MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

A Comparison of Extremum Seeking Algorithms Applied to Vapor Compression System Optimization

Guay, M.; Burns, D.J.

TR2014-039 June 2014

Abstract

In recent years, a number of extremum seeking algorithms have been proposed. While each approach aims to estimate the gradient of a performance metric in realtime and steer inputs to values that optimize the metric, the way in which each method accomplishes this goal can have practical implications that depend on the application. In this paper, we compare the performance of traditional perturbation-based extremum seeking to time-varying extremum seeking in the context of optimizing the energy efficiency of a vapor compression system. In order to benchmark these algorithms, we simulate their performance using a moving-boundary model of a vapor compression machine that has been tuned and calibrated to data gathered from a multi-split style room air conditioner operating in cooling mode. We show that while perturbation-based extremum seeking appears simplest to tune, some challenging minima are not obtained. Also, we find that time-varying extremum seeking converges faster and more reliably than the other method tested.

American Control Conference (ACC), 2014

A Comparison of Extremum Seeking Algorithms Applied to Vapor Compression System Optimization

Martin Guay and Daniel J. Burns

Abstract— **In recent years, a number of extremum seeking algorithms have been proposed. While each approach aims to estimate the gradient of a performance metric in realtime and steer inputs to values that optimize the metric, the way in which each method accomplishes this goal can have practical implications that depend on the application.**

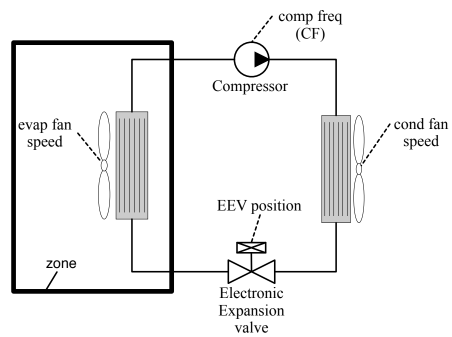
In this paper, we compare the performance of traditional perturbation-based extremum seeking to time-varying extremum seeking in the context of optimizing the energy efficiency of a vapor compression system. In order to benchmark these algorithms, we simulate their performance using a moving-boundary model of a vapor compression machine that has been tuned and calibrated to data gathered from a multi-split style room air conditioner operating in cooling mode. We show that while perturbation-based extremum seeking appears simplest to tune, some challenging minima are not obtained. Also, we find that time-varying extremum seeking converges faster and more reliably than the other method tested.

I. Introduction

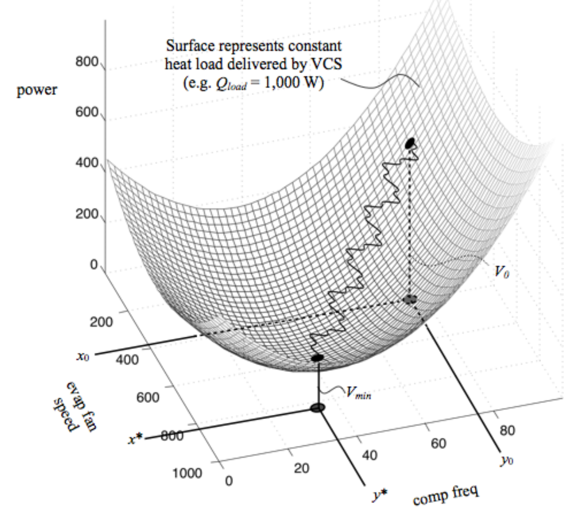
Vapor compression systems (VCS), such as heat pumps, refrigeration and air-conditioning systems, are widely used in industrial and residential applications. The introduction of variable speed compressors, electronically-positioned valves, and variable speed fans to the vapor compression cycle has greatly improved the flexibility of the operation of such systems (Fig. 1A). This increased flexibility allows the heat delivered by the machine to be directly matched to the load, and this design has proved to be more energy efficient than the duty cycling characteristic of vapor compression systems with fixed speed compressors. Further, the combination of commanded inputs to the VCS that delivers a particular amount of heat is often not unique, and various combinations of inputs (*e.g.* compressor speed, fan speeds, etc.) constitute sets of inputs that each cause the VCS to consume different amounts of energy (Fig. 1B). As the boundary conditions vary (*e.g.* heat load, outdoor air temperature), the inputs to the VCS necessarily must change to drive various regulated quantities to their setpoints. By operating the VCS with sets of inputs that have been determined to be energy optimal for a particular boundary condition, the aggregate efficiency as measured by the coefficient of performance (COP) can be dramatically improved.

However, determining these energy-optimal sets of inputs is not straightforward. Models of the vapor compression system that attempt to describe the influence of commanded inputs on thermodynamic behavior and power consumption are often low in fidelity, and while they may have useful predictive capabilities over the conditions in which they were calibrated, the environments into which these systems are deployed are so diverse as to render comprehensive calibration and model tuning intractable. Therefore, relying on model-based strategies for realtime (online) optimization is tenuous.

Recently, model-free methods that operate in realtime and aim to optimize a cost have received increased attention



A.



B.

Fig. 1. The vapor compression system under study consists of a compressor, condensing heat exchanger, electronically controlled expansion valve, and evaporating heating exchanger. The inputs to the VCS that are manipulated by the control system include (i) the compressor frequency, (ii) the condenser fan speed, (iii) the EEV position, and (iv) the evaporator fan speed.

M. Guay is with the Department of Chemical Engineering, Queens' University, Kingston, ON, Canada. martin.guay@chee.queensu.ca

D. J. Burns is with the Mechatronics Group at Mitsubishi Electric Research Laboratories, 201 Broadway, Cambridge, MA 02139. burns@merl.com, and is the author to whom correspondence should be addressed.

and have demonstrated improvements in the optimization of vapor compression systems and other HVAC applications [1], [2], [3]. To date, the dominant extremum seeking algorithm that appears in the HVAC research literature is the traditional perturbation-based algorithm first developed in the 1920s and re-popularized in the late 1990s. However, new extremum seeking approaches have been developed that offer different characteristics.

Extremum seeking control (ESC) has been the subject of considerable research effort over the last decade. This approach, which dates back to the 1920s [4], is an ingenious mechanism by which a system can be driven the optimum of a measured variable of interest [5]. ESC can be viewed as an empirical real-time optimization (RTO) implementation in which no exact model descriptions is required but where the objective function of the RTO problem must be available from process measurements. Extremum-seeking control provides an effective control system design technique that can be used to steer an unknown dynamical system to an equilibrium that optimizes a cost function. Unfortunately, developments in this area have struggled with the poor performance of existing algorithms which has limited to applicability of the technique.

Over the last few years, many researchers have considered various approaches to overcome the limitations of ESC. In [6], the performance limitations associated with ESC were considered in detail. Tighter bounds on the tuning parameters as well as more precise statements on the guarantees of convergence were proposed. The non-local properties on ESC was studied in [7]. This work extends the work in [8] by considering the case where the fast dynamics can be assumed to be uniformly global asymptotically stable along the equilibrium manifold. More precise statements concerning the dependence of the stability properties on the tuning parameters are provided. In [9], [10] and [11], an alternative ESC algorithm is considered where an adaptive control and estimation approach is used. The results in [12] unify the approaches based on singular perturbation and parameter estimation by considering the case where the objective function is parameterized in a known fashion. Recent work reported in [13] have proposed a Newton-based extremum-seeking technique that provides an estimate of the inverse of the Hessian of the cost function. This technique can effectively alleviate the convergence problems associated with the increase of the gain of the Newton update. In [14], a time-varying estimation-based extremum seeking control technique is proposed. This technique attempts to overcome the limitations of the ESC associated with the difficulty in tuning the algorithm and the corresponding transient performance deficiencies. In this approach, the ESC problem is solved using a time-varying gradient estimation procedure that avoids the limitations associated with the averaging nature of ESC.

This paper considers the problem of selecting an extremum seeking algorithm in order to optimize the energy efficiency of a vapor compression system. Toward that end, a physics-based model of a VCS previously developed is used as a platform to evaluate the performance of various ESC algorithms. A description of the plant under study and details about comparison criteria are provided in Section II. A description of the ESC algorithms considered here is given in Section III, and Section IV provides simulated results and a brief discussion. Finally, concluding remarks are offered in Section V.

II. Realtime Optimization of Vapor Compression System Performance

A. Evaluation Criteria

Minimizing realtime power consumption of vapor compression systems represent a strenuous and important test for evaluating the performance of extremum seeking algorithms. Considering the ubiquity of these systems deployed in space cooling/heating and refrigeration applications, relatively small improvements in cycle efficiency can have measurable impacts in national energy consumption. Additionally, convexity in the equilibrium map between the control inputs and energy consumption—a required system property for extremum seeking—has recently been demonstrated [1].

The long time constants associated with the machine thermodynamics and the interacting room air temperature dynamics indicate that the equilibrium map can only be considered as static when viewed on time scales of tens of minutes. In order to satisfy the time scale separation requirement of perturbation-based extremum seeking, previously reported results [1] have shown that convergence time on the order of hours is routine for realistic initial conditions. Therefore, we emphasize convergence rate performance in the subsequent simulations.

Also, whereas traditional studies of the vapor compression cycle usually consider single-evaporator, single-condenser systems, more complex and interconnected systems are becoming prevalent. Systems with multiple evaporators or multiple condensers are already commercially available, some of which offer simultaneous heating and cooling functions wherein an indoor heat exchanger may be operated either as a condenser or an evaporator depending on the occupant's preferences. Multiple heat exchangers introduce more inputs to be used in the optimization, and the reconfigurability of individual heat exchangers introduce permutations that could not be easily captured with simple model-based techniques. While we do not consider such systems in this paper, extensions to multiple heat exchanger systems are planned, and we therefore test the performance of multi-input extremum seeking controllers with a two-input case where both the indoor and outdoor (evaporator and condenser) fans are simultaneously varied.

Finally, the sensitivity of optimal energy consumption to small changes in input values may be high for some operating points, and we therefore choose operating points of the employed model that are challenging to reach.

B. Simulation Environment

In order to evaluate the performance of various extremum seeking algorithms, a model of vapor compression system

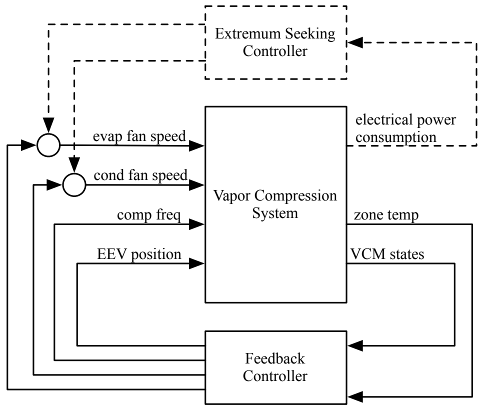


Fig. 2. Extremum seeking algorithms evaluated are configured to optimize electrical power consumption by modulating both evaporator and condenser fan speeds while the feedback controller regulates zone temperature and internal VCS states.

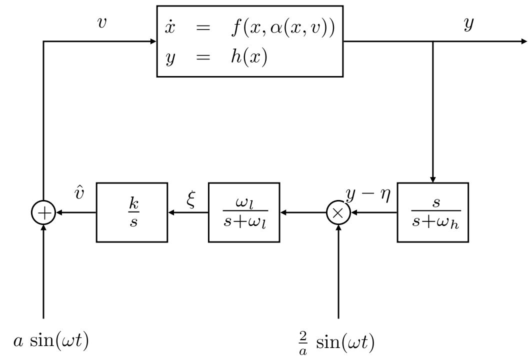


Fig. 3. Standard perturbation-based extremum seeking control.

has been developed based on the Thermosys toolbox for MATLAB/Simulink [15]. This model captures pertinent dynamics through a moving-boundary approximation to the heat exchanger dynamics. The parameters used in this model have been calibrated to data obtained from a 2.6 *kW* single-zone room air conditioner operating in cooling mode.

Typically, vapor compression machines with variable actuators are controlled with feedback regulators that command compressor speeds and electronic expansion valve (EEV) positions. Additionally, indoor and outdoor heat exchanger fans may be controlled to a small number of discrete speeds. In this context, the extremum seeking controllers considered here will continuously modulate both heat exchanger fans speeds in order to minimize electrical power consumption (see Fig. 2). The room temperature regulation and internal vapor compression machine state regulation are performed by the feedback controller, with the extremum seeking controller serving to relocate the steady state operation on the equilibrium manifold to the point that minimizes power.

Disturbances to the VCS include heat load (thermal power removed from the indoor space by the evaporator and rejected to the outdoor space by the condenser) and the outdoor air temperature. For each combination of these two disturbances, an optimal set of four inputs minimized VCS power consumption.

III. Extremum Seeking Control Algorithms

In this study, two classes of ESC techniques are considered to solve the air conditioning problem. We first consider the standard perturbation-based extremum seeking control. We will then consider the estimation based techniques. This class of technique attempt to estimate the gradient directly without any averaging. A third class worth mentioning but not tested here, due to space restrictions, are optimization-based techniques.

A. ESC problem statement

Before presenting the ESC approaches that are considered in this study, we provide a specific statement of the ESC problem.

Consider a nonlinear system

$$\dot{x} = f(x, \alpha(x, v)) \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, v is the vector of input variables taking values in $\mathscr{U} \subset \mathbb{R}^p$ and $y \in \mathbb{R}$ is the variable to be minimized. The function $\alpha(x, v)$ is smooth. It denotes the potential existence of a state-feedback that may be required to stabilize the process at its equilibrium. The state-feedback is parametrized by the input variable v. It is assumed that $f(x, \alpha(x, v))$ is a smooth vector valued functions of x and v and that $h(x)$ is a smooth function of x.

The objective is to steer the system to the equilibrium x^* and v^* that achieves the minimum value of $y(= h(x^*))$. The equilibrium (or steady-state) map is the n dimensional vector $\pi(v)$ which is such that:

$$f(\pi(v), \alpha(\pi(v), v)) = 0.$$

The equilibrium cost function is given by:

$$y = h(\pi(v)) = \ell(v) \tag{3}$$

Thus, at equilibrium, the problem is reduced to finding the minimizer v^* of $y^* = \ell(v^*)$.

Some basic assumption are required to ensure that this problem is well-posed.

Assumption 1: The equilibrium cost (3) is such that

1) $\frac{\partial \ell(v^*)}{\partial v} = 0$
2) $\frac{\partial^2 \ell}{\partial v \partial v^T} > \alpha I, \ \forall v \in \mathscr{U}$.

where α is a strictly positive constant.

B. Perturbation-based ESC

The standard perturbation-based ESC is shown in Figure 3. The fundamental element of the ESC mechanism is the dither signal $d(t) = a \sin(\omega t)$ with amplitude a and frequency ω. The dither signal multiplies the output function y filtered by a high-pass filter with bandwidth ω_h. It then enters a low-pass filter with bandwidth ω_l. The filtered signal ξ is integrated and multiplied by the optimization k. The dither signal is then added to the output and the resulting input v is finally fed back to unknown nonlinear system. The ESC loop can be written as the following dynamical system.

$$\begin{aligned}\dot{x} &= f(x,\alpha(x,\hat{v}(t)+a\sin(\omega t)))\\ \dot{\hat{v}} &= \omega k\xi\\ \dot{\xi} &= -\omega\omega_l\xi+\omega\frac{\omega_l}{a}(h(x)-\eta)\sin(\omega t)\\ \dot{\eta} &= -\omega\omega_h\eta+\omega\omega_h h(x).\end{aligned}$$

The stability and performance of this closed-loop is now well understood. The tuning of the ESC requires that the frequency ω be chosen to be small enough to ensure that a time-scale separation exists between the system's dynamics and the optimization (or quasi steady-state) dynamics.

C. Estimation-based ESC

Estimation-based ESC are based on the direct estimation of the gradient of the steady-state cost function. To make the ideas clear, let us consider the static map:

$$y(t)=\ell(v(t)) \tag{4}$$

If one differentiates (4) with respect to time, the following dynamics are obtained:

$$\dot{y}=\frac{\partial\ell}{\partial v}^T\dot{v}.$$

Defining $\theta=\frac{\partial\ell}{\partial v}$, one can therefore write the following dynamical system:

$$\dot{y}=\theta^T\dot{v}. \tag{5}$$

The design of the extremum seeking routine is based on the dynamics (5). The first step consists in the estimation of the time-varying parameters θ. In the second step, we define a suitable controller that achieves the extremum-seeking task. One possible formulation can be summarized as follows.

We consider the unknown nonlinear system expressed in the optimization quasi steady-state time-scale:

$$\begin{aligned}\varepsilon\dot{x} &= f(x,\alpha(x,v))\\ y &= h(x)\end{aligned}$$

where ε is small positive time-scale separation parameter. Let $\hat{y}$ represent the predicted output for a given value of the parameter estimates $\hat{\theta}$. The output prediction error is denoted by $e=y-\hat{y}$ and the parameter estimation error is given by $\tilde{\theta}=\theta-\hat{\theta}$. We consider the following prediction dynamics:

$$\dot{\hat{y}}=\dot{v}^T\hat{\theta}+Ke+c^T\dot{\hat{\theta}}, \tag{6}$$

where K is a positive constant to be assigned and where the time varying parameter $c\in\mathbb{R}^p$ is the solution of the differential equation:

$$\dot{c}^T=-Kc^T+\dot{v}^T \tag{7}$$

with initial conditions $c(0)=0$. Let $\Sigma\in\mathbb{R}^{p\times p}$ be the solution to the following matrix differential equation

$$\dot{\Sigma}=cc^T-k_T\Sigma+\delta I \tag{8}$$

with initial conditions $\Sigma(0)=\alpha_1 I\succ 0$, where α_1, δ and k_T are strictly positive constants to be assigned. The inverse of Σ is then given as the solution to the matrix differential equation:

$$\dot{\Sigma}^{-1}=-\Sigma^{-1}cc^T\Sigma^{-1}+k_T\Sigma^{-1}-\delta\Sigma^{-2} \tag{9}$$

with initial condition $\Sigma^{-1}(0)=\frac{1}{\alpha}I$. Let the variable $\hat{\eta}$ be defined by the dynamical system:

$$\dot{\hat{\eta}}=-K\hat{\eta}. \tag{10}$$

Then, based on (6),(7) and (10), one considers the following parameter update law proposed in [16]:

$$\dot{\hat{\theta}}=\mathrm{Proj}(\Sigma^{-1}(c(e-\hat{\eta})-\sigma\hat{\theta}),\hat{\theta}),\qquad \hat{\theta}(0)=\theta^0\in\Theta^0, \tag{11}$$

where σ is a positive constant. $\mathrm{Proj}\{\phi,\hat{\theta}\}$ denotes a Lipschitz projection operator [17] such that

$$-\mathrm{Proj}\{\phi,\hat{\theta}\}^T\tilde{\theta}\le-\phi^T\tilde{\theta}, \tag{12}$$

$$\hat{\theta}(0)\in\Theta^0 \implies \hat{\theta}\in\Theta,\forall t\ge 0 \tag{13}$$

where $\Theta\triangleq B(\hat{\theta},z_\theta)$, where $B(\hat{\theta},z_\theta)$ is the ball centered at $\hat{\theta}$ with radius z_θ.

Following standard arguments, it will be shown in the next section that the filter parameter must be satisfy the following assumption.

Assumption 2: There exists constants $\alpha_2>0$ and $T>0$ such that

$$\int_t^{t+T}c(\tau)c(\tau)^T d\tau\ge\alpha_2 I \tag{14}$$

$\forall t>0$.

The extremum-seeking controller considered is given by:

$$\dot{v} = -k\hat{\theta}+d(t) \tag{15}$$

where $d(t)$ is a bounded dither signal with $\|d(t)\|\le D$ and $k>0$. The main advantage of this technique over perturbation based technique is that they are not dependent on the choice of dither signal. In addition, they provide some additional freedom to tune and further improve performance of the ESC system. As in perturbation-based ESC, the application to unknown dynamical systems requires the use of a time-scale separation parameter. However, the choice of this parameter does not affect the choice of dither, as long as the assumption given by Equation (14) is met.

IV. Simulation Results

A. One-input case

1) Perturbation-based ESC: In this first section, we consider the performance of the ESC techniques described in the previous section. The function is convex and contains a unique minimum in the operating region of interest. The minimum occurs at a fan motor voltage of 3.404 V which results in overall power consumption of the VCS at 508 W. For the purpose of simulation, it is assumed that the system is

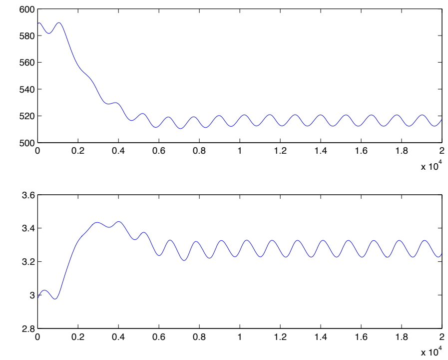


Fig. 4. Optimization of electrical power using the perturbation-based ESC by manipulating the indoor fan speed. The top plot shows the power as a function of time. The bottom plot shows the indoor fan speed.

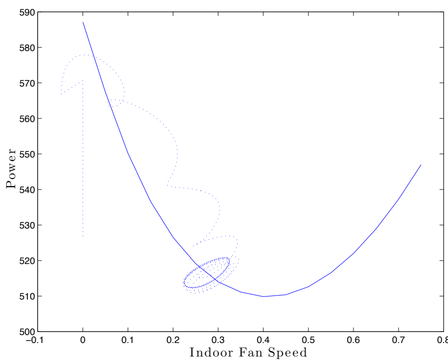


Fig. 5. Optimization of electrical power using the perturbation-based ESC by manipulating the indoor fan speed. The full line shows the steady-state input-output relationship for the system. The ESC input-output trajectory is shown as the dotted line.

initiated at the steady-state occurring at an indoor fan motor voltage of 3 V[1].

We first consider the performance of the perturbation based ESC. The best performance was obtained for the following value of the parameters:

$$a = 0.05,\ \omega = 0.005,\ k = 0.0001,\ \omega_h = 0.005,\ \omega_l = 0.001.$$

The corresponding results are shown in Figures 4 and 5.

The perturbation-based ESC is shown to converge to what it perceives as the optimum quickly. However, the resulting conditions are not optimal. Efforts to increase or reduce the optimization gain or the amplitude do not yield significant improvements in the performance. Reduction of the dither frequency is shown to improve the accuracy of the optimization at the cost of slowing the convergence rate. Consequently, the convergence is longer and the transient response is slower.

2) Estimation-based ESC: The time-varying estimation-based ESC technique is considered for the solution of the steady-state optimization problem. In this application, the pivotal parameter is the time-scale separation parameter, ε. It is set at 0.005. The remaining parameters are set as:

$$k_g = 0.01,\ K = k_T = 20,\ \delta = 10^{-6}.$$

The dither signal is set to $d(t) = 0.05\sin(0.005t)$. The results are shown in Figures 6 and 7. The main advantage of this technique is that the choice of tuning parameters provides a much wider range of suitable parameters. The transient performance of this technique is particularly good. The final value reached is slightly biased as in the perturbation-base ESC technique. However, one can rely on additional tools to improve the performance since the choice of dither is not tied to the time-scale separation.

[1]In the simulation model used, the indoor unit fan is driven by a variable speed DC motor. The input is a signal that represents the motor voltage, and therefore the units that ultimately drive the indoor fan speed is specified in volts.

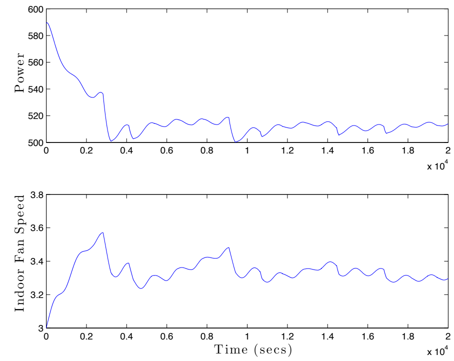


Fig. 6. Optimization of electrical power using the time-varying ESC by manipulating the indoor fan speed. The top plot shows the power as a function of time. The bottom plot shows the indoor fan speed.

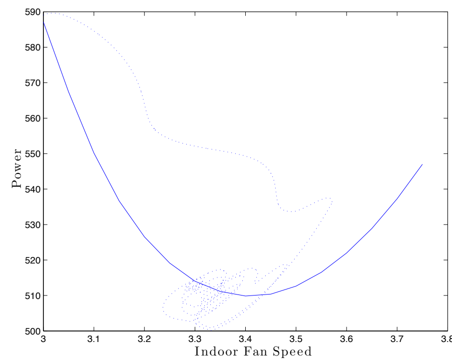


Fig. 7. Optimization of electrical power using the time-varying ESC by manipulating the indoor fan speed. The full line shows the steady-state input-output relationship for the system. The ESC input-output trajectory is shown as the dotted line.

B. Two-input case

In this section, we consider the problem where both indoor and outdoor fan speed to optimize power consumption. We first consider the perturbation-based approach with the same tunings as above for the loop involving the indoor fan ESC loop. We add the outdoor fan speed loop with the following parameters:

$$a = 20,\ \omega = 0.008,\ k = 0.1,\ \omega_h = 0.008,\ \omega_l = 0.003.$$

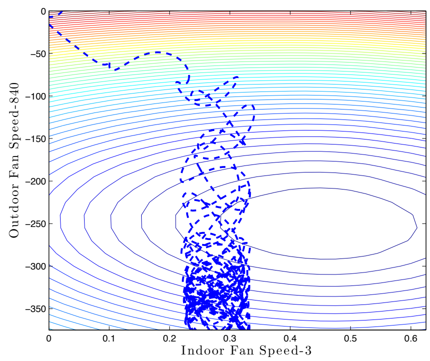


Fig. 8. Optimization of electrical power using the perturbation-based ESC by manipulating the indoor and outdoor fan speeds. The contour lines of the steady-state input-output relationship for the system are shown as full lines. The ESC input trajectory is shown as the dotted line.

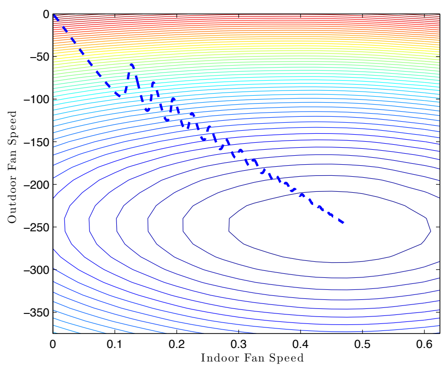


Fig. 9. Optimization of electrical power using the time-varying ESC by manipulating the indoor and outdoor fan speeds with alternative tuning. The contour lines of the steady-state input-output relationship for the system are shown as full lines. The ESC input trajectory is shown as the dotted line.

We show the progress of the ESC technique over a contour plot of the steady-state cost in Figure 8. The perturbation-based succeeds in minimizing the cost, however it fails to identify the correct minimizer. Attempts to minimize the bias is extremely difficult in this case.

We then test the time-varying ESC technique with a singular perturbation parameter $\varepsilon = 0.001$. We remove the dither for the indoor fan speed and introduce a dither in the outdoor fan given by $d(t) = 20\sin(0.008t)exp(-0.0005t)$ as in the perturbation-based case but with a exponential decay. The optimization gain is given by $k_{g_1} = 0.01$ for the indoor fan loop and $k_{g_2} = 0.1$ for the outdoor fan loop. The estimation gains are set to $K = k_T = 50$ with $\delta = 10^{-6}$. The results are shown in Figure 9. The simulation results demonstrate the effectiveness of the time-varying technique in the multi-input case. The perceived bias in the location of the optimal is minimized by a proper choice of optimization gain, k_g, and perturbation parameter ε.

V. Conclusion

The paper presents a study of the application of extremum-seeking control to the optimization of the energy efficiency of a vapor compression system. Two classes of ESC techniques were investigated: Perturbation-based ESC and time-varying ESC. The advantages and disadvantages of both techniques were assessed to study their applicability to minimizing the steady-state power consumption of a vapor compression system. While the perturbation-based method was a bit easier to tune, in some cases, the optimum was not obtained. The time-varying estimation-based ESC was shown to outperform perturbation-based techniques in terms of transient performance and accuracy.

References

[1] D. Burns and C. Laughman, "Extremum seeking control for energy optimization of vapor compression systems," in *International Refrigeration and Air Conditioning Conference*, 2012.

[2] H. Sane, C. Haugstetter, and S. Bortoff, "Building hvac control systems - role of controls and optimization," in *Proceedings of the 2006 American Controls Conference*, 2006.

[3] P. Li, Y. Li, and J. E. Seem, "Efficient Operation of Air-Side Economizer Using Extremum Seeking Control," *JOURNAL OF DYNAMIC SYSTEMS MEASUREMENT AND CONTROL-TRANSACTIONS OF THE ASME*, vol. 132, no. 3, MAY 2010.

[4] M. Leblanc, "Sur l'*é*lectrification des chemins de fer au moyen de courants alternatifs de fr*é*quence *é*lev*é*e," Revue G*é*n*é*rale de l'Electricit*é*, 1922.

[5] Y. Tan, W. Moase, C. Manzie, D. Nesic and, and I. Mareels, "Extremum seeking from 1922 to 2010," in *29th Chinese Control Conference (CCC)*, july 2010, pp. 14 –26.

[6] M. Krstic, "Performance improvement and limitation in extremum seeking control," *Systems and Control Letters*, vol. 39, no. 5, pp. 313–326, 2000.

[7] Y. Tan, D. Nesic, and I. Mareels, "On non-local stability properties of extremum seeking control," *Automatica*, vol. 42, no. 6, pp. 889 – 903, 2006.

[8] M. Krstic and H. Wang, "Stability of extremum seeking feedback for general dynamic systems," *Automatica*, vol. 36, no. 4, pp. 595–601, 2000.

[9] V. Adetola and M. Guay, "Parameter convergence in adaptive extremum-seeking control," *Automatica*, vol. 43, no. 1, pp. 105 – 110, 2007.

[10] M. Guay, D. Dochain, and M. Perrier, "Adaptive extremum seeking control of continuous stirred tank bioreactors with unknown growth kinetics," *Automatica*, vol. 40, no. 5, pp. 881 – 888, 2004.

[11] P. Cougnon, D. Dochain, M. Guay, and M. Perrier, "On-line optimization of fedbatch bioreactors by adaptive extremum seeking control," *Journal of Process Control*, vol. 21, no. 10, pp. 1526 – 1532, 2011.

[12] D. Nesic, A. Mohammadi, and C. Manzie, "A systematic approach to extremum seeking based on parameter estimation," in *Decision and Control (CDC), 2010 49th IEEE Conference on*, dec. 2010, pp. 3902 –3907.

[13] A. Ghaffari, M. Krstic, and D. Nesic, "Multivariable newton-based extremum seeking." *Automatica*, vol. 48, no. 8, pp. 1759–1767, 2012.

[14] M. Guay, S. Dhaliwal, and D. Dochain, "A time-varying extremum-seeking control approach," in *American Control Conference (ACC), 2013*, June 2013, pp. 2643–2648.

[15] Alleyne, A., *et. al.*, "THERMOSYS 4 Toolbox," University of Illinois at Urbana-Champaign. `http://arg.mechse.illinois.edu/thermosys`, 2012.

[16] V. Adetola and M. Guay, "Robust adaptive mpc for systems with exogeneous disturbances," in *American Control Conference, St Louis, MO*, 2009.

[17] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adaptive Control Design*, S. Haykin, Ed. Wiley: Toronto, 1995.