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*CIRP Conference on Modeling of Machining Operations (CIRP CMMO)*

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## Cutter Workpiece Engagement Calculations for Five-axis Milling using Composite Adaptively Sampled Distance Fields

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### Abstract

Composite adaptively sampled distance fields are a new approach to shape representation that is well suited for Numerically Controlled (NC) milling simulation. In NC milling, as the milling tool moves along the tool path, it carves out a swept volume and a portion of the workpiece is removed. During the milling tool motion, it is in contact with the workpiece over an instantaneous common surface which is called Cutter Workpiece Engagement (CWE) surface. In order to model the process mechanics and dynamics accurately, it is important to have a precise geometric representation of the CWE surface. One of the fundamental difficulties has been the accurate and computationally efficient determination of this surface along the tool path for five-axis milling applications. In this paper, we provide a brief introduction to distance fields for swept volumes and describe a new method for determining the CWE surface for general and complex five-axis NC milling. The combination of high simulation speed, high accuracy and modest memory requirements provide significant improvements over existing approaches.

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### 1. Introduction

Simulating the process of NC milling is of fundamental importance in computer aided design (CAD) and computer aided manufacturing (CAM). Virtual simulation of NC milling processes has started to become more important in order to minimize the discrepancies between the desired and actual machined surfaces, and one of the key technologies for advancing the productivity and quality of machining process is to design, test and produce the parts in a virtual environment. In NC milling, as the tool moves along the tool path, it is in contact with the workpiece over a common surface called the cutter workpiece engagement (CWE) surface. One of the steps in simulating the machining operations is the accurate extraction of the CWE surface between the tool and in-process workpiece.

It is necessary to have a precise geometric representation of the engagement surface in order to

model the process mechanics and dynamics accurately, such as the prediction of cutting forces, torque, power, tool deflections and vibrations. It is through this engagement surface that the forces are applied between the tool and the workpiece. Besides using the geometric properties of engagement surfaces in physics based models, they can be used in the selection of process parameters such as axial and radial depth of cuts, tool parameters and the determination of surface form errors due to tool deflections. More often the milling tool and the workpiece geometries are complex during the machining of free-form surfaces for example, die-mold, aerospace and aerospace parts milled by five-axis motions.

### 2. Related Work and Goals

Various approaches to NC milling simulation have been described in the literature, and they can be categorized into three major approaches: solid modeling, spatial partitioning and discrete vectors. An extensive

review appears in [1]. Solid modeling approaches use boundary representation (B-rep) [2] which is not practical for complex milling tools and long tool paths because of high computational cost and data storage. Another most common approach is cell decomposition using spatial partitioning approaches such as Z-buffer, G-buffer, dixel, Graf-tree, voxel and octree [3-5], etc. The third approach, called the point vector method, approximates the machined surface by a discrete set of points and vectors. The cutting is simulated by calculating the intersection of these vectors with the cutter swept volumes [6]. A new method uses composite adaptively sampled distance fields (*cADF*) to represent the machined surface enabling fast and accurate simulation, and compact data structure [1].

One of the fundamental difficulties has been the accurate and computationally efficient determination of the CWE surface in most of these approaches. It is a challenging task due to the complex and changing tool/workpiece intersection during NC milling. For example, B-rep based methods can analytically compute this surface for simple milling tools and 2.5 axis tool paths [2, 7]. However, they are impractical for complex motions due to high computational costs. Polygon based methods also receive some attention; however the accuracy of these methods [8-9] is limited by the polygonal representation of the object model. Thus, they have prohibitive processing times and memory requirements for calculating high precision CWE surface. Another approach is to use distance fields to calculate the cutter workpiece engagements accurately, robustly and efficiently for 3-axis motions [10].

Five axis milling refers to the ability of the NC milling tool to perform movement about five different axes simultaneously. The machine's NC data known as cutter location (CL) points, consist of a sequence of tool configurations specified by  $(x, y, z, \theta_1, \theta_2)$ , where  $x, y$  and  $z$  are translation coordinates and,  $\theta_1$  and  $\theta_2$  are rotation angles around primary and secondary axis respectively. In the literature, a few methods have been employed for five-axis milling. A new solid modeling based approach called parallel slicing method [11] has been developed for five-axis flank milling operations where the removed volume is sliced into a number of parallel planes along the intermediate axis of cutter locations.

When all the existing methods are considered, there is a need for a space and time efficient method for determining a high precision CWE surface for tools moving along five-axis tool paths. These needs become more important especially in physical modelling of machining process. In this paper, we propose a new and efficient approach for computing the CWE surface between the milling tool and the workpiece for general 5-axis motions. *cADFs* are used to implicitly represent

the in-process workpiece, and analytically or procedurally defined Euclidean distance fields are used to represent the swept volumes of the milling tool corresponding to each CL point of the tool path. The new method can handle complex and general multi-axis motions for general milling tools. The results are compared with a solid modeler based method in terms of accuracy and speed.

### 3. Distance Fields Based NC Milling Simulation

We have proposed a new approach [1] to NC milling simulation that can rapidly generate a highly accurate representation of the milled workpiece in order to overcome the shortcomings of the previous methods. In this new representation each surface is implicitly represented by a signed Euclidean distance field (1).

$$d_S(P) = \begin{cases} \inf_{\forall q \in \partial S} \|P - q\|_2 & P \in S \\ - \inf_{\forall q \in \partial S} \|P - q\|_2 & P \notin S \end{cases} \quad (1)$$

This function yields the minimum Euclidean distance from a point  $P$  to the closest in the boundary of the set  $\partial S$ . An octree bounding volume hierarchy is used to obtain spatial localization of geometric operations.

#### 3.1. Distance fields of 5-axis motions

In this section, we develop a mathematical formulation for computing the swept volumes of general surfaces of revolution. The CWE surface is the instantaneous intersection between the tool at final position and in-process workpiece as given in Fig 1. Sweeping an arbitrary set of points  $S$  along a motion  $M$  in a space is usually formulated as an infinite union operation expressed formally as,

$$sweep(S, M) = \bigcup_{q \in M} S^q \quad (2)$$

where  $S^q$  denotes the set  $S$  positioned according to a configuration  $q$  of motion  $M(t)$ , within a normalized interval.  $M(t)$  is a one parameter family of rigid body transformations in  $E^3$ . The distance from a point  $P$  in the space to the boundary of swept volume is defined in the world coordinate frame;

$$d_S(P, sweep(S, M(t))) = \inf_{q \in \partial sweep(S, M(t))} \|P - q\|_2 \quad (3)$$

Finding the distance field of a swept volume requires computing the envelopes of the swept volume. This process is difficult for general 5-axis motions [12]. Instead the computation of distance field is handled by an inverted trajectory approach where the problem is

solved in tool coordinate frame instead of world coordinate frame [1, 13].

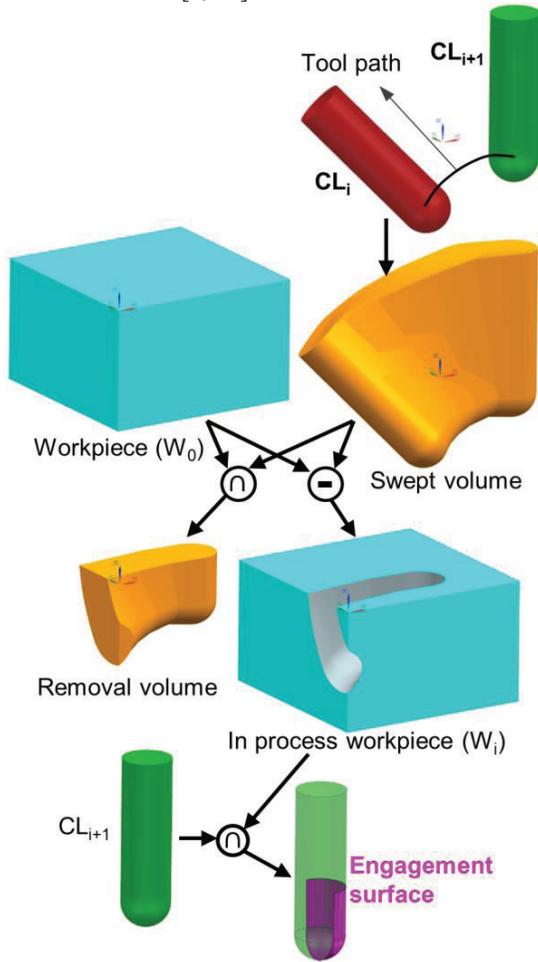


Fig. 1. Calculation of the CWE surface using Boolean operations

When the test point  $P$  is viewed in tool coordinate frame, it moves along an inverted trajectory,  $\hat{T}_P$  which is defined according to the inverse of the motion  $M(t)$ . In this tool coordinate frame, the distance field is now defined by,

$$dist(S, \hat{T}_P) = \min_{y \in \partial S, z \in \hat{T}_P} \|y - z\|_2 \quad (4)$$

In 3-axis milling case, the tool axis is always constant in one direction, translates in space and only rotates around its own axis, however, in 5-axis milling case, addition of two rotational axes allow to machine variety of different workpieces and motions. Compared to 3-axis machining, the inverted trajectory has a more complex geometry because of the rotational effects as seen in Fig 2. Besides three translational movements, the tool can also be rotated around two axes. The minimum distance between the inverted trajectory and tool can be

computed by using numerical search methods. Direct analytical solution of the minimum distance function is rather difficult; hence it is cast as a one-dimensional minimization problem. It is solved by employing an iterative numerical method which combines the golden section search and inverse quadratic interpolation.

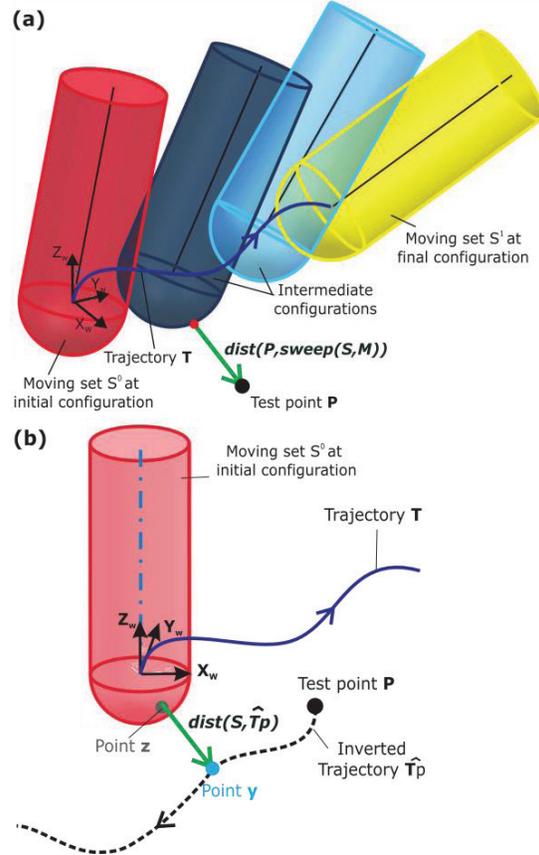


Fig. 2. 5-axis motion and inverted trajectory

#### 4. Cutter Workpiece Engagement

The CWE is the instantaneous contact surface as seen in Fig 3. Since the geometry of in-process workpiece varies, the cutter-workpiece intersection geometry is evaluated at discrete time steps corresponding to CL points along the tool path. Our method is a sampling based approach, where the set of sampling points are generated by using various sampling strategies such as cylindrical, spherical, geodesic sampling approaches, etc. The set of points,  $P_{test,G}$  are generated on the boundary of the tool positioned at origin, then these points are transformed according to motion  $M(t)$ .

$$M(t) = \begin{bmatrix} Rc(t) & T(t) \\ 0 \dots 0 & 1 \end{bmatrix}, \quad Rc(t) = R(\theta_2).R(\theta_1) \quad (5)$$

Where the translations are defined in terms of time dependent translation vector,  $T(t)$  and the rotations are defined by combining the primary ( $\theta_1$ ) and secondary ( $\theta_2$ ) angles of 5-axis motion,  $Rc(t)$ .

The transformed points  $P_{test,L}$  on the boundary of tool in local coordinate frame are now tested against the in-process workpiece,  $W_i$ . The contact points corresponding to the CWE surface are found based on the distance values between the points  $P_{test,L}$  and the in-process workpiece,

$$P_{contact,L} \subset P_{test,L}$$

$$s.t. \quad abs(dist(P_{test,L}, W_i)) \leq \varepsilon \quad (6)$$

A subset of the points,  $P_{contact,L}$  having the distance values below a certain distance threshold, form the CWE surface as seen in Fig 3. In order to determine the angle of engagement between cutter and workpiece, the points,  $P_{contact,L}$  in local coordinate frame are transformed by the inverse motion,  $M(t)^{-1}$ . The angle of engagement is defined in clockwise direction and measured from the normal vector perpendicular to instant tangent tool path vector. The entry angle is the angle at which the tool enters the workpiece, and exit angle is the angle at which the tool leaves the workpiece. The angle of engagement is basically defines the limits of CWE surface as a function of depth of cut along the tool axis as seen in Fig 4. The milling tool actually removes material and creates cutting forces within limits of CWE surface.

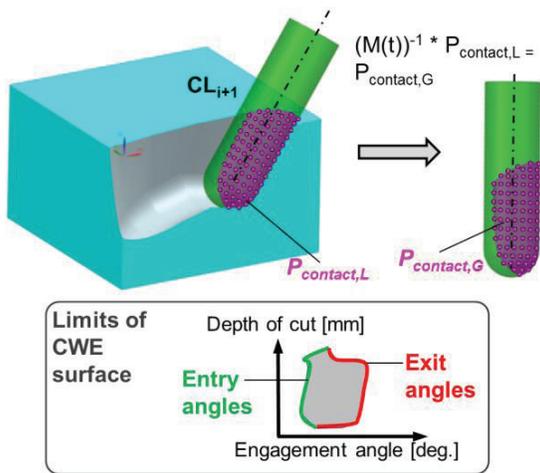


Fig. 3. Determination of contact points corresponding to the cutter workpiece engagement surface

### 5. Results

In order to demonstrate the capabilities of our approach to CWE computation, we have simulated three different examples. Two of them are the fabrication of

different impeller parts which are one of the most complex 5-axis machining tasks. The last example shows a complex 5-axis motion for ball-end mill cutting a block. The simulated surfaces are shown in Fig 5. The simulation was performed using a single core of a 2.8 GHz Intel Core i7 with 8 GB of DRAM.

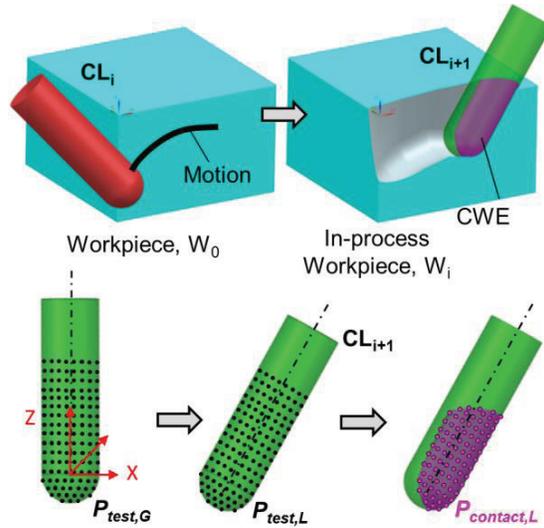


Fig. 4. The extraction of engagement angles from data points corresponding to CWE points

#### 5.1. Verification by a solid modeler based system

Currently the most popular schemes used in solid modelers are B-rep and Constructive Solid Geometry (CSG). In B-rep systems, an object is represented by both its boundaries defined by faces, edges, vertices and the connectivity information. We have developed a test system based on a B-rep solid modeler library to test the accuracy of our CWE results. The tool movements are subtracted from the workpiece model by using Boolean functions in order to find the in-process machined surface. Once the in-process workpiece is obtained for each CL point, the CWE surface is extracted. This surface is analyzed at slices perpendicular to tool axis in order to determine the angles of engagement.

#### 5.2. Examples

The first example shows a 2 mm radius ball-end mill cutting a workpiece which was previously machined by flat-end mill as shown in Fig 6. The ball-end mill moves according to BC type (rotations are around Y and Z axis respectively) 5-axis motion. Different views of CWE surfaces as well as the angle of engagements are shown for different CL points (CL#30 and CL#50) for the detail-A shown in Fig 5. For the CL#30, both the front and back faces of ball-end mill are in contact with the

workpiece. However, for the CL#50, only the front face of ball-end mill is in contact with the workpiece. The angle of engagements can change a lot for the neighboring tool path passes.

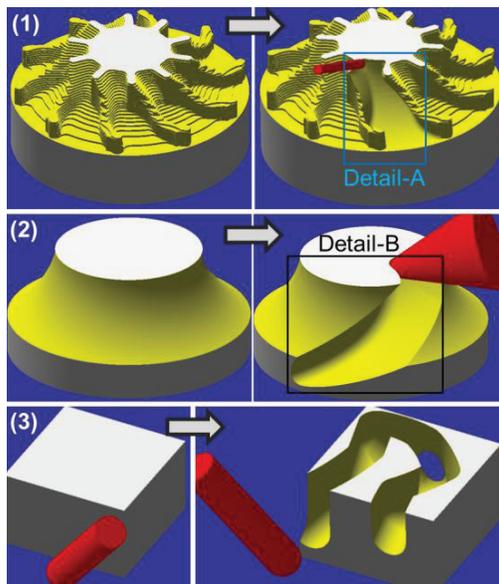


Fig. 5. Parts machined by ball-end and tapered ball-end mills

In our second example shown in Fig 7, the taper ball-end mill tool of 5 mm radius and 20 degree taper angle moves according to AC type (rotations are around X and Z axis respectively) 5-axis motion and removes the material from a workpiece. Different views of CWE surfaces as well as the angle of engagements are shown for different CL points (CL#45 and CL#135) for the detail-B shown in Fig 5. The tip (ball part) of the taper ball-end mill is in full contact (0 to 360 deg. angle of engagement) with the workpiece for the CL#45; however the tip of the milling tool is not in full contact with the workpiece for the CL#135 because of rotation angles. The milling tool has also less angle of engagement for this CL point, because it gets very close to top surface of workpiece. In the last example, ball-end mill tool moves according to BC type 5-axis motion as shown in Fig 8. For the given CL point, the angles of engagement can change dramatically along the depth of cut. The CWE surface has a hole because the tool penetrates through the workpiece boundary. The in-process workpiece and the corresponding tool instance are shown in two different views.

Our approach is demonstrated for five-axis milling of different workpieces by different tools. We have compared our CWE results to those obtained from B-rep based test system. The angle of engagements obtained from the two methods agrees very well, and the difference in angles between our method and B-rep

based method are less than 1%. However, the computational time and memory requirements are much higher for solid modeler based methods. In the first example in Fig 6, the tool path for the machined surface requires 1658 CL points.

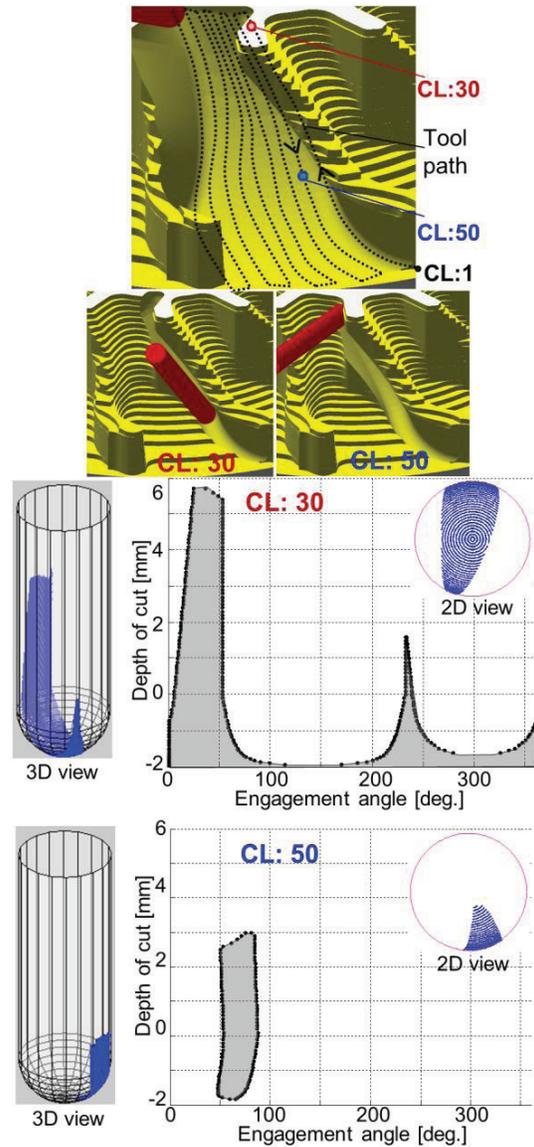


Fig. 6. The CWE surfaces for Ball-end mill tool

Our method can calculate the angle of engagements in 1 minute for all CL points, however B-rep based method cannot simulate the all CL points, only for 324 CL points in 62 minutes. In the second example in Fig 7, our method calculates the angle of engagement for one pass (consisting of 180 CL points) of tool path in 29 seconds, however the B-rep based method calculates in 280 seconds. In all the examples for the angle

comparisons, our method is around 6 to 20 times faster than B-rep based method.

## 6. Conclusion

In this paper, we have described a new method for calculating the CWE surface for milling tools moving according to 5-axis motions using a new shape representation, composite adaptively sampled distance fields (*cADF*). The high accuracy provided by this approach to NC milling simulation enables fast, accurate, efficient and robust calculation of the geometric properties of CWE surface which is an important input to physical and geometric models. The conventional geometric simulation methods have usually some serious problems to apply into CWE surface calculations for multi-axis milling applications. Our method can be generalized to general tools with different distance formulas for complex five axis motions.

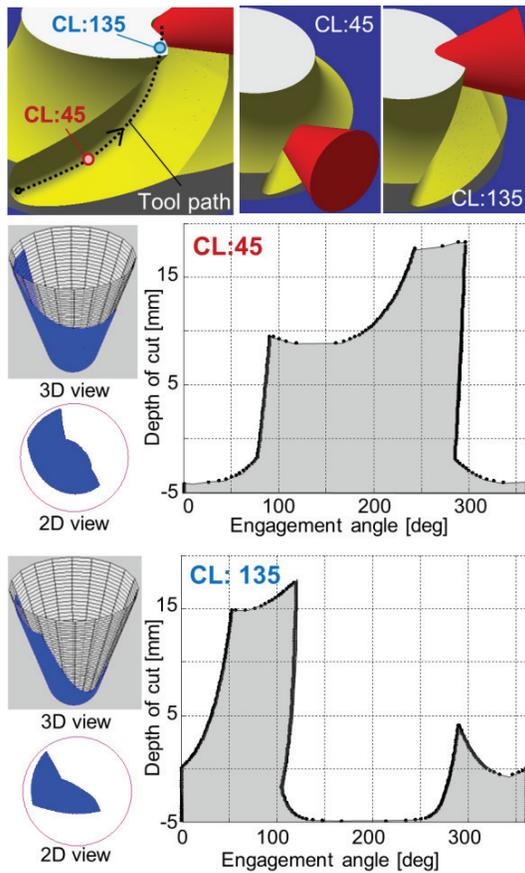


Fig. 7. The CWE surfaces for tapered ball-end mill tool

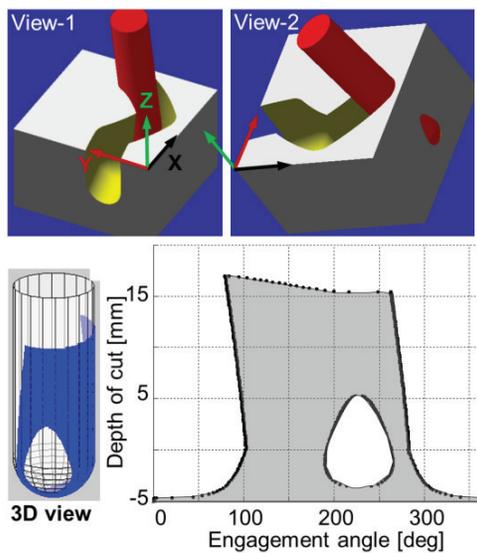


Fig. 8. The CWE surface for tapered ball-end mill tool.

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