac{\left(1 - \exp(-\lambda_3 \mu)\right)^N \prod_{t=1}^{N} \left(1 - \exp(-\lambda_1 \rho t)\right)}{\lambda_3 \mu \gamma_{th} P \exp(-\lambda_3 \mu \gamma_{th} P)}$$

$$V_2 = \int_{\mu}^{+\infty} \left[\frac{\lambda_3 \exp(-\lambda_3 x)}{\lambda_1 \exp(-\lambda_1 \rho t x)} \prod_{t=1}^{N} \left(1 - \exp(-\lambda_1 \rho t x)\right) dx \right. \times \prod_{t=1}^{N} \left(1 - \exp(-\lambda_1 \rho t x)\right)] dx. \quad (15)$$

Having expanded $\prod_{t=1}^{N} \left(1 - \exp(-\lambda_1 \rho t x)\right)$ by the binomial identity, we have the following expression for $V_2$

$$V_2 = \frac{\lambda_3 \exp(-\lambda_3 \mu - \sum_{t=1}^{N} \lambda_1 \rho t) + \sum_{v=1}^{M-N} \sum_{j_1, j_2, \ldots, j_v \in \bar{q}} \frac{(-1)^v \lambda_3 \mu \gamma_{th} P}{\lambda_3 + \sum_{t=1}^{N} \lambda_1 \rho t} \times \exp(-\lambda_3 \mu - \sum_{t=1}^{N} \lambda_1 \rho t + \sum_{t=1}^{N} \lambda_1 \rho t)] (16)$$

Collecting (14) and (16), the closed-form expression of the exact outage probability (13) is given by

$$P_{out}^1 = \left(1 - \exp(-\lambda_3 \mu)\right)^N \prod_{t=1}^{N} \left(1 - \exp(-\lambda_1 \rho t)\right) \times \prod_{t=1}^{M-N} \left(1 - \exp(-\lambda_1 \rho t)\right)$$

$$+ \sum_{v=1}^{M-N} \sum_{j_1, j_2, \ldots, j_v \in \bar{q}} \frac{(-1)^v \lambda_3 \mu \gamma_{th} P}{\lambda_3 + \sum_{t=1}^{N} \lambda_1 \rho t + \sum_{t=1}^{N} \lambda_1 \rho t} \times \exp(-\lambda_3 \mu - \sum_{t=1}^{N} \lambda_1 \rho t + \sum_{t=1}^{N} \lambda_1 \rho t)] (17)$$

Note that in the derivation of (17), we used $\rho t = \gamma_{th} / (\mu \gamma_{th} P)$. Next, we calculate the term $P_{out}^2$ in (11). We first rewrite it as follows

$$P_{out}^2 = \sum_{\bar{q_1}, \bar{q_2} \in \bar{q}} \Pr(\Psi_{1\bar{q}_1} \geq \gamma_{th}, \ldots, \Psi_{1N} \geq \gamma_{th}, \Psi_{1N+1} < \gamma_{th}, \ldots, \Psi_{1M} < \gamma_{th}) \sum_{t=1}^{N} \Psi_{2\bar{q}_2}(\gamma_{th}) (18)$$

With the same manner as in the derivation of (17), and using (7), we can obtain the closed-form expression of $P_{out}^2$ as in (19) at the top of next page.


\[ P_1^{\text{out}} = \sum_{Q_1, Q_2 \leq K \leq N} \Pr(\gamma_3 < \mu, \gamma_{1j_1} \geq \rho P, \ldots, \gamma_{1j_N} \geq \rho P, \gamma_{1j_{N+1}} < \rho P, \ldots, \gamma_{1j_M} < \rho P) \]

\[ + \Pr(\gamma_3 \geq \mu, \gamma_{1j_1} \geq \rho P \gamma_3, \ldots, \gamma_{1j_N} \geq \rho P \gamma_3, \gamma_{1j_{N+1}} < \rho P \gamma_3, \ldots, \gamma_{1j_M} < \rho P \gamma_3) \]

\[ \Delta = \sum_{Q_1, Q_2 \leq K \leq N} (V_1 + V_2), \]

(13)

\[
P_2^{\text{out}} = \sum_{Q_1, Q_2 \leq K \leq N} (1 - \exp(-\lambda_3 \mu)) \prod_{t=1}^{M \text{th}} \exp(-\lambda_{1j_t} \gamma_{1j_t}/\gamma_P) \prod_{t=N+1}^{M} \left(1 - \exp(-\lambda_{1j_t} \gamma_{1j_t}/\gamma_P)\right) + \\
\frac{\lambda_3 \mu \gamma_P}{\lambda_3 \mu \gamma_P + \sum_{t=1}^{N} \lambda_{1j_t} \gamma_{1j_t}} \exp\left(-\lambda_3 \mu - \sum_{t=1}^{N} \lambda_{1j_t} \gamma_{1j_t}/\gamma_P\right) + \\
\sum_{v=1}^{M-N} \sum_{j_1 < j_2 < \ldots < j_v} \sum_{c=1}^{M} \sum_{W_1, W_2} \exp\left(-\lambda_3 \mu \gamma_P \right) \frac{\lambda_{2j_1, 2j_2, \ldots, 2j_v} \lambda_{4j_1} \gamma_{4j_1} \gamma_P}{\lambda_{2j_1} \lambda_{2j_2} \gamma_P + \lambda_{4j_1} \gamma_P} \exp\left(-\lambda_{2j_1, 2j_2, \ldots, 2j_v} \gamma_P - \lambda_{4j_1} \mu \right) - \\
\frac{\lambda_{2j_1} \lambda_{2j_2} y \exp\left(-\lambda_{2j_1, 2j_2} \gamma_P - \lambda_{4j_1} \mu \right)}{\lambda_{2j_1} \gamma_P + \lambda_{4j_1} \gamma_P} \prod_{v=1}^{K-1} \left[ \exp\left(-\lambda_{2j_{v+1}} \gamma_P\right) - \lambda_{2j_v} y \exp\left(-\lambda_{2j_v} \gamma_P - \lambda_{4j_v} \mu \right) \right] \\
\prod_{v=K+1}^{N} \left[1 - \exp\left(-\lambda_{2j_{v+1}} \gamma_P\right) + \lambda_{2j_v} y \exp\left(-\lambda_{2j_v} \gamma_P - \lambda_{4j_v} \mu \right) \right] dy. \]

(19)

C. Exact Outage Probability of Homogeneous Networks

In these networks, we assume that \( \lambda_{1i} = \lambda_1, \lambda_{2i} = \lambda_2, \) and \( \lambda_{4i} = \lambda_4 \) for all \( i, \) so that (13) can be rewritten as follows

\[
P_1^{\text{out}} = \sum_{N=0}^{K-1} \left( \begin{array}{c} M \\ N \end{array} \right) (V_1^H + V_2^H) \]

(20)

where

\[
V_1^H = (1 - \exp(-\lambda_3 \mu)) \exp(-N \lambda_1 \rho P) \left(1 - \exp(-\lambda_1 \rho P)\right)^{M-N} \\
F_{\Psi_{2j_v}}(x) = \sum_{t=1}^{K} \left( \begin{array}{c} N \\ t \end{array} \right) \left( F_{\Psi_{2\lambda_1}}(x) \right)^{N-t+1} \times (1 - \Psi_{2j_v} F_{\Psi_{2\lambda_1}}(x))^{t-1}. \]

(23)

By using the results obtained in (4) and (23), we can express \( P_2^{\text{out}} \) by (24) which is given at the top of next page. Finally, by adding either \( P_1^{\text{out}} \) and \( P_2^{\text{out}} \) or \( P_1^{\text{H, out}} \) and \( P_2^{\text{H, out}} \), we obtain a closed-form expression for the outage probability. To see an asymptotic outage diversity gain, we will make an asymptotic outage probability analysis next.
D. Asymptotic Outage Probability

We shall derive the expressions of \( P_{\text{out}} \) in the high \( \rho_p \) region. By choosing only first two terms of Maclaurin expansion series for \( g(x) \) a function of \( x \), we have \( g(x) \approx g(0) + \left( \frac{dg}{dx} \right)_{x=0} x \). Applying this result for \( \exp(-x) \), we obtain \( \exp(-x) \approx 1 - x \) and \( \lim_{x \to \infty} \exp(-x) = 0 \). Thus, at a very high \( \gamma_p \) value, i.e., \( \gamma_p \to +\infty \) (or \( \rho_p \to 0 \)), we can, respectively, obtain asymptotic (25) and (26) from (14) and (15) as follows

\[
V_1 \approx (1 - \exp(-\lambda_3 \mu)) \left( \prod_{i=N+1}^{M} \lambda_{1j_i} \right) \rho_p^{M-N} \tag{25}
\]

and

\[
V_2 \approx \lambda_3 \left( \prod_{i=N+1}^{M} \lambda_{1j_i} \right) \frac{\rho_p}{\mu}^{M-N} \times \int_{0}^{\infty} x^{M-N} \exp(-\lambda_3 x) dx
\]

\[
= \left( \prod_{i=N+1}^{M} \lambda_{1j_i} \right) \Gamma(M-N+1, \lambda_3 \mu) \left( \frac{\rho_p}{\lambda_3 \mu} \right)^{M-N} \tag{26}
\]

where \( \Gamma(a, x) = \int_{x}^{\infty} t^{a-1} \exp(-t) dt \) denotes the incomplete gamma function [11].

Similarly, an asymptotic CDF of \( \Psi_{2i} \) is given by

\[
F_{\Psi_{2i}}(x) \approx \lambda_2 \lambda_{1j_i} \frac{\lambda_2 u \exp(-\lambda_4 u)}{\lambda_{4i} u} \frac{x}{\gamma_p} \tag{27}
\]

which results in the asymptotic PDF of \( \Psi_{2i} \) in the following form:

\[
f_{\Psi_{2i}}(x) \approx \lambda_2 \lambda_{1j_i} \lambda_{2j_i} \frac{\lambda_2 u \exp(-\lambda_4 u)}{\lambda_{4i} u} \frac{1}{\gamma_p} \tag{28}
\]

Now using Eqs. (27) and (28), we can first obtain

\[
\Pr(\Psi_{2i} < \gamma_{th}) \approx \sum_{c=1}^{N} \sum_{W_1, W_2} \lambda_2 \lambda_{1j_i} \lambda_{2j_i} \frac{\lambda_2 u \exp(-\lambda_4 u)}{\lambda_{4i} u} \frac{1}{\gamma_p} \left( \frac{\rho_p}{\lambda_3 \mu} \right)^{N-K+1} \times \int_{0}^{\gamma_{th}} x^{N-K} dx \tag{29}
\]

which becomes

\[
\Pr(\Psi_{2i} < \gamma_{th}) \approx \sum_{c=1}^{N} \left( \lambda_2 \lambda_{1j_i} \lambda_{2j_i} \frac{\lambda_2 u \exp(-\lambda_4 u)}{\lambda_{4i} u} \frac{1}{\gamma_p} \right) \left( \frac{\rho_p}{\lambda_3 \mu} \right)^{N-K+1} \times \int_{0}^{\gamma_{th}} x^{N-K} dx \tag{30}
\]

Theorem 1: Using Eqs. (25) and (26), and (30), asymptotic outage probabilities for non-homogeneous and homogeneous networks are, respectively, given by

\[
\Phi_{\text{out}}^{\gamma_{th} \to \infty} \approx (D_1 + D_2) \left( \frac{\gamma_{th}}{\gamma_p} \right)^{M-K+1} \tag{31}
\]

where we defined

\[
D_1 \triangleq \sum_{c=1}^{N} \lambda_1 \left( 1 - \exp(-\lambda_3 \mu) + \frac{\Gamma(M-K+2, \lambda_3 \mu)}{(\lambda_3 \mu)^{M-K+1}} \right),
\]

\[
D_2 \triangleq \sum_{c=1}^{N} \lambda_1 \left( 1 - \exp(-\lambda_3 \mu) + \frac{\Gamma(M-N+1, \lambda_3 \mu)}{(\lambda_3 \mu)^{M-N}} \right) \times \frac{1}{N-K+1} \times \left( \frac{\rho_p}{\mu} \right)^{N-K+1} \times \frac{\lambda_2 u \exp(-\lambda_4 u)}{\lambda_{4i} u}
\]

\[
D_3 \triangleq \left( \frac{M}{K-1} \right) (1 - \exp(-\lambda_3 \mu)) \frac{\lambda_1}{\lambda_3 \mu} \times \Gamma(M-K+2, \lambda_3 \mu) (\lambda_3 \mu)^{M-K+1}
\]

\[
D_4 \triangleq \left( \frac{M}{N-k} \right) (1 - \exp(-\lambda_3 \mu)) \frac{\lambda_1}{\lambda_3 \mu} \times \Gamma(M-N+1, \lambda_3 \mu) (\lambda_3 \mu)^{M-N}
\]

From (31), we can see that the outage diversity gain is \( G_d = M - K + 1 \).

Proof: A proof of this theorem is provided in Appendix A.

Note that from Theorem 1, \( G_d \) is in the range of [1, M].
E. Ergodic Channel Capacity

The channel capacity of the proposed protocol can be expressed as

\[ C(\Psi_{2j_e}) = \begin{cases} 
0, & \text{if } N < K, \\
\frac{1}{2} \log_2(1 + \Psi_{2j_e}), & \text{if } N \geq K.
\end{cases} \quad (3) \]

From (32), the average channel capacity can be formulated as

\[ C_{\text{avg}} = \frac{1}{2} \sum_{q_1,q_2 \leq K \leq N} \Pr(\Psi_{1j_1} \geq \gamma_{\text{th}}, \ldots, \Psi_{1j_M} \geq \gamma_{\text{th}}) \times \int_0^\infty \log_2(1 + x)f_{\Psi_{2j_e}}(x)dx. \quad (3) \]

Combining results obtained in (10) and (19), the exact expression of (33) is given by (34) (see the top of next page).

IV. NUMERICAL RESULTS

In this section, we present various Monte Carlo simulations to verify the theoretical results derived above. In a two-dimensional network, we assume that the co-ordinates of the source, the destination, the relay and the primary user are (0, 0), (1, 0), (x_i, 0), and (x_p, y_p), respectively. Hence, the distances are calculated as \( d_{1i} = x_i, \ d_{2i} = 1 - x_i, \ d_3 = \sqrt{x_p^2 + y_p^2} \) and \( d_{4i} = \sqrt{(x_i - x_p)^2 + y_p^2} \). In all of the simulations, we assume the path-loss exponential \( \beta \) equals 3, the threshold \( \gamma_t \) equals 1 and the ratio \( \mu \) between \( I_{th} \) and \( P_{th} \) equals 1.

In Fig. 1, we present the outage probability as a function of \( \bar{\gamma} \) in dB. In this simulation, the number of relays \( M \) equals 3, the co-ordinates \( x_{R_1} \) of relays are 0.3, 0.4, and 0.5, and the positions of the primary user are (0.4, 0.4). As expected, the outage probability is best when the best relay is chosen and that is worst once the worst relay is selected (K=3).

In Fig. 2, we present the outage probability as a function of \( \bar{\gamma}_p \) in dB in the i.i.d network with \( x_{R_1} = 0.5, \ x_p = y_p = 0.5 \) and \( K = 2 \). It can be observed that the outage performance is better when increasing the number of relays \( M \). Figure presents the average capacity as a function of \( \bar{\gamma}_p \) in dB. In this figure, the parameters are fixed as follows: \( \mu = 1, \ M = 3 \) and \( x_{R_1} \in \{0.4, 0.6\} \). It can be seen that the average capacity increases when the better relay is chosen or the primary user is further the secondary network. From Figs. 1-3, we can see that the simulation results match very well with the theoretical results. In addition, it is also seen that the diversity order obtained equals to \( M - K + 1 \).

V. CONCLUSIONS

In this paper, we considered the N-th best reactive DF relay selection for CR to enhance the performance of licensed user under a stringent constraint from licensed networks. In particular, the exact expressions for outage probability and ergodic capacity have been derived over i.n.i.d fading channels, which enable us to evaluate the impact of using reactive DF relays in improving the CR networks performance. The asymptotic outage probability has been also obtained to reveal two important high SNR performance metrics, i.e., diversity and coding gains. Finally, the numerical results have been provided to illustrate the correctness of our analysis.

**APPENDIX A: A DETAILED DERIVATION OF (31)**

Using Eqs. (25), (26), and (30), the corresponding asymptotic outage probability of (17) is given by

\[
\hat{P}_1 \approx \sum_{q_1,q_2 \leq K} \prod_{l=1}^{M} \lambda_{1j_l}(\rho)^{M-K+1} \\
\times \left(1 - \exp(-\lambda_{3j}) + \frac{\Gamma(M-K+2,\lambda_{3j})}{\lambda_{3j}^{M-K+1}}\right). \quad (A.1)
\]
\[ C_{\text{avg}} = \frac{1}{2} \sum_{Q_1, Q_2 \geq K} \left[ \left( 1 - \exp(-\lambda_3 \mu) \right) \prod_{t=1}^{N} \exp(-\lambda_{1j_t} \gamma_{th} / \gamma_p) \prod_{t=N+1}^{M} \left( 1 - \exp(-\lambda_{1j_t} \gamma_{th} / \gamma_p) \right) + \frac{\lambda_3 \mu \gamma_p}{\lambda_3 \mu \gamma_p + \sum_{t=1}^{N} \lambda_{1j_t} \gamma_{th}} \exp(-\lambda_3 \mu - \sum_{t=1}^{N} \lambda_{1j_t} \gamma_{th} / \gamma_p) + \sum_{v=1}^{M-N} \sum_{j_1, j_2, \ldots, j_v = N+1}^{M} \sum_{c=1}^{N} \sum_{W_1, W_2}^{N} \sum_{v=1}^{K} \prod_{u=1}^{v} \left( \lambda_{2j_u} + \lambda_{2j_u} \exp(-\lambda_{4j_u} u) / \lambda_{4j_u} u \right) \rho_p^{M-K+1} \right]. \] 

Using again Eqs. (A.1) and (30) for (19), we get (A.2) as

\[ \hat{P}_{\text{out}} \approx \sum_{Q_1, Q_2 \geq K} \prod_{t=1}^{N} \lambda_{1j_t} \left( 1 - \exp(-\lambda_3 \mu) \right) + \Gamma \left( M - N + 1, \lambda_3 \mu \right) \prod_{t=1}^{N} \lambda_{1j_t} \left( 1 - \exp(-\lambda_3 \mu) \right) + \sum_{c=1}^{N} \left( \lambda_{2j_c} + \lambda_{2j_c} \exp(-\lambda_{4j_c} u) / \lambda_{4j_c} u \right) \rho_p^{M-K+1}. \] 

Thus, combining (A.1) and (A.2), we can obtain \( \hat{P}_{\text{out}} \). Similarly, an asymptotic expression for \( \hat{P}_{\text{out}} \) in the homogeneous networks can be readily computed as \( \hat{P}_{\text{out}} \) in (31).

REFERENCES