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Two-Way Cognitive Relay Networks with Multiple Licensed Users

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***Abstract*—This paper tackles the important question of how to compensate the inherent spectrum efficiency loss in cognitive relay networks. Particularly, by considering two-way cognitive relaying, we seek to enhance the performance of the secondary network in terms of the reliability due to limited transmit power, and the spectral efficiency of the half-duplex dual-hop relay transmission. We derive new closed-form expressions for the outage probability of a cognitive relay network with two-way communications in the presence of multiple primary users. Our expressions accurately take into account the impact of the maximum allowable interference constraint at the primary users on the secondary network.**

I. Introduction

With the exponentially increasing demand of mobile multimedia applications, wireless operators are faced with the challenge of a shortage of radio frequency spectrum. However, it has been shown in recent measurement campaigns that the frequency spectrum is not being efficiently utilized. Cognitive radio technology, proposed by Mitola over a decade ago [1], is an excellent solution to alleviate this frequency spectrum under-utilization. The most commonly used spectrum-sharing approach in cognitive radio networks is the underlay approach, where the secondary user (SU) is allowed to concurrently occupy the same frequency band as the primary user (PU) provided that the SU does not inflict harmful interference on the PU. With this strategy, the maximum transmit power at the SU is governed to remain below a predefined threshold. As such, the secondary network suffers a loss in performance compared to its non-cognitive counterpart, especially in severe pathloss and shadowing environments.

Against this background, cooperative communications is a promising solution to enhance wireless reliability and coverage [2], [3]. Due to the transmit power constraints at the secondary networks, satisfying the high QoS demand for multimedia services at the SU while limiting the interference at the PU is daunting. By interconnecting cooperative communications and cognitive radio networks, it is envisioned that the important problem of contemporary wireless networks can be solved, i.e., performance enhancement for wireless systems under scarce spectrum usage. As a result, the concept of cognitive cooperative communications has attracted both academic and industry attention as an efficient means to arrest the signal degradation in the secondary network.

The performance of cognitive cooperative communications has attracted much attention in recent works. Among them, the performance of cognitive cooperative communications with decode-and-forward (DF) relays was considered in [4], [5]. In addition, the outage probability of cognitive relay networks with amplify-and-forward (AF) relaying was presented for Rayleigh and Nakagami-m fading channels in [6] and [7], respectively. Although the one-way cognitive cooperative communications can cope with the reliability degradation of secondary networks, its spectral efficiency is limited by the pre-log factor of $\frac{1}{2}$ due to half-duplex dual-hop communications.

The two-way relaying technique has been discovered to compensate this loss by allowing two users to simultaneously communicate with each other through the assistance of a half-duplex relay [8]. Although two-way relaying has gained a lot attention, its impact has been only considered in non-cognitive networks [9]–[14]. As such, in this paper, we take a step forward to examine the advantage of two-way relaying in cognitive networks with a three-fold purpose to: i) enhance the spectrum utilization using cognitive radio, ii) enhance the reliability due to limited transmit power at the SU using relay networks, and iii) enhance the spectral efficiency of half-duplex cognitive relay networks using two-way transmission. With this in mind, we propose cognitive relay networks with two-way communication and derive new closed-form expressions for the outage probability under the maximum allowable interference constraint at the primary network with multiple primary users.

Notation: The superscript $(\cdot)^H$ denotes complex conjugate transposition; $E\{\cdot\}$ denotes expectation; $\mathcal{CN}\left(\mu, \sigma^2\right)$ denotes the complex Gaussian distribution with mean μ and variance σ^2; $F_\varphi(\gamma)$ denotes the cumulative distribution function (CDF) of the random variable (RV) φ; The probability density function (PDF) of φ is denoted by $f_\varphi(x)$. The binomial coefficient is denoted by ${}_nC_k \stackrel{\triangle}{=} \frac{n!}{(n-k)!k!}$.

II. System and Channel Model

We propose a two-way cognitive relay network with K multiple PUs equipped with single antennas coexistent in the same frequency band. We consider two scenarios in the secondary network: 1) two users U_1 and U_2 communicating via an intermediate relay R, and 2) two user groups $\mathbb{X}_1 = \{\mathrm{U}_{1,1}, ..., \mathrm{U}_{1,M}\}$ and $\mathbb{X}_2 = \{\mathrm{U}_{2,1}, ..., \mathrm{U}_{2,N}\}$ communicating via an intermediate relay R. In these scenarios, we consider the following assumptions.

Assumption 1: All channels in the proposed network follow independent Rayleigh fading, such that the channel power $|h|^2$ for a fading channel h has a chi-squared distribution with two degrees of freedom. We denote $|h|^2 \sim \chi^2(2, \lambda_{|h|^2})$, where $\lambda_{|h|^2} \triangleq E\{|h|^2\}$. The PDF and the CDF of $\gamma_{|h|^2}$ are, respectively, given as

$$\begin{aligned} f_{|h|^2}(x) &= \frac{1}{\lambda_{|h|^2}} e^{-\frac{x}{\lambda_{|h|^2}}} \mathrm{U}(x) \text{ and} \\ F_{|h|^2}(x) &= 1 - e^{-\frac{x}{\lambda_{|h|^2}}} \mathrm{U}(x) \end{aligned} \quad (1)$$

where $\mathrm{U}(\cdot)$ denotes the discrete unit step function.

Assumption 2: For two user groups $\mathbb{X}_1 = \{\mathrm{U}_{1,1}, ..., \mathrm{U}_{1,M}\}$ and $\mathbb{X}_2 = \{\mathrm{U}_{2,1}, ..., \mathrm{U}_{2,N}\}$, the channel between the users in $\mathbb{X}_1$ and R is assumed to experience identically distributed fading. The channel between the users in $\mathbb{X}_2$ and R is also assumed to experience identically distributed fading.

A. Transmission under maximum allowable interference constraint with two users U_1 and U_2

Under a maximum allowable interference constraint I_p at all the PUs, the transmit power allocation at $\mathrm{U}_1 \triangleq \mathrm{U}_{1,1}$, $\mathrm{U}_2 \triangleq \mathrm{U}_{2,1}$, and R are defined as

$$P_{\mathrm{U}_1} = \frac{I_p}{\max_{j=1,...,K}\{|g_1^j|^2\}},\ P_{\mathrm{U}_2} = \frac{I_p}{\max_{j=1,...,K}\{|g_2^j|^2\}}, \text{ and } P_{\mathrm{R}} = \frac{I_p}{\max_{j=1,...,K}\{|h_0^j|^2\}}. \quad (2)$$

The received signal at the relay in the first multiple access channel (MAC) phase is given by

$$y_{\mathrm{R}} = \sqrt{\frac{P_{\mathrm{U}_1}}{2}} h_1 x_1 + \sqrt{\frac{P_{\mathrm{U}_2}}{2}} h_2 x_2 + n_{\mathrm{R}} \quad (3)$$

where x_1 and x_2 are transmission symbols from U_1 and U_2, respectively. We also assume that $E\{x_1\} = E\{x_2\} = 0$, and $E\{|x_1|^2\} = E\{|x_2|^2\} = 1$. The additive noise at the relay node is denoted by $n_{\mathrm{R}} \sim \mathcal{CN}(0; \sigma_n^2)$.

The received signals, respectively, at U_1 and U_2 in the second broadcast channel (BC) phase, are given by

$$\begin{aligned} y_{\mathrm{U}_1} &= h_1 G \sqrt{\frac{P_{\mathrm{U}_2}}{2}} h_2 x_2 + h_1 G n_{\mathrm{R}} + n_{\mathrm{U}_1} \text{ and} \\ y_{\mathrm{U}_2} &= h_2 G \sqrt{\frac{P_{\mathrm{U}_1}}{2}} h_1 x_1 + h_2 G n_{\mathrm{R}} + n_{\mathrm{U}_2} \end{aligned} \quad (4)$$

where G is the relay gain of the AF protocol, $n_{\mathrm{U}_1} \sim \mathcal{CN}(0; \sigma_n^2)$, and $n_{\mathrm{U}_2} \sim \mathcal{CN}(0; \sigma_n^2)$. Note that in (4), we assume channel reciprocity and perfect self interference cancellation for exposition simplicity. The AF relay gain is computed as follows:

$$G = \sqrt{\frac{P_{\mathrm{R}}}{P_{\mathrm{U}_1}|h_1|^2/2 + P_{\mathrm{U}_2}|h_2|^2/2 + \sigma_n^2}}. \quad (5)$$

Applying (5) into (4), the end-to-end signal-to-noise ratios (e-SNRs) for links $\mathrm{U}_1 \to \mathrm{R} \to \mathrm{U}_1$ and $\mathrm{U}_2 \to \mathrm{R} \to \mathrm{U}_2$ are given by

$$\begin{aligned} \gamma_{\mathrm{U}_1} &\triangleq \frac{P_{\mathrm{R}}|h_2|^2 P_{\mathrm{U}_1}|h_1|^2/2}{P_{\mathrm{R}}|h_2|^2\sigma_n^2 + \frac{P_{\mathrm{U}_1}}{2}|h_1|^2\sigma_n^2 + \frac{P_{\mathrm{U}_2}}{2}|h_2|^2\sigma_n^2 + \sigma_n^4} \text{ and} \\ \gamma_{\mathrm{U}_2} &\triangleq \frac{P_{\mathrm{R}}|h_1|^2 P_{\mathrm{U}_2}|h_2|^2/2}{P_{\mathrm{R}}|h_1|^2\sigma_n^2 + \frac{P_{\mathrm{U}_1}}{2}|h_1|^2\sigma_n^2 + \frac{P_{\mathrm{U}_2}}{2}|h_2|^2\sigma_n^2 + \sigma_n^4}. \end{aligned} \quad (6)$$

B. Transmission under maximum allowable interference constraint with two user groups $\mathbb{X}_1$ and $\mathbb{X}_2$

To choose two individual users from $\mathbb{X}_1$ and $\mathbb{X}_2$, we propose partial best user selection (PBUS), which uses channel information from users to the relay. Our proposed scheme is less complicated compared to other schemes such as [15], [16] which require the end-to-end dual-hop instantaneous channel state information (CSI) for the selection. Note that in PBUS, same training symbols are used by all users. According to PBUS, two users U_1 and U_2 are selected using the following selection criterion

$$\begin{aligned} p^* &= \max\arg_{p\in[1,M]}(|h_{1,p}|^2), \mathrm{U}_1 \triangleq \mathrm{U}_{1,p^*}, \text{ and} \\ q^* &= \max\arg_{q\in[1,N]}(|h_{2,q}|^2), \mathrm{U}_2 \triangleq \mathrm{U}_{2,q^*} \end{aligned} \quad (7)$$

where $h_{1,p}$ and $h_{2,q}$ denote channels from $\mathrm{U}_{1,p}$ to R and $\mathrm{U}_{2,q}$ to R, respectively.

III. Distribution of the SNR of the Two-Way Cognitive Relay Network

In the high SNR region, we have the following approximated forms and upper bounds on the e-SNRs

$$\begin{aligned} \gamma_{\mathrm{U}_1} &\approx \frac{P_{\mathrm{R}}}{\frac{P_{\mathrm{U}_1}}{2} + P_{\mathrm{R}}} \frac{(\frac{P_{\mathrm{U}_1}}{2} + P_{\mathrm{R}})X_1 \frac{P_{\mathrm{U}_2}}{2} X_2}{(\frac{P_{\mathrm{U}_1}}{2} + P_{\mathrm{R}})X_1 + \frac{P_{\mathrm{U}_2}}{2} X_2} \\ &\le \min\left(P_{\mathrm{R}} X_1, \frac{P_{\mathrm{R}} P_{\mathrm{U}_2}}{P_{\mathrm{U}_1} + 2P_{\mathrm{R}}} X_2\right) \triangleq \gamma_{\mathrm{U}_1}^{\mathrm{up}} \text{ and} \\ \gamma_{\mathrm{U}_2} &\approx \frac{P_{\mathrm{R}}}{\frac{P_{\mathrm{U}_2}}{2} + P_{\mathrm{R}}} \frac{(\frac{P_{\mathrm{U}_2}}{2} + P_{\mathrm{R}})X_2 \frac{P_{\mathrm{U}_1}}{2} X_1}{(\frac{P_{\mathrm{U}_1}}{2} + P_{\mathrm{R}})X_1 + \frac{P_{\mathrm{U}_2}}{2} X_2} \\ &\le \min\left(P_{\mathrm{R}} X_2, \frac{P_{\mathrm{R}} P_{\mathrm{U}_1}}{P_{\mathrm{U}_2} + 2P_{\mathrm{R}}} X_1\right) \triangleq \gamma_{\mathrm{U}_2}^{\mathrm{up}} \end{aligned} \quad (8)$$

where $X_1 \triangleq \frac{|h_1|^2}{\sigma_n^2}$ and $X_2 \triangleq \frac{|h_2|^2}{\sigma_n^2}$. Applying (2) into (8), we have

$$\begin{aligned} \gamma_{\mathrm{U}_1}^{\mathrm{up}} &= \min\left(\tilde{I}_p \frac{H_2 G_1}{(H_0 + 2G_1)G_2}, \tilde{I}_p \frac{H_1}{H_0}\right) \text{ and} \\ \gamma_{\mathrm{U}_2}^{\mathrm{up}} &= \min\left(\tilde{I}_p \frac{H_1 G_2}{(H_0 + 2G_2)G_1}, \tilde{I}_p \frac{H_2}{H_0}\right) \end{aligned} \quad (9)$$

where $\tilde{I}_p \triangleq \frac{I_p}{\sigma_n^2}$. Using *Assumption 1*, we proceed to derive the CDFs of $\gamma_{\mathrm{U}_1}^{\mathrm{up}}$ and $\gamma_{\mathrm{U}_2}^{\mathrm{up}}$. To simplify our notation, we also define the following

$$\begin{aligned} &H_0 \triangleq \max\{|h_0^j|^2\}_{j=1}^K, H_1 \triangleq |h_1|^2, H_2 \triangleq |h_2|^2, \\ &G_1 \triangleq \max\{|g_1^j|^2\}_{j=1}^K, \text{ and } G_2 \triangleq \max\{|g_2^j|^2\}_{j=1}^K. \end{aligned} \quad (10)$$

We first derive the particular distributions for the RVs H_0, G_1, and G_2. The corresponding CDFs is presented in the following lemma.

Lemma 1: When $|g_1^j|^2 \sim \chi^2(2, \lambda_{G_1^j})$ with $\lambda_{G_1^j} \triangleq E\{|g_1^j|^2\}$, the CDF of G_1 is given by

$$F_{G_1}(x) = 1 - \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} e^{-\sum_{t=1}^{K} \frac{1}{\lambda_{G_1^j}} x} = 1 - \widetilde{\sum}_1 \left[e^{-\frac{1}{\lambda_{G_1}} x} \mathrm{U}(x) \right] \quad (11)$$

where $|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k|$ denotes the cardinality of the union of k indices. Also, we define $\widetilde{\sum}_1 [\cdot] \triangleq \sum_{k=1}^{K} \frac{(-1)^{k+1}}{k!} \underbrace{\sum_{n_1=1}^{K} \cdots \sum_{n_k=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_k| = k} \Big[\cdot\Big]$ and $\frac{1}{\lambda_{G_1}} \triangleq \sum_{t=1}^{k_1} \frac{1}{\lambda_{G_1^{n_{k_t}}}}$. Similarly, the CDFs of G_2 and H_0 are respectively given by

$$F_{G_2}(x) = 1 - \widetilde{\sum}_2 \left[e^{-\frac{1}{\lambda_{G_2}} x} \mathrm{U}(x) \right] \text{ and}$$
$$F_{H_0}(x) = 1 - \widetilde{\sum}_3 \left[e^{-\frac{1}{\lambda_{H_0}} x} \mathrm{U}(x) \right] \quad (12)$$

where $\widetilde{\sum}_2 [\cdot]$ and $\widetilde{\sum}_3 [\cdot]$ are similarly defined as $\widetilde{\sum}_1 [\cdot]$ with $\frac{1}{\lambda_{H_0}} \triangleq \sum_{t=1}^{k_2} \frac{1}{\lambda_{H_0^{n_{k_t}}}}$ and $\frac{1}{\lambda_{G_2}} \triangleq \sum_{t=1}^{k_3} \frac{1}{\lambda_{G_2^{n_{k_t}}}}$, where $\lambda_{H_0^j} \triangleq E\{|h_0^j|^2\}$ and $\lambda_{G_2^j} \triangleq E\{|g_2^j|^2\}$.

Proof: The proof is provided in Appendix A. ∎

From *Lemma 1*, the PDFs of G_1, G_2, and H_0 are respectively given by

$$f_{H_0}(x) = \widetilde{\sum}_3 \left[\frac{e^{-\frac{1}{\lambda_{H_0}} x} \mathrm{U}(x)}{\lambda_{H_0}} \right], f_{G_1}(x) = \widetilde{\sum}_1 \left[\frac{e^{-\frac{1}{\lambda_{G_1}} x} \mathrm{U}(x)}{\lambda_{G_1}} \right],$$
$$\text{and } f_{G_2}(x) = \widetilde{\sum}_2 \left[\frac{e^{-\frac{1}{\lambda_{G_2}} x} \mathrm{U}(x)}{\lambda_{G_2}} \right]. \quad (13)$$

A. Derivation of the distribution of the e-SNR with two users U_1 *and* U_2

Due to the symmetric upper bounds on the e-SNRs in the functions of (H_2, G_1) and (H_1, G_2), we will focus on the derivation of the CDF of $\gamma_{\mathrm{U}_1}^{\mathrm{up}}$, which is defined as

$$F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma) = E_{H_0}\{F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma|H_0)\}$$
$$= 1 - \left(1 - E_{H_0}\left\{F_{\gamma_1}\left(\gamma|H_0\right)\right\}\right)\left(1 - E_{H_0}\left\{F_{\gamma_2}\left(\gamma|H_0\right)\right\}\right) \quad (14)$$

where $\gamma_1 \triangleq \tilde{I}_p \frac{H_2 G_1}{(H_0 + G_1) G_2}$ and $\gamma_2 \triangleq \tilde{I}_p \frac{H_1}{H_0}$.

Next, we compute the conditional CDF $F_{\gamma_1}(\gamma|H_0)$, which is evaluated as

$$F_{\gamma_1}(\gamma|H_0) = Pr\left(H_2 < (\gamma G_2 (H_0 + 2G_1)/\tilde{I}_p G_1) \middle| H_0\right)$$
$$= 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \widetilde{\sum}_3 \Big[\beta_1(\gamma) - \alpha_{11}(\gamma) \beta_1(\gamma) H_0 e^{\alpha_{11}(\gamma) H_0} \Gamma\left(0, H_0 \alpha_{11}(\gamma)\right) \Big] \quad (15)$$

where $\Gamma(\cdot,\cdot)$ denotes the incomplete gamma function. Also, we define $\alpha_{11}(\gamma) \triangleq \frac{\lambda_{G_2} \gamma}{\lambda_{G_1}(2\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)}$ and $\beta_1(\gamma) \triangleq \frac{\lambda_{H_2}\tilde{I}_p}{(2\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)}$. The derivation of (15) is further detailed in Appendix B. We also derive the conditional CDF $F_{\gamma_2}(\gamma|H_0)$ as

$$F_{\gamma_2}(\gamma|H_0) = Pr\left((H_1 \tilde{I}_p)/H_0 < \gamma \middle| H_0\right)$$
$$= Pr\left(H_1 < (\gamma H_0)/\tilde{I}_p \middle| H_0\right)$$
$$= 1 - e^{-\frac{\gamma H_0}{\lambda_{H_1} \tilde{I}_p}}. \quad (16)$$

Using (15) and (16), $F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma|H_0)$ is evaluated as

$$F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma|H_0) = 1 - \left(1 - F_{\gamma_1}(\gamma|H_0)\right)\left(1 - F_{\gamma_2}(\gamma|H_0)\right)$$
$$= 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \widetilde{\sum}_3 \Big[\beta_1(\gamma) e^{-\frac{\gamma H_0}{\lambda_{H_1}\tilde{I}_p}} - \alpha_{11}(\gamma)\beta_1(\gamma) H_0 e^{\alpha_{11}(\gamma) H_0} \Gamma\left(0, H_0 \alpha_{11}(\gamma)\right) e^{-\frac{\gamma H_0}{\lambda_{H_1}\tilde{I}_p}} \Big]. \quad (17)$$

Based on the property of $F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma) = E_{H_0}\{F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma|H_0)\}$, we present our result for $F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma)$ in the following theorem.

Theorem 1: The CDF of an upper bound on user e-SNR, achieved by two-way transmission is given in (18) at the top of the next page. In (18), we defined $\alpha_{12}(\gamma) \triangleq \frac{1}{\lambda_{H_0}} + \frac{\gamma}{\lambda_{H_1}\tilde{I}_p}$.

Proof: The proof is provided in Appendix C. ∎

According to *Theorem 1*, $F_{\gamma_{\mathrm{U}_2}^{\mathrm{up}}}(\gamma)$ can be easily derived by defining $\beta_2(\gamma) \triangleq \frac{\lambda_{H_1}\tilde{I}_p}{(2\gamma\lambda_{G_1} + \lambda_{H_1}\tilde{I}_p)}$, $\alpha_{21}(\gamma) \triangleq \frac{\lambda_{G_1}\gamma}{\lambda_{G_2}(2\gamma\lambda_{G_1} + \lambda_{H_1}\tilde{I}_p)}$, and $\alpha_{22}(\gamma) \triangleq \frac{1}{\lambda_{H_0}} + \frac{\gamma}{\lambda_{H_2}\tilde{I}_p}$.

B. Derivation of the distribution of the e-SNR with two user groups $\mathbb{X}_1$ *and* $\mathbb{X}_2$

Considering PBUS, the CDFs of $H_1 \triangleq \max\{|h_{1,p}|^2\}_{p=1}^{M}$ and $H_2 \triangleq \max\{|h_{2,q}|^2\}_{q=1}^{N}$ are given by

$$F_{H_1}(x) \triangleq 1 - \sum_{k=1}^{M} {}_M C_k (-1)^{k+1} e^{-\frac{kx}{\lambda_{H_1}}} \mathrm{U}(x) \text{ and}$$
$$F_{H_2}(x) \triangleq 1 - \sum_{l=1}^{N} {}_N C_l (-1)^{l+1} e^{-\frac{lx}{\lambda_{H_2}}} \mathrm{U}(x) \quad (19)$$

where $\lambda_{H_1} \triangleq E\{|h_{1,p}|^2\}, \forall p$ and $\lambda_{H_2} \triangleq E\{|h_{2,q}|^2\}, \forall q$. Based on the above, the CDF of $F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma)$ is given in the following theorem.

Theorem 2: The CDF of the upper bound on the user e-SNR, achieved by two-way transmission is given in (20). In (20), we define $\tilde{\beta}_1(l\gamma) \triangleq \frac{\lambda_{H_2}\tilde{I}_p}{(2l\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)}$, $\tilde{\alpha}_{11}(l\gamma) \triangleq \frac{l\lambda_{G_2}\gamma}{\lambda_{G_1}(2l\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)}$, and $\tilde{\alpha}_{12}(k\gamma) \triangleq \frac{1}{\lambda_{H_0}} + \frac{k\gamma}{\lambda_{H_1}\tilde{I}_p}$.

Proof: Following the proof provided in Appendix B and Appendix C, we can readily derive (20). ∎

$$F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma) = 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \widetilde{\sum}_3 \left[\frac{1}{\lambda_{H_0}} \beta_1(\gamma)\alpha_{12}(\gamma)^{-1} - \frac{\beta_1(\gamma)\alpha_{11}(\gamma)}{\alpha_{11}(\gamma) - \alpha_{12}(\gamma)} \Big(\frac{1}{\alpha_{12}(\gamma)} + \frac{1}{\alpha_{11}(\gamma) - \alpha_{12}(\gamma)} \log(\frac{\alpha_{12}(\gamma)}{\alpha_{11}(\gamma)}) \Big) \right]. \tag{18}$$

$$F_{\gamma_{\mathrm{U}_{\mathrm{PBUS},1}}^{\mathrm{up}}}(\gamma) = 1 - \sum_{k=1}^{M} \sum_{l=1}^{N} {}_M C_k {}_N C_l (-1)^{k+l+2} \widetilde{\sum}_1 \widetilde{\sum}_2 \widetilde{\sum}_3 \Bigg[\frac{1}{\lambda_{H_0}} \tilde{\beta}_1(l\gamma)\tilde{\alpha}_{12}(k\gamma)^{-1} - \frac{\tilde{\beta}_1(l\gamma)\tilde{\alpha}_{11}(l\gamma)}{\tilde{\alpha}_{11}(l\gamma) - \tilde{\alpha}_{12}(k\gamma)} \Big(\frac{1}{\tilde{\alpha}_{12}(k\gamma)} + \frac{1}{\tilde{\alpha}_{11}(l\gamma) - \tilde{\alpha}_{12}(k\gamma)} \log(\frac{\tilde{\alpha}_{12}(k\gamma)}{\tilde{\alpha}_{11}(l\gamma)}) \Big) \Bigg]. \tag{20}$$

IV. Outage Probability Analysis

In this section, we define two outage probabilities: 1) user outage probability for U_1 or U_2, and 2) system outage probability.

A. Outage probability with two users U_1 *and* U_2 *under maximum allowable interference constraint*

The user outage probability is given by

$$\begin{aligned} P_{\mathrm{U}_1}^{\mathrm{out}}(\gamma_{th}) &= Pr\left(\gamma_{\mathrm{U}_1} < \gamma_{th}\right) \geq Pr\left(\gamma_{\mathrm{U}_1}^{\mathrm{up}} < \gamma_{th}\right) \\ &= F_{\gamma_{\mathrm{U}_1}}^{\mathrm{up}}(\gamma_{th}) \triangleq P_{\mathrm{U}_1}^{\mathrm{out,lo}}(\gamma_{th}) \end{aligned} \tag{21}$$

where γ_{th} is a fixed e-SNR threshold causing an outage event. Similarly, we can have $P_{\mathrm{U}_2}^{\mathrm{out}}(\gamma_{th}) \geq F_{\gamma_{\mathrm{U}_2}}^{\mathrm{up}}(\gamma_{th}) \triangleq P_{\mathrm{U}_2}^{\mathrm{out,lo}}(\gamma_{th})$. Using *Theorem 1*, lower bounds on the user outage probabilities are given in (22) at the top of the next page. The exact system outage occurs when the minimum e-SNR between U_1 and U_2 is below γ_{th}. Thus,

$$P_{\mathrm{sys}}^{\mathrm{out}} \triangleq Pr\left(\min(\gamma_{\mathrm{U}_1}, \gamma_{\mathrm{U}_2}) < \gamma_{th}\right). \tag{23}$$

Since the exact CDFs for γ_{U_1} and γ_{U_2} are dependent on each other, it is infeasible to derive the system outage in (23). Thus, we will compute the system outage probability numerically in the simulations.

B. Outage probability with two user groups $\mathbb{X}_1$ *and* $\mathbb{X}_2$ *under maximum allowable interference constraint*

Based on the CDFs of the upper bounds on the user e-SNR for PBUS, the lower bound on the user outage probability can be derived by applying a similar approach as in the derivation of $P_{\mathrm{U}_1}^{\mathrm{out,lo}}(\gamma_{th})$ in (22).

V. Numerical Examples

In the numerical examples, we assume that U_1 and U_2 are coordinated at $[0,0]$ and $[0,1]$, respectively. The relay node R is located between U_1 and U_2. The PUs are located at $[P_x, P_y]$. As a pathloss model, we adopt an exponentially decaying model whose channel mean power is inversely proportional to the distance between two nodes. We also consider a pathloss exponent of four. In the following figures, the curves obtained from actual link simulations are denoted by **Ex**, whereas analytically derived curves are denoted by **An**. To obtain the outage probability, we consider a fixed $\gamma_{th} = 1$ dB.

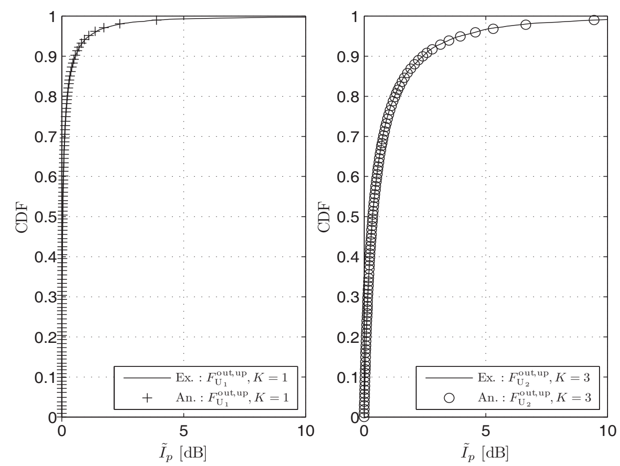

Fig. 1. CDFs of upper bounds on user e-SNRs for $K = 1$ and $K = 3$.

In Fig. 1, we show the accuracy of the derived CDFs for user e-SNRs at $I_p = 3$ dB. In this figure, we assume $K = 1$ and $K = 3$ PUs coordinated at $\mathrm{PU}_1 = [0.7, 0.3]$, and $\{\mathrm{PU}_1 = [0.7, 0.3], \mathrm{PU}_2 = [0.3, 0.7], \mathrm{PU}_3 = [0.5, 0.9]\}$. The relay node is coordinated at $[0.5, 0]$. With nonidentical distance between U_1 and PUs and U_2 and PUs, $F_{\gamma_{\mathrm{U}_1}^{\mathrm{up}}}(\gamma)$ is seen to be different from $F_{\gamma_{\mathrm{U}_2}^{\mathrm{up}}}(\gamma)$.

In Fig. 2, we show the outage probabilities for various number of PUs. The PUs are assumed to be coordinated at the same places as in Fig. 1. This figure shows a good match between simulations and analysis. As the number of PUs increases, the outage probability increases. For $K = 1$, since PU_1 is near U_2, the outage probability of U_2 is worse than that of U_1.

In Fig. 3, we compare the system outage probability $P_{\mathrm{sys}}^{\mathrm{out}}$ for $K = 1$ and $K = 3$ with $P_{\mathrm{U}_1}^{\mathrm{out}}(\gamma_{th})$ and $P_{\mathrm{U}_2}^{\mathrm{out}}(\gamma_{th})$. Tight lower bounds for $P_{\mathrm{U}_1}^{\mathrm{out,lo}}(\gamma_{th})$ and $P_{\mathrm{U}_2}^{\mathrm{out,lo}}(\gamma_{th})$ can be seen from this figure. As K increases, discrepancy between $\min(P_{\mathrm{U}_1}^{\mathrm{out,lo}}(\gamma_{th}), P_{\mathrm{U}_2}^{\mathrm{out,lo}}(\gamma_{th}))$ and $P_{\mathrm{sys}}^{\mathrm{out}}$ increases. This figure shows a good match between simulation and analysis. As the number of PUs increases, a worse outage probability is obtained. For $K = 1$, since the PU_1 is near U_2, the outage

$$P_{\mathrm{U}_1}^{\mathrm{out,lo}}(\gamma_{th}) = 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \widetilde{\sum}_3 \left[\frac{\beta_1(\gamma_{th})}{\lambda_{H_0}\alpha_{12}(\gamma_{th})} - \frac{\beta_1(\gamma_{th})\alpha_{11}(\gamma_{th})}{\lambda_{H_0}(\alpha_{11}(\gamma_{th}) - \alpha_{12}(\gamma_{th}))} \left(\frac{1}{\alpha_{12}(\gamma_{th})} + \frac{\log(\frac{\alpha_{12}(\gamma_{th})}{\alpha_{11}(\gamma_{th})})}{\alpha_{11}(\gamma_{th}) - \alpha_{12}(\gamma_{th})} \right) \right]. \quad (22)$$

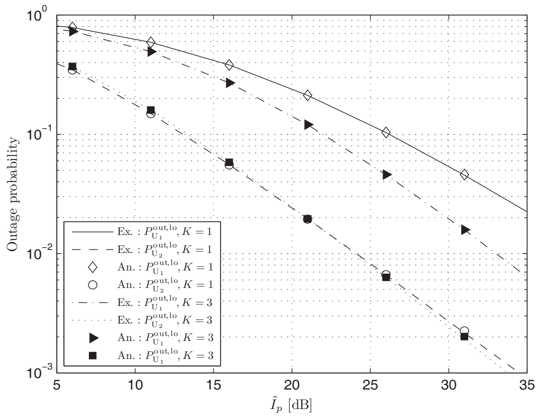

Fig. 2. User outage probability for various values of K.

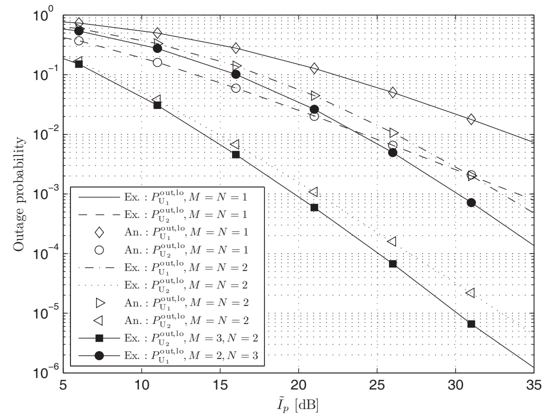

Fig. 4. User outage probability for various values of M and N. Two PUs are assumed in the system.

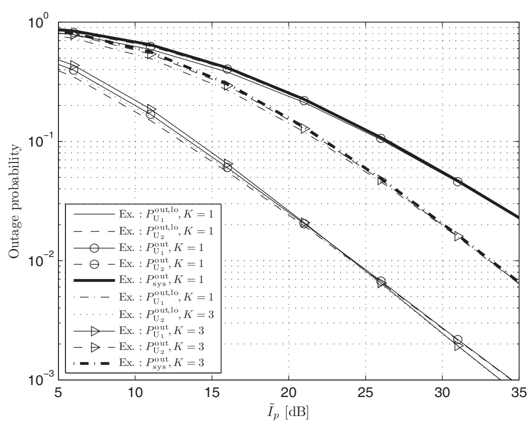

Fig. 3. User outage probability for various values of K.

probability of U_2 is worse than than of the U_1.

In Fig. 4, we show the outage probability for various values of M and N. We highlight that for $K = 2$ as either M or N increases, a lower outage probability is obtained using PBUS.

VI. Conclusions

We derived the user outage probability for two-way relay transmissions in cognitive spectrum sharing with multiple primary users. Under a maximum allowable interference constraint at the PUs, upper bounds on the user e-SNRs were derived for two scenarios in the secondary network: 1) two users communicating via an intermediate relay, and 2) two user groups with PBUS communicating via an intermediate relay. Based on these, lower bounds on the user outage probabilities were derived and verified via simulations.

Appendix A: Detailed Derivation of Lemma 1

Due to independent fading for all links from U_1 to the kth PU, the CDF of $G_1 = \max_{k=1,\cdots,K}\{|g_1^1|^2, ..., |g_1^K|^2\}$ is given by $F_{G_1}(x) = \prod_{j=1}^{K} F_{G_1^j}(x)\mathrm{U}(x) = \prod_{j=1}^{K}(1-x_j)\mathrm{U}(x)$, where $x_j \stackrel{\triangle}{=} e^{-\frac{1}{\lambda_{G_1^j}}x}$. With some manipulations, we can see that

$$\prod_{j=1}^{K}(1-x_j) = 1 + \sum_{j=1}^{K}\frac{(-1)^j}{j!} \underbrace{\sum_{n_1=1}^{K}\cdots\sum_{n_j=1}^{K}}_{|n_1 \bigcup n_2 \bigcup \cdots \bigcup n_j|=j} \prod_{t=1}^{j} x_{n_t}. \quad \text{(A.1)}$$

Replacing x_j with its definition, we can prove (11).

Appendix B: Detailed Derivation of (15)

We start the computation of

$$\begin{aligned}
&Pr\left(H_2 < \gamma G_2(H_0 + 2G_1)/\tilde{I}_p G_1 | H_0\right) \\
&= \int_0^\infty \int_0^\infty F_{H_2}(\gamma b(H_0 + 2a)/\tilde{I}_p a) f_{G_1}(a) f_{G_2}(b)\, da db \\
&= 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \Big[\frac{1}{\lambda_{G_2}\lambda_{G_1}} \\
&\int_0^\infty \int_0^\infty e^{-\frac{1}{\lambda_{H_2}}\frac{\gamma b(H_0+2a)}{\tilde{I}_p a}} e^{-\frac{a}{\lambda_{G_1}}} e^{-\frac{b}{\lambda_{G_2}}} da db \\
&= 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \left[\frac{\lambda_{H_2}\tilde{I}_p}{\lambda_{G_1}} \right. \\
&\left. \int_0^\infty \left(\frac{a e^{-\frac{a}{\lambda_{G_1}}}}{a(2\gamma\lambda_{G_2}\lambda_{H_2}\tilde{I}_p) + \lambda_{G_2}H_0\gamma} \right) da \right]. \qquad \text{(B.1)}
\end{aligned}$$

To compute (B.1), we need to use $F_1 = \int_0^\infty \frac{xe^{-ax}}{cx+d}dx$, where $a, c, d > 0$. Using [17, Eqs. (3.353.3), (8.350.2), and (8.211.1)], F_1 becomes

$$F_1 = \frac{1}{ac} + \frac{de^{\frac{adx}{c}}}{c^2}E_1\left(-\frac{adx}{c}\right) = \frac{1}{ac} - \frac{de^{\frac{ad}{c}}}{c^2}\Gamma\left(0, \frac{ad}{c}\right) \quad \text{(B.2)}$$

where $E_1(\cdot)$ denotes the exponential integral function of order 1. Using (B.2), (B.1) is equivalent to the following:

$$\begin{aligned} &Pr\left(H_2 < \gamma G_2(H_0 + 2G_1)/\tilde{I}_p G_1 | H_0\right) = 1- \\ &\widetilde{\sum}_1 \widetilde{\sum}_2 \left[\frac{\lambda_{H_2}\tilde{I}_p}{(2\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)} - \frac{I_p\lambda_{H_2}\lambda_{G_2}H_0\gamma}{\lambda_{G_1}(2\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)^2}\right. \\ &\left. e^{\frac{\lambda_{G_2}H_0\gamma}{\lambda_{G_1}(2\gamma\lambda_{G_2}+\lambda_{H_2}\tilde{I}_p)}}\Gamma\left(0, \frac{\lambda_{G_2}H_0\gamma}{\lambda_{G_1}(2\gamma\lambda_{G_2} + \lambda_{H_2}\tilde{I}_p)}\right)\right] \\ &= 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \Big[\beta_1(\gamma) - \\ &\alpha_{11}(\gamma)\beta_1(\gamma)H_0 e^{\alpha_{11}(\gamma)H_0}\Gamma\left(0, H_0\alpha_{11}(\gamma)\right)\Big]. \end{aligned} \quad \text{(B.3)}$$

Appendix C: Proof of Theorem 1

Since $F_{\gamma_{U_1}^{up}}(\gamma) = E_{H_0}\{F_{\gamma_{U_1}^{up}}(\gamma | H_0)\}$, we have

$$\begin{aligned} F_{\gamma_{U_1}^{up}}(\gamma) = 1 - \widetilde{\sum}_1 \widetilde{\sum}_2 \Bigg[&\underbrace{\int_0^\infty \beta_1(\gamma) e^{-\frac{\gamma g}{\lambda_{H_1}\tilde{I}_p}} \frac{1}{\lambda_{H_0}} e^{-\frac{g}{\lambda_{H_0}}} dg}_{I_1} - \\ &\underbrace{\int_0^\infty \frac{\alpha_{11}(\gamma)\beta_1(\gamma) g e^{\alpha_{11}(\gamma)g}\Gamma\left(0, g\alpha_{11}(\gamma)\right)}{\lambda_{H_0}} e^{-\frac{\gamma g}{\lambda_{H_1}\tilde{I}_p}} e^{-\frac{g}{\lambda_{H_0}}} dg}_{I_2}\Bigg] \end{aligned} \quad \text{(C.1)}$$

where I_1 becomes

$$I_1 = \frac{\beta_1(\gamma)}{\lambda_{H_0}}\left(\frac{\gamma}{\lambda_{H_1}\tilde{I}_p} + \frac{1}{\lambda_{H_0}}\right)^{-1}. \quad \text{(C.2)}$$

The computation of I_2 can be evaluated by applying the following integration:

$$F_2 = \int_0^\infty axe^{(a-b)x}\Gamma(0, ax)dx = \frac{a}{2b^2}{}_2F_1\left(1, 2, 3; \frac{b-a}{b}\right) \quad \text{(C.3)}$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ denotes the Gauss hypergeometric function. Since $z\,{}_2F_1(1, 2; 3; z) = -2\left(1 + 1/z\log(1-z)\right)$, we have an alternative form for F_2 using an elementary function as follows:

$$F_2 = a/(b(a-b)) + (a\log(b/a))/(a-b)^2. \quad \text{(C.4)}$$

Using (C.4), I_2 is given by

$$\begin{aligned} I_2 = &\frac{\beta_1(\gamma)}{\lambda_{H_0}} \frac{\alpha_{11}(\gamma)}{(\alpha_{11}(\gamma) - \alpha_{12}(\gamma)))}\left[\frac{1}{\alpha_{12}(\gamma)} + \right. \\ &\left. \frac{1}{(\alpha_{11}(\gamma) - \alpha_{12}(\gamma))}\log\left(\frac{\alpha_{12}(\gamma)}{\alpha_{11}(\gamma)}\right)\right]. \end{aligned} \quad \text{(C.5)}$$

Collecting eqs. (C.2) and (C.5), we can arrive at the closed-form expression of $F_{\gamma_{U_1}^{up}}(\gamma)$. Similarly, we can derive $F_{\gamma_{U_2}^{up}}(\gamma)$ by symmetry between $F_{\gamma_{U_2}^{up}}(\gamma)$ and $F_{\gamma_{U_1}^{up}}(\gamma)$.

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