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### Abstract

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# Fractionally-Spaced Statistical Equalizer for Fiber Nonlinearity Mitigation in Digital Coherent Optical Systems

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**Abstract:** We propose a fractionally-spaced equalizer with trained second-order statistics to deal with nonlinear impairment in coherent optical communications. The proposed 3-tap equalizer improves Q-factor by more than 2 dB for long-haul transmissions of 5,230 km.

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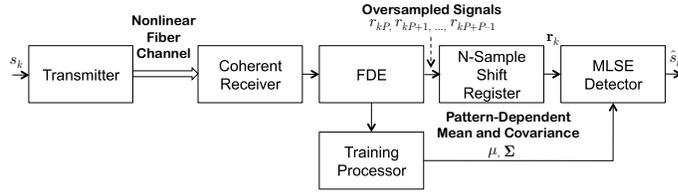
## 1. Introduction

Digital coherent optical transmissions have been used in practical systems to increase data rates with dual-polarized phase-shift keying and quadrature-amplitude modulation (DP-PSK/QAM). However, fiber nonlinearity can significantly degrade the advantage of coherent transmission systems over non-coherent transmission systems as such spectrally efficient modulation formats require higher signal power which causes higher nonlinear distortion [1]. Therefore, to mitigate fiber nonlinearity has been of great importance in optical communication research. Recently, it was shown that digital back-propagation (DBP) proposed in [2, 3] offers a substantial performance gain for nonlinear compensation. However, the DBP requires high-complexity processing using split-step Fourier methods (SSFM). Although there exist several reduced-complexity methods [4, 5], the overall performance is susceptible to the SSFM parameters. We focus on another method based on statistical sequence equalizers, studied in [6], which mitigates data-pattern dependent nonlinearity. The statistical sequence equalizer achieves good performance with low complexity for short-memory channels, and could be combined with other methods including DBP. In this paper, we extend the original sequence equalizer in several directions: i) we propose the use of trained second-order statistics (i.e., not only mean but also covariance), ii) we adopt a fractionally-spaced signal processing, and iii) we use an excess windowing for pattern matching. We obtain more than 2 dB improvement using the proposed equalizer for coherent optical communications with 40 Gbps non-return-to-zero (NRZ) dual-polarized quaternary PSK (DP-QPSK) or differential QPSK (DP-DQPSK) signals after 5,230 km transmissions. The achieved performance was better than that of the original DBP in low local dispersion channels, and was comparable to the DBP in high local dispersion channels.

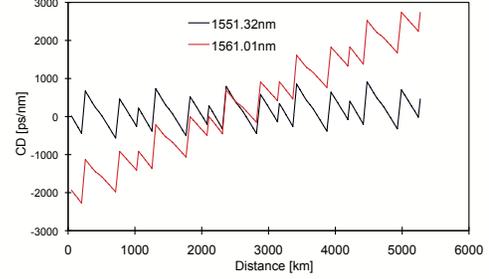
## 2. Fractionally-Spaced Statistical Sequence Equalizer

Fig. 1(a) shows the schematic of the proposed statistical sequence equalizer. As shown in [6], intra-channel nonlinear distortion highly depends on the transmitted data pattern. The statistical sequence equalizer first acquires such data-pattern-dependent distortion characteristics by averaging the received sequence with training data or on-line learning process. The trained mean signals are then used to decode by searching for the minimum Euclidean distance from the received sequence. In this paper, we propose the use of the second-order statistics (covariance) in addition to the first-order statistics (mean) to mitigate residual nonlinear noise. Furthermore, we introduce fractionally-spaced processing with expanded window size to improve the performance by exploiting the correlation over adjacent received samples.

Consider  $M$  consecutive transmission data,  $\mathbf{s}_k = [s_{k-\lfloor(M-1)/2\rfloor}, \dots, s_k, \dots, s_{k+\lfloor M/2\rfloor}]^T \in \mathbb{C}^M$ , where  $s_k$  denotes the transmission data at the  $k$ -th symbol. Here  $\lfloor \cdot \rfloor$ ,  $[\cdot]^T$ , and  $\mathbb{C}$  denote a floor function, a transpose operation, and a complex-number set, respectively. For  $M = 3$  and QPSK constellations, there exist  $4^M = 64$  possible data patterns (for one polarization). For each pattern, the oversampled receiving sequence is analyzed to obtain the empirical statistics over transmissions. We consider a window size of  $N$  samples centered around the target transmission data to build statistics. Let  $\mathbf{r}_k = [r_{kP-\lfloor(N-1)/2\rfloor}, \dots, r_{kP}, \dots, r_{kP+\lfloor N/2\rfloor}]^T \in \mathbb{C}^N$  be the  $k$ -th chunk of the oversampled receiving sequence within the window of length  $N$ , where  $P$  denotes the oversampling factor. We propose to use an excess window size ( $N > MP$ )



(a) Coherent system with fractionally-spaced statistical equalizer.



(b) Fiber channel dispersion maps.

Fig. 1. Coherent fiber-optic system and dispersion map.

to enhance performance. Letting  $\mathbf{s}$  be one of possible data patterns, the statistics, namely pattern-dependent mean vectors  $\boldsymbol{\mu}(\mathbf{s}) \in \mathbb{C}^N$  and pattern-dependent covariance matrices  $\boldsymbol{\Sigma}(\mathbf{s}) \in \mathbb{R}^{2N \times 2N}$ , are obtained as follows:

$$\boldsymbol{\mu}(\mathbf{s}) = \frac{1}{\mathcal{N}(\mathbf{s})} \sum_{j: \mathbf{s}_j = \mathbf{s}} \mathbf{r}_j, \quad \boldsymbol{\Sigma}(\mathbf{s}) = \frac{1}{\mathcal{N}(\mathbf{s}) - 1} \sum_{j: \mathbf{s}_j = \mathbf{s}} \begin{bmatrix} \Re[\mathbf{r}_j - \boldsymbol{\mu}(\mathbf{s})] \\ \Im[\mathbf{r}_j - \boldsymbol{\mu}(\mathbf{s})] \end{bmatrix} \begin{bmatrix} \Re[\mathbf{r}_j - \boldsymbol{\mu}(\mathbf{s})] \\ \Im[\mathbf{r}_j - \boldsymbol{\mu}(\mathbf{s})] \end{bmatrix}^T, \quad (1)$$

where  $\mathcal{N}(\mathbf{s})$  is the total number of occurrences that the data pattern  $\mathbf{s}$  appeared in the past. Here,  $\Re$ ,  $\Re[\cdot]$  and  $\Im[\cdot]$  denote a real-number set, the element-wise real part and imaginary part operations, respectively. With the statistics, the expected likelihood of the oversampling received signals  $\mathbf{r}_k$  inside the excess window given a data pattern  $\mathbf{s}$  is calculated as follows:

$$\Pr(\mathbf{r}_k | \mathbf{s}) = \frac{1}{\sqrt{\det[2\pi\boldsymbol{\Sigma}(\mathbf{s})]}} \exp\left(-\frac{1}{2} \begin{bmatrix} \Re[\mathbf{r}_k - \boldsymbol{\mu}(\mathbf{s})] \\ \Im[\mathbf{r}_k - \boldsymbol{\mu}(\mathbf{s})] \end{bmatrix}^T \left(\boldsymbol{\Sigma}(\mathbf{s})\right)^{-1} \begin{bmatrix} \Re[\mathbf{r}_k - \boldsymbol{\mu}(\mathbf{s})] \\ \Im[\mathbf{r}_k - \boldsymbol{\mu}(\mathbf{s})] \end{bmatrix}\right). \quad (2)$$

Note that it reduces to a simplified function of the Euclidean distance,  $\|\mathbf{r}_k - \boldsymbol{\mu}(\mathbf{s})\|$ , when no information of the covariance is available. The expected benefit of the second-order statistics is twofold: i) less-noisy samples are prioritized via diagonal variance information and ii) correlated nonlinear noise is effectively whitened via off-diagonal correlation information. Using the likelihood described above, the statistical equalizer employs the maximum-likelihood sequence estimation (MLSE) to detect the transmission data  $s_k$  through the Viterbi algorithm. Since the computational complexity of MLSE grows exponentially with the channel memory, more specifically  $\mathcal{O}[N4^M]$ , we may use a channel shortening equalizer including frequency-domain chromatic dispersion compensation or reduced-complexity DBP. We obtained higher-than 2 dBQ with a short memory MLSE using just  $M = 3$  taps, that can outperform the DBP.

### 3. Performance Evaluations

For simulations, we used the fiber link configuration corresponding to the experimental setup used in [4]. The channel under test is a 10GBaud NRZ QPSK or DQPSK signal with a center wavelength of 1551.32nm or 1561.01nm. Fig. 1(b) gives the dispersion maps of both channels. After pre-dispersion compensation the signal was propagated through 5 loops of 18 spans of non-zero dispersion shift fiber (NZ-DSF) and 3 spans of standard single-mode fiber (SSMF) with compensating erbium-doped fiber amplifiers (EDFAs) (5 dB noise figure), post-dispersion compensation and an optical filter (4th order Gaussian filter with a bandwidth of  $2.5 \times 10$  GHz). Coherent detection was performed using a hybrid mixer and balanced photo-detectors. The electric transmit filter used a 25ps rise-time Gaussian pulse, and receive filter used a 4th order Bessel filter with a cutoff of 75% of the symbol rate. After digitizing to 2 samples per symbol the residual dispersion was removed using a linear frequency-domain equalizer (FDE) and the  $P$ -times oversampling signal is fed into the statistical sequence equalizer. The loop length was 1,046 km. The Q-factor was calculated by error counting. We assumed no polarization mode dispersion (PMD) for simulations, and used a circular polarization basis so that two parallel equalizers for each polarization work individually.

Figs. 2(a) and 2(b) show the simulation results of Q-factor evaluations for DP-DQPSK in low local dispersion channels (1551.32nm wavelength) and high local dispersion channels (1561.01nm wavelength) respectively, after 5 loops (5,230 km). Comparing to 1-tap phase compensation method, one can see that a significant improvement of

higher than 2 dBQ was achieved by the proposed fractionally-spaced equalizer using  $M = 3$  taps,  $N = 9$  excess window, and  $P = 2$  oversampling. It should be noted that such a large gain was not provided when we use the conventional equalizer [6] with such a small number of taps, since the scheme does not exploit the second-order statistics and oversampling signals. Moreover, the proposed equalizer could outperform the original DBP (using 1 step per span, requiring 210 Fourier transform operations), in low local dispersion channels. Even for high local dispersion channels, the fractionally-spaced 3-tap equalizer achieved comparable performance in peak Q factor to the DBP. The analogous behavior is seen in Figs. 2(d) and 2(e) where DP-QPSK signals are used.

Figs. 2(c) and 2(f) show Q values versus the fiber distance for DP-DQPSK and DP-QPSK at a launch power of  $-7$  dBm, with incrementing the fiber distance further from 5,230 km to 10,460 km by 1,046 km loop each. It is observed that the proposed statistical sequence equalizer maintains 2 dBQ improvement even at 10,460 km for low local dispersion case, whereas the improvement is considerably reduced for high local dispersion case. It suggests that such a low tap equalizer shall work with other channel shortening methods for long-haul transmissions.

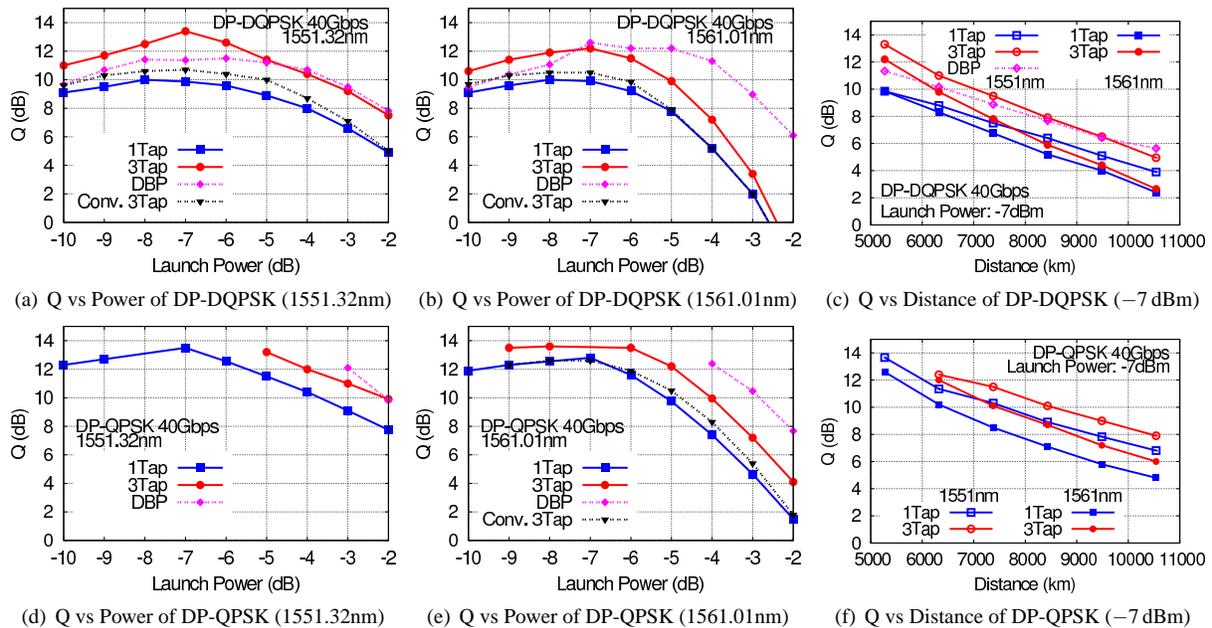


Fig. 2. Q-factor performance. “1Tap” denotes a phase compensation filter with no memory. “3Tap” denotes the proposed fractionally-spaced sequence equalizer with  $M = 3$  taps,  $N = 9$  excess window and  $P = 2$  oversampling. “DBP” denotes the original DBP with hundreds of SSFM iterations using manually optimized parameters. “Conv. 3Tap” denotes the conventional 3-tap statistical equalizer [6] without using the second-order statistics and oversampling.

#### 4. Summary

We proposed the fractionally-spaced sequence equalizer with the second-order statistics to mitigate nonlinear impairment that is dependent on transmission data pattern. A significant performance improvement of Q-factor of more than 2 dB was obtained for long-haul coherent fiber-optic communication systems over 5,230 km. It was verified that oversampling processing with higher-order statistics can provide a significant gain compared to the existing statistical equalizer. More importantly, a short-memory equalizer with just 3 taps could achieve better performance than the DBP in low local dispersion conditions.

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