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Method for Locating of Single-phase-to-ground Faults in Ungrounded Distribution Systems

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***Abstract*— This paper proposes a new method for determining locations of single-phase-to-ground faults in ungrounded distribution system using pre-fault and during-fault fundamental-frequency measurements. The pre-fault measurements along with the load profiles are used to determine load demands for each individual load in a feeder section. The during-fault measurements are used to determine possible faulted area and location. The faulted feeder and faulted feeder section are first determined based on the reactive power factors of residual powers determined for the feeder breakers and switches with sensors. The line segments in the faulted feeder section are then tested for a sign change of phase-to-ground voltage on the fault phase to determine the faulted line segment. At last, the location of the fault is determined by finding a location along the faulted line segment having a zero phase-to-ground voltage on the faulted phase. Numerical examples are given to demonstrate the effectiveness of proposed method.**

***Index Terms*-- Distribution line, distribution system, single-phase-to-ground fault, fault location, ungrounded.**

I. INTRODUCTION

Ungrounded distribution systems are widely used, especially at medium voltage levels. Compared with the grounded systems, the ungrounded systems do not have neural wires to connect with the ground, and they are connected to ground through phase-to-ground capacitances of power lines. When a single-phase-to-ground fault occurs, the fault currents of ungrounded systems are less than normal load currents, thus the system can continue to operate until the fault is corrected. However, as a result of the fault, the lines of the distribution systems experience over-voltages, which can damage the lines when the fault is not corrected in a timely manner. Thus, fast and accurate fault location analysis is important for the safe and stable operation of ungrounded distribution systems.

Several methods have been proposed for locating single-phase-to-ground faults in ungrounded distribution systems, including signal injection methods [1], and transient measurement based methods [2]. Those methods have their own limitations when applied to real time applications of distribution systems. Reference [1] uses a zero-sequence signal generator that operates at a distinct frequency other than the fundamental frequency to aid in locating the ground fault, and the method is not suitable for on-line applications. Reference [2] estimates the fault distance by analyzing the initial transients of the ground fault, and a much higher sampling rate is required for an accurate estimation.

This paper proposes a new method for determining locations of single-phase-to-ground faults in ungrounded distribution systems based on pre-fault and during-fault fundamental-frequency measurements. The pre-fault and during -fault measurements collected from feeder breakers at the roots of feeders and switches with sensors along the feeders are used for estimating load demands of a feeder section, and the location of fault respectively.

The power consumption of each load in a feeder section is determined as a product of a scaling factor determined for all loads in the section, and a base power defined by a load profile of the load. The scaling factors for the feeder section is determined through a load flow based iterative procedure. The buses of importing and exporting measuring devices of the section are modeled as constant voltage bus, and constant power bus respectively. The scaling factors are updating iteratively and converged solutions are obtained when the solved active powers determined by a load flow solution are matched with the target active powers determined by the measurements at the importing measuring device. During the iterations, the mismatches of active powers for each phase are converted into incremental phase-to-phase powers through equivalent resistance WYE-DELTA conversion, and then those phase-to-phase powers are used to determine the incremental changes of scaling factors for DELTA-connected loads.

The proposed method has decomposed the fault locating task into a set of sub-tasks to be achieved, including determining the feeder having the fault, determining the faulted section of the faulted feeder, determining the faulted line segment of the faulted section, and determining the exact location of the fault in the faulted line segment. Based on the during -fault measurements, reactive power factors of residual powers at the roots of feeders and boundaries of section of feeders are calculated, and then used to determine the faulted feeder, and faulted feeder section. The faulted line segment is identified by checking a sign change of voltage on the faulted phase for each line segment in the faulted feeder section using during-fault measurements at the boundaries of faulted feeder section, and load demands determined based on load profiles for each individual load and the pre-fault measurements for the faulted feeder section. Knowing the faulted line segment, the location of the fault is determined by finding a location along the faulted line segment having a zero voltage on the faulted phase. The test results for a sample system are given to demonstrate the effectiveness of the proposed method.

II. PROPOSED METHOD

A. Ungrounded Distribution Systems and Measurements

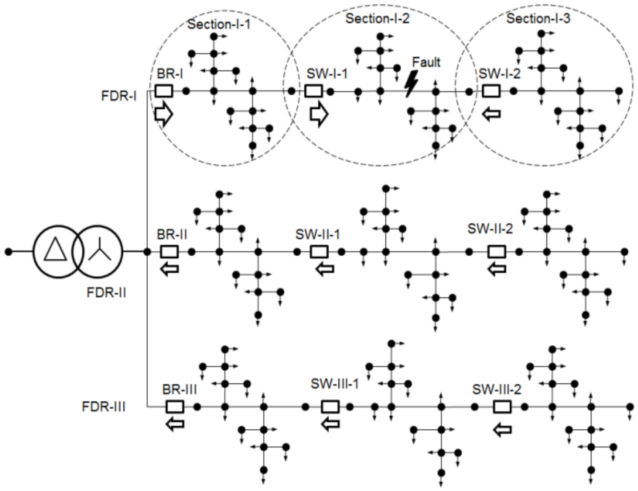

Fig.1. Model of a line segment with a fault

Fig. 1 shows an example of ungrounded distribution system with a single-phase-to-ground fault. It includes a distribution substation in which a three-phase transformer with ungrounded winding connections is connected with downstream feeders, and each downstream feeder has one feeder breaker and several switches with sensors. Based on the location of switches along the feeder, a feeder can be partitioned into several sections, and each feeder section has one breaker or switch with sensor at its root as an importing measuring device, and may have one or multiple switches with sensors at its downstream boundaries as exporting measuring devices. The feeder section is defined as all buses and devices between the importing and exporting devices. In Fig. 1, the system has three feeders, and each feeder has one feeder breaker, two switches with sensors, and three feeder sections.

The buses of a feeder section can be grouped into several layers according to the number of devices connected between each bus and the importing measuring device. The higher a layer number is, the farther the buses of the layer are from the importing device.

The measurements collected from feeder breakers and switches with sensors include the currents flowing through the device downstream on phase a, b and c, $I_{ps,a}$, $I_{ps,b}$ and $I_{ps,c}$, and the phase-to-ground voltages on phase a, b and c, $V_{p,a}$, $V_{p,b}$ and $V_{p,c}$, where bus p and s are the terminal buses of the device, and bus p is upstream to bus s. The residual voltages and residual currents for the device can be determined by summing up the measured phase-to-ground voltages and phase currents of all phases as:

$$v_p^{res} = V_{p,a} + V_{p,b} + V_{p,c} \quad (1)$$

$$i_{ps}^{res} = I_{ps,a} + I_{ps,b} + I_{ps,c} \quad (2)$$

where, v_p^{res} is the residual voltage at bus p, i_{ps}^{res} is the residual current flowing from bus p to bus s and measured at bus p.

The apparent residual power for a device between bus p and s can be determined based on the residual voltage and residual current on the device according to:

$$s_{ps}^{res} = v_p^{res} i_{ps}^{res*} \quad (3)$$

where, s_{ps}^{res} is the apparent residual power at the device. The apparent residual power includes a real component, active residual power, and an imaginary component, reactive residual power. Accordingly, a reactive power factor of residual power, qf_{ps}^{res} can be determined as the ratio of reactive residual power over the magnitude of apparent residual power, and also determined as the sine of the phase angle difference between the residual voltage and residual current according to:

$$qf_{ps}^{res} = \sin(\angle v_p^{res} - \angle i_{ps}^{res}) \quad (4)$$

B. *Determination of faulted phase,faulted feeder, and faulted feeder section*

The phase-to-ground voltages measured at the secondary side of the substation transformer, i.e. the upstream bus of feeder breakers are used to determine the faulted phase for a single-phase-ground fault.

When a bolted single-phase-to-ground fault occurs at a feeder, the voltage on the faulted phase of the faulted feeder, and adjacent feeders that connected to the same substation transformer as the faulted feeder instantaneously drops to a small value close to zero, and meanwhile the voltages on the un-faulted phases instantaneously rise to values close to 1.73 times of normal operation voltages. A phase x is determined as faulted phase, if the following conditions are met:

$$|V_{sub,x}| \leq \underline{V} \quad x \in \{a, b, c\} \quad (5)$$

$$|V_{sub,y}| \geq \overline{V} \quad y \in \{a, b, c\}, y \neq x \quad (6)$$

where, $V_{sub,x}$ and $V_{sub,y}$ are the voltages measured at the secondary side of the substation transformer on the phase x and y respectively, and $\underline{V}$ and $\overline{V}$ are the lower and upper thresholds of voltage magnitude used for abnormal voltage determination. For examples, the lower and upper thresholds can be set as 0.30, and 1.40 per unit respectively.

The faulted feeder and faulted feeder section are determined by examining the reactive power factor of residual powers determined by residual voltage and residual current measurements. Ignoring the asymmetry of power lines, the residual currents of an ungrounded distribution system are mainly contributed from the phase-to-ground capacities of un-faulted phases of the faulted and un-faulted feeders, and the residual powers are dominated by reactive powers, that is the magnitude of reactive power factor for the residual power is close to 1.0.

For a substation connected with multiple feeders, a feeder is determined as a faulted one if the reactive power factor of residual power at the feeder breaker is close to +1 as described in Eq. (7):

$$qf_{fdr}^{res} > \underline{qf} \quad (7)$$

where, qf_{fdr}^{res} is the reactive power factor of residual power at the feeder breaker fdr, $\underline{qf}$ is a pre-determined lower threshold of reactive power factor. For example, $\underline{qf}$ is 0.9. If the substation contains only a single feeder, the reactive power factor of residual power at the feeder breaker cannot be determined due to the zero value of residual current at the breaker. Thus, Eq. (5) and (6) are used instead to determine whether the feeder has a single-phase-to-ground fault.

A feeder section is determined to be a faulted one when the reactive power factors of residual powers are close to +1 at its importing measuring device, and close to -1 at one of its exporting measuring device as described as follows:

$$qf_{im}^{res} > \underline{qf} \quad (8)$$

$$qf_{ex}^{res} < -\underline{qf} \quad (9)$$

where, qf_{im}^{res} and qf_{ex}^{res} are the reactive power factors of residual powers at the importing device im and the exporting device ex respectively. If the feeder section does not have any exporting measuring devices, only the condition described in Eq. (8) is used to determine whether the section is a faulted one. For a single-feeder substation, if the feeder contains multiple feeder sections, only Eq. (9) is used to determine whether the section connected with the feeder breaker is a faulted one, due to the zero residual current at its importing measuring device.

C. Estimating of Load Demands of the Faulted Feeder Section

During a fault, the currents flowing in the faulted feeder section are contributed from individual loads of the feeder, the short circuit fault, and the shunt capacitances of distribution lines. The load demands for individual loads are not measured, but determined based on the pre-fault measurements at the measuring devices and load profiles for each individual load.

The loads in the feeder section include fixed loads and scalable loads. The fixed loads refer to the equivalent WYE-connected loads consumed by downstream feeder sections connected to the faulted feeder section through the exporting measuring devices. The scalable loads refer to individual loads in the faulted section, which are DELTA-connected.

For an exporting measuring device, the equivalent power consumptions can be determined using the phase-to-ground voltage and phase current measurements at the measuring device:

$$S_{ex_i,x} = V_{ex_i,x} I_{ex_i,x}^{*} \qquad x \in \{a, b, c\} \quad (10)$$

where, $S_{ex_i,x}$, $V_{ex_i,x}$ and $I_{ex_i,x}$ are the equivalent power consumption, measured phase-to-ground voltage, and measured phase current on phase x of the exporting measuring device ex_i respectively.

The power consumptions of individual loads can be defined as:

$$S_{p,xy} = \alpha_{p,xy} S_{p,xy}^{base} \qquad xy \in \{ab, bc, ca\} \quad (11)$$

where, $S_{p,xy}^{base}$ is the base power consumption of load connected to bus p between phase x and phase y given by load profile for the time interval of the fault, $\alpha_{p,xy}$ is the scaling factor for the load connected to bus p between phase x and phase y.

Each individual load may have its own scaling factors $\alpha_{p,xy}$ at the moment of fault occurring. In this paper, a set of uniform scaling factors are used for all loads in the faulted section, such that individual loads between phase x and phase y use the same scaling factor α_{xy} to determine power consumption of the loads.

The scaling factors for the faulted feeder section are determined using the pre-fault measurements at the importing and exporting measuring devices of the feeder section. A load flow based iterative procedure is used to determine the scaling factors for the section. The scaling factors are initialized with a set of initial values and the power demands are determined for each individual load. Then, the buses of individual loads and exporting measuring devices are treated as constant active power and reactive power buses, i.e., PQ buses. The upstream buses of importing measuring devices are treated as a constant voltage and phase angle bus, i.e., the swing bus. The voltages of the swing bus are set as the measured voltages at the importing measuring device. A load flow procedure, such as a backward/forward sweep method can be used to determine the power flows of the feeder section. Based on the solved power flow results, a calculated active power at the importing measuring device is determined according to:

$$P_{im,x}^{calculated} = |V_{im,x}||I_{im,x}^{calculated}| \cos(\angle V_{im,x} - \angle I_{im,x}^{calculated}) \qquad x \in \{a, b, c\} \quad (12)$$

where, $P_{im,x}^{calculated}$ and $V_{im,x}$ are the calculated active power and measured voltage on phase x at the importing measuring device, and $I_{im,x}^{calculated}$ is the calculated phase current on phase x flowing through the importing measuring device into the feeder section. The calculated active power is then checked against a target active power determined by the voltage and current measurements at the importing measuring device as:

$$P_{im,x} = |V_{im,x}||I_{im,x}| \cos(\angle V_{im,x} - \angle I_{im,x}) \qquad x \in \{a, b, c\} \quad (13)$$

where, $P_{im,x}$ is the target active power on phase x at the importing measuring device, $I_{im,x}$ is the measured phase current of phase x at the importing measuring device. If the calculated active powers and the target power active powers for all phases are close enough, then the current set of scaling factors are the final solution. Otherwise, the scaling factors are adjusted iteratively until the difference is below threshold.

An incremental equivalent resistance based method is used to adjust the scaling factors. It uses the phase active power mismatch at importing device to determine equivalent WYE-connected incremental resistances for all three phases at the importing device, and then converts the WYE-connected incremental resistances into DELTA-connected incremental resistances, and based on those DELTA-connected incremental resistances, determines set of incremental phase-to-phase active powers. The required incremental scaling factors are determined based on the incremental phase-to-phase active powers and base values provided by the load profiles.

The active power mismatches of all phases at the importing measuring device are determines according to

$$\Delta P_{im,x} = P_{im,x} - P_{im,x}^{calculated} \qquad x \in \{a,b,c\} \quad (14)$$

where, $\Delta P_{im,x}$ is the active power mismatch of phase *x* at the importing measuring device. The equivalent WYE-connected resistances can be determined as:

$$\Delta R_{im,x} = \left|V_{im,x}\right|^2 / \Delta P_{im,x} \quad x \in \{a,b,c\} \qquad (15)$$

where, $\Delta R_{im,x}$ is the equivalent resistance for phase *x* at the importing device. Using the WYE-DELTA transformation of resistances, a set of DELTA-connected equivalent resistances can be determined according to:

$$\Delta R_{im,ab} = \frac{\Delta R_{im,a}\Delta R_{im,b}+\Delta R_{im,b}\Delta R_{im,c}+\Delta R_{im,c}\Delta R_{im,a}}{\Delta R_{im,c}} \quad (16)$$

$$\Delta R_{im,bc} = \frac{\Delta R_{im,a}\Delta R_{im,b}+\Delta R_{im,b}\Delta R_{im,c}+\Delta R_{im,c}\Delta R_{im,a}}{\Delta R_{im,a}} \quad (17)$$

$$\Delta R_{im,ca} = \frac{\Delta R_{im,a}\Delta R_{im,b}+\Delta R_{im,b}\Delta R_{im,c}+\Delta R_{im,c}\Delta R_{im,a}}{\Delta R_{im,b}} \quad (18)$$

where, $\Delta R_{im,ab}, \Delta R_{im,bc}$ and $\Delta R_{im,ca}$ are the equivalent DELTA-connected resistances between phase *a* and phase *b*, phase *b* and phase *c*, phase *c* and phase *a* respectively. Based on these DELTA-connected resistances, the corresponding incremental active phase-to-phase powers are determined as:

$$\Delta P_{im,xy} = \left|V_{im,xy}\right|^2 / \Delta R_{im,xy} \quad xy \in \{ab,bc,ca\} \quad (19)$$

where $\Delta P_{im,xy}$ is the required incremental active power between phase *x* and phase *y*, and $V_{im,xy}$ is the measured phase-to-phase voltage between phase *x* and phase *y*. The required incremental scale factors can be determined according to:

$$\Delta\alpha_{xy} = \frac{\Delta P_{im,xy}}{\sum_{p \in LD} P_{p,xy}^{base}} \qquad xy \in \{ab,bc,ca\} \qquad (20)$$

where, $\Delta\alpha_{xy}$ is the required incremental scale factor for loads between phase *x* and phase *y*, $P_{p,xy}^{base}$ is the base active power of load connected to bus p between phase *x* and phase *y* provided by the load profiles, LD is the set of load buses in the feeder section.

D. Estimating of current distribution on the un-faulted devices of the Faulted Feeder Section

In order to differentiate the faulted line segment from the un-faulted ones, the currents on the un-faulted devices in the faulted feeder section needs to be determined first. Those currents can be determined by ignoring the fault in the feeder section, and using measured voltages at the importing measuring device, measured voltages and currents at the exporting measuring devices, and load demands of the faulted feeder section.

In the faulted feeder section, only the voltages at the buses of importing and exporting measuring devices are known. The voltages of all unmeasured buses have to be estimated based on those measured voltages.

For any bus residing in the connectivity path between each pair of the importing measuring device and one of the exporting measuring devices, its phase-to-ground voltages are determined based its distances to two measuring devices and measured voltages at two measuring devices. The estimated phase-to-ground voltage of bus p is determined according to:

$$\tilde{V}_p = \frac{d_{p-ex}}{d_{im-p}+d_{p-ex}} V_{im} + \frac{d_{im-p}}{d_{im-p}+d_{p-ex}} V_{ex} \qquad (21)$$

where, $\tilde{V}_p$ is the vector of estimated phase-to-ground voltages of bus p, V_{im} and V_{ex} are the measured phase-to-ground voltages at the importing measuring device im and the exporting measuring device ex, d_{im-p} and d_{p-ex} are the sum of length of line segments residing at the path between the importing device im and bus p, and bus p and the exporting measuring device ex respectively. If there are multiple exporting measuring devices existing, and common buses between different paths, the voltages of those common buses are set as the average of estimated voltages for all paths.

For any bus not residing on any paths between the measuring devices, but fed from one of buses in the paths, its estimated voltage is set as the estimated voltage of the feeding bus on the paths. If a feeder section does not have any exporting measuring device, the voltages of all buses within the section are set as the measured voltages at the importing measuring device.

A backward sweep procedure is used to determine the current distribution of the faulted feeder section if the fault within the section is ignored. The procedure starts at the devices connected upstream to the last layer, then moves to next layer upstream, and ends at the devices connected downstream to the first layer of the section.

Fig. 2 shows an example of a line segment between an upstream bus *p* and a downstream bus *s*. The line segment is modeled by a series phase impedance matrix Z_{ps}^{se}, and a shunt admittance matrix Y_{ps}^{sh} partitioned into two terminal buses. I_{ps} and $I_{ps'}$ are the currents entering the line segment through the upstream bus *p,* and the currents leaving the segment through the downstream bus *s.*

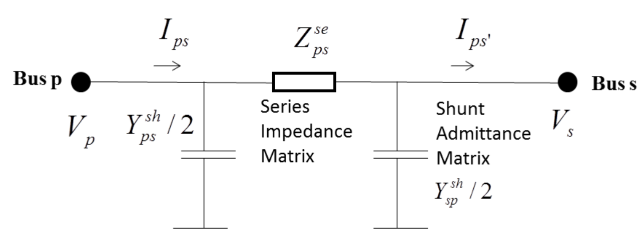

Fig.2. Model of a line segment

For any line segment between bus *p* and *s*, the phase currents leaving the line segment through bus *s*, $\tilde{I}_{ps'}$ can be determined based on the equivalent currents of loads connected to the bus *s*, and the currents entering into all downstream devices through the bus *s*, according to:

$$\tilde{I}_{ps'} = I_s^{load} + \sum_{t \in DD_s} \tilde{I}_{st} \qquad (22)$$

where, I_s^{load} is the vector of equivalent currents of loads connected to bus s, and defined as:

$$I_s^{load} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{s,ab}^{load} \\ I_{s,bc}^{load} \\ I_{s,ca}^{load} \end{bmatrix} \quad (23)$$

$I_{s,xy}^{load}$ is the equivalent load current between phase *x* and *y*, and determined as:

$$I_{s,xy}^{load} = \frac{S_{s,xy}^*}{\tilde{V}_{s,x}^* - \tilde{V}_{s,y}^*} \qquad xy \in \{ab, bc, ca\} \quad (24)$$

$S_{s,xy}$ is the power consumption of load connected between phase *x* and *y* at bus *s*, $\tilde{V}_{s,x}$ and $\tilde{V}_{s,y}$ are the estimated phase-to-ground voltage on phase *x* and phase *y* of bus *s*. DD_s is the set of buses that directly connected with bus *s* through a line segment or switch. $\tilde{I}_{st}$ is the vector of phase currents entering a device between bus *s* and *t* through bus *s*. If *s* is the upstream bus of an exporting measuring device, $\tilde{I}_{st}$ is set as the corresponding measured phase currents directly.

The estimated currents entering the line segment between the upstream bus *p* and the downstream bus *s* through bus *p*, $\tilde{I}_{ps}$ are determined according to:

$$\tilde{I}_{ps} = \tilde{I}_{ps'} + \frac{1}{2} Y_{ps}^{sh} \left(\tilde{V}_p + \tilde{V}_s \right) \quad (25)$$

Eq. (24) and (25) can also be used to calculate the currents flowing on a switch between bus *p* and bus *s* by setting the shunt admittance Y_{ps}^{sh} as zero.

E. Determining the Faulted Line Segment of the Faulted Feeder Section

When a bolted single-phase-to-ground fault occurs at a location within a line segment, the phase-to-ground voltage on the faulted phase is substantially equals zero at the faulted location, so the voltage of faulted phase at the downstream bus of the line segment with fault would have different sign than the voltage at the upstream terminal bus, if the currents entering the line segment through the upstream bus did not enter into the ground through the fault point, but flew into the downstream bus through the line segment.

A forward sweep procedure is used to determine the possible faulted line segments by sequentially assuming a fault is downstream to a line segment and examining the phase-to-ground voltage change at its terminal buses. The procedure starts from the devices connected downstream to the buses at the first layer, and then proceeds to the devices in the next downstream layers, until all possible line segments are evaluated. If a line segment is determined as a faulted segment, then the devices downstream are not checked. If the line segment is the only device in the layer, then the procedure is completed. If there are other line segments in the same layer, the procedure continues to check those segments to see whether a fault is possibly occurring within the segment.

For a device between an upstream bus *p* and a downstream bus *s*, the current entering the device through bus *p*, I_{ps} is:

$$I_{ps} = I_{up'} - I_p^{load} - \sum_{t \in DD_p, t \neq s} \tilde{I}_{pt} \quad (26)$$

where, $I_{up'}$ is the vector of currents leaving a device between bus *u* and bus *p* through bus *p*, and bus *u* is upstream to bus *p*, I_p^{load} is the equivalent currents of loads at bus *p*, DD_p is the set of buses that directly connected to bus *p* downstream, $\tilde{I}_{pt}$ is the vector of estimated currents entering a device between bus *p* and bus *t* through bus *p*, and bus *t* is downstream to bus *p*. If bus *u* is the upstream terminal bus of importing measuring device, $I_{up'}$ is set as the current measured at the importing measuring device directly.

If the device is a line segment between bus *p* and bus *s*, the phase-to-ground voltages of the downstream bus *s* can be determined as:

$$V_s = V_p - Z_{ps}^{se} \left(I_{ps} - \frac{1}{2} Y_{ps}^{sh} V_p \right) \quad (27)$$

If it is a switch, then the phase-to-ground voltages of the downstream bus *s* are set the same as the upstream ones. If bus *p* is the terminal bus of importing measuring device, V_p is set as the measured voltage at the importing measuring device directly.

After the voltages of the downstream bus of a line segment are determined, the voltage at faulted phase is checked to see whether it is close to zero by using Eq. (28), or the sign of the voltage at downstream bus is different from the voltage of the upstream bus by using Eq. (29):

$$\left| V_{s,x} \right| \leq \underline{V_0} \quad (28)$$

$$\cos(\angle V_{p,x} - \angle V_{s,x}) \leq \underline{pf} \quad (29)$$

where, $V_{s,x}$ and $V_{p,x}$ are the voltages at the faulted phase *x* of bus *s* and bus *p* respectively. $\underline{V_0}$ is a threshold for zero voltage checking, such as 0.00001 per unit. $\underline{pf}$ is a threshold for voltage sign change checking, for example -0.5.

If one of above conditions is true, the line segment is a possible faulted one. If not, this line segment is not faulted, and the currents leaving from the segment through the downstream bus are determined for subsequent analyses of the downstream layers. For a line segment between bus *p* and *s*, the currents leaving the segment through bus *s* can be determined as:

$$I_{ps'} = I_{ps} - \frac{1}{2} Y_{ps}^{sh} \left(V_p + V_s \right) \quad (30)$$

If the device is a switch, then the currents leaving are the same as ones entering.

F. Determining the Faulted Location within the Faulted Line Segment

After the possible faulted line segment is determined, the possible faulted location along the line segment can be determined by finding a location having zero phase-to-ground voltage on the faulted phase.

Assumed a single-phase-to-ground fault occurs on a line segment between an upstream bus *p* and a downstream bus *s*, and the fault is located at point *f* as shown in Fig. 3. The line segment can be divided into two sub-segments according to the location of the fault, one is between bus *p* and the fault location *f*, and the other is between the fault location *f* and bus *s*.

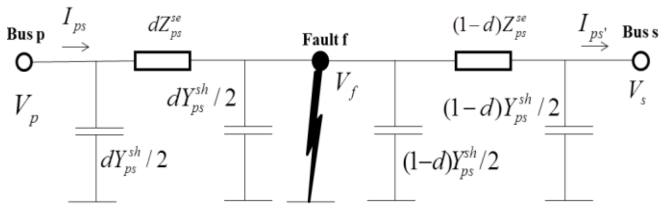

Fig.3. Model of a line segment with a fault

If d is the ratio of distance between the fault location f and the upstream bus p over total length of the line segment between bus p and bus s, the sub-segment between bus p and fault location f can be modeled with a series impedance dZ_{ps}^{se}, and a shunt admittance dY_{ps}^{sh} split into two terminal buses, p and f, and the sub-segment between fault location f and bus s can be modeled with a series impedance $(1-d)Z_{ps}^{se}$, and a shunt admittance $(1-d)Y_{ps}^{sh}$ split into two terminal buses, f and s.

The voltage at the faulted phase x of bus f, $V_{f,x}$ is determined as:

$$V_{f,x} = V_{p,x} - dZ_{ps,x}^{se}(I_{ps} - \frac{1}{2}dY_{ps}^{sh}V_p) \quad (31)$$

where, $Z_{ps,x}^{se}$ is the vector of series impedance matrix elements corresponding to the row of the faulted phase x.

The magnitude of voltage $V_{f,x}$ becomes zero when the point f has a bolted single-phase-to-ground fault. Thus, the location of fault can be determined by solving the root of a complex quadratic equation of distance ratio d:

$$\left(\frac{1}{2}Z_{ps,x}^{se}Y_{ps}^{sh}V_p\right)d^2 - \left(Z_{ps,x}^{se}I_{ps}\right)d + V_{p,x} = 0 \quad (32)$$

The equation (32) can be solved by a root solving method, such as the bisection method, or Newton's method within a feasible range [0,1] of variable d. After the ratio d is obtained, the exact geographic locations can be derived when the geographic coordinates of two terminal buses of the faulted line segment are known.

III. NUMERICAL EXAMPLES

The proposed method has been tested with several sample ungrounded systems, and satisfactory results are obtained.

Fig. 4 gives an example of test distribution systems. The system includes 1 three-phase transformer, and 3 downstream feeders. Each feeder contains 1 feeder breaker, 2 switches with sensors, 18 three-phase line segments, and 22 three-phase buses. Both the breakers and switches are equipped with measuring units for voltages and currents. Each line segment is 1.25 miles long, and the maximum length of a single feeder is 15 miles.

Table I gives the test results on six different fault cases. Each case represents a single-phase-to-ground fault at a specific bus. As shown in Table I, the maximum prediction error of the proposed method is 0.1528 miles. Considering the feeder length is 15 miles long, the maximum prediction error is 1.02 % of feeder length.

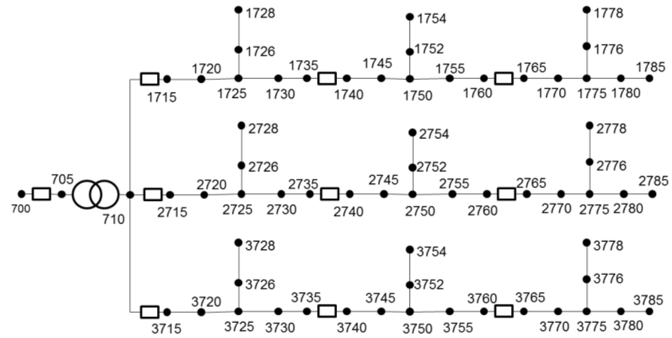

Fig.4 A sample ungrounded distribution system

TABLE I. TEST RESULTS ON A SAMPLE SYSTEM

Fault Location	Prediction Error(miles)	Prediction Error (%)
2720	0.0035	0.023
2726	0.1528	1.019
2745	0.0065	0.043
2750	0.0088	0.058
2775	0.0041	0.027
2780	0.0463	0.309

IV. CONCLUSIONS

This paper has proposed a method for locating of single-phase-to-ground faults in ungrounded distribution system using pre-fault and during-fault fundamental-frequency measurements at feeder breakers and switches with sensors. The fault locating task has been decomposed into a set of sub-tasks to be achieved, including determining the feeder having the fault, determining the faulted section of the faulted feeder, determining the fault line segment of the faulted section, and determining the exact location of the fault in the faulted line segment.

The pre-fault measurements and load profiles are used to determine the load demands for the fault feeder section. The power consumption of each load is determined as a product of uniform scaling factors for all loads of the feeder section and base power defined by load profiles for individual loads. The during-fault measurements are used to determine the faulted feeder and feeder section by examining the reactive power factors of residual powers at roots of feeders and boundaries of feeder sections, and also the fault line segment and fault location by examining the phase-to-ground voltage on the fault phase along line segments.

The preliminary results on the test system have proven the effectiveness of proposed method.

REFERENCES

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