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***Index Terms*— motion control; hybrid systems; reset control; PID control.**

I. Introduction

Hybrid control systems have been the subject of significant research interest due to their promise in enabling higher performance, versatility, and autonomy.

A specific topic within hybrid control of synthesis nature, which has received considerable attention in recent years is integrator resetting and more generally first order reset elements (FORE), see [3], [10] and references therein. The Clegg integrator and FORE elements, first developed in [5], [8], [9], are being revisited within the hybrid systems framework yielding many interesting formal hybrid analysis [3], [10]. This has allowed for a better understanding of reset control systems. These control algorithms are found to be very promising as there have been demonstrations that they can overcome fundamental limitations associated with linear feedback, such as overshoot and settling time bounds, see [3]. However, due to conservatism of existing sufficient conditions for stability of hybrid systems, recent papers have focused on constructive synthesis results and in depth understanding for resetting controllers for simple plants such as an integrator when combined with Clegg integrators and FORE's [3], [10]. Therefore, it is desired to extend the synthesis and design of reset controllers for more classes of problems where constructive results can be obtained.

This paper presents a new integral and lag resetting control technique for use with PID and lag control for first and second order dominant plants, which reduces to a Clegg integrator for integral control of a first order plant. The developed resetting controller is particularly different from existing designs for second order dominant systems, which is important for motion control. For such systems, Lyapunov stability analysis of the system are presented as well as different modifications to improve steady-state disturbance rejection are discussed.

The paper is organized as follows. Section II presents the proposed resetting PID control designs including different extensions in order to maintain constant disturbance rejection. Whereas, synthesis of lag and some class of higher order resetting controllers are discussed in Section III. Case study simulations and experiments for motion control are presented in Section IV. Conclusions and future work are given in Section V.

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II. Resetting PID Control

A hybrid resetting (impulsive) system is given by:

$$\begin{aligned} \dot{x} &= f_c(x) \quad \text{if } (t,x) \notin S_r \\ x^+ &= f_d(x), \quad \text{if } (t,x) \in S_r \end{aligned} \tag{1}$$

Where x is the state, the Lipschitz continuous function f_c describes the continuous-time dynamics and f_d describes the resetting law with S_r being the resetting set that defines the resetting condition based on time and/or state, see [13], [6] for more background and details. It is assumed that the set of resetting times t_k is well defined and distinct and thus the system above is non-zeno with well defined solutions.

Assumption 2.1: The resetting times t_k are well defined and distinct and $\exists\ \epsilon > 0$ such that $t_k - t_{k-1} \geq \epsilon$.

This can be achieved by imposing a sampling on the resetting condition, as would be in a practical implementation, or also in the temporal regularization method as done in [14]. Also note that the resetting conditions are represented by disjoint sets as done in [6]; see [14] for alternative representations and more formal discussions of such matters and its effect on existence and uniqueness of system solutions.

Let us consider the class of state-dependent reset control systems given by:

$$\begin{aligned} \dot{x} &= f_c(x) \quad \text{if } x \notin S_r \\ x^+ &= f_d(x), \quad \text{if } x \in S_r \end{aligned} \tag{2}$$

Which is a special case of the class given by Equation (1). Another assumption that is needed for invariance set type of statements for reset systems is the quasi-continuous dependence on initial conditions property, see [6] for background and details.

Assumption 2.2: The system given by Equation (2) is a left continuous dynamical system by means of the quasi-continuous dependence on initial conditions property [6].

First some main results in stability of reset systems are summarized and restated for completeness as they will be later used.

Theorem 1: Let $V(x)$ be a continuously differentiable radially unbounded positive definite function such that $V(0) = 0$ and

$$\begin{aligned} \dot{V}(x) &\leq 0 \qquad \text{if } x \notin S_r \\ \Delta V &= V(x^+) - V(x) \leq 0 \quad \text{if } x \in S_r \end{aligned}$$

Then under assumption 2.1 :
(i) The $x = 0$ solution of the hybrid system given by Equation (1) is globally Lyapunov stable.
(ii) The $x = 0$ solution of the hybrid system given by Equation (2) is globally asymptotically stable if assumption 2.2 is satisfied and in addition either

$$\dot{V}(x) < 0 \quad \text{if } x \notin S_r$$

or the set $S = \{x \notin S_r \dot{V} = 0\} \cup \{x \in S_r \Delta V = 0\}$ contains no invariant set other then the set $\{0\}$.
(iii) The $x = 0$ solution of the hybrid system given by Equation (1) is globally exponentially stable if V is quadratic and $\exists\, \alpha > 0$ such that $\dot{V} \leq -\alpha V$.

The proof of these statements can be found in [13], [6] and particularly part (ii), which is the extension of the invariant set theorem to resetting systems, can be found in [6]. Note that the statement of part (ii) applies only to state-dependent hybrid systems given by Equation (2) and following assumption 2.2 unlike parts (i) and (iii).

Consider the following class of plants consisting of a chain of integrators:

$$a\, y^{(n)} = f(y, \ldots, y^{(n-1)}) + u \tag{3}$$

Where $y^{(n)}$ is the n^{th} derivative of the targeted output y, where n is the order of the system. In this paper it is assumed that $n \leq 2$ since this is reasonable for the dominant dynamics of most practical control systems. Whereas, the known constant parameter $a > 0$ is the high frequency gain. It is assumed that signals $y, ..y^{(n-1)}$ are available, i.e. y for $n = 1$ and $y, \dot{y}$ for $n = 2$. This is typical for PID control even if only y is measured as $\dot{y}$ is usually obtained through some type of filtered differentiation in practice. Furthermore, the reference trajectory r and its first n derivatives $r^{(1)}, \ldots, r^{(n)}$ are known, bounded and, piecewise continuous. This means the above system is either a first order system:

$$a\, \dot{y} = f(y) + u$$

or a second order system:

$$a\, \ddot{y} = f(y, \dot{y}) + u$$

Define the following generalized error variable:

$$z = -(d/dt + K_{pp})^{n-1} e = y^{(n-1)} - z_r$$

Which is similar to that used in sliding mode control and some adaptive controllers, see for instance [11]. Where $K_{pp} > 0$ is a chosen scalar, $e = r - y$ is the tracking error for a desired reference r. Consider the following control law:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a\dot{z}_r - f(y, \ldots, y^{(n-1)}) \quad (4) \\ \dot{z}_c &= z, \quad \text{if } z z_c > 0 \\ z_c^+ &= 0, \quad \text{if } z z_c \leq 0 \end{aligned}$$

Where $K_{pv} > 0$ is a proportional gain, $K_{iv} > 0$ is an integral gain. Whereas, the "feedforward" signal, $\dot{z}_r$ is such that for $n = 1$, we have $\dot{z}_r = \dot{r}$, whereas $\dot{z}_r = \ddot{r} + K_{pp}\dot{e}$ for $n = 2$. Substituting Equation (4) into Equation (3) yields the following hybrid resetting closed loop system:

$$\begin{aligned} \dot{x}_{cl} &= A_c x_{cl}, \quad \text{if } z z_c > 0 \\ x_{cl}^+ &= A_d x_{cl}, \quad \text{if } z z_c \leq 0 \end{aligned} \tag{5}$$

Where $x_{cl} = [z, z_c]^T$ and

$$\begin{aligned} A_c &= \begin{bmatrix} -K_{pv}/a & -K_{iv}/a \\ 1 & 0 \end{bmatrix} \\ A_d &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Consider the following Lyapunov function:

$$V = a\, z^2 + K_{iv} z_c^2$$

Computing $\dot{V}$ for the continuous-time part of system (5) yields:

$$\dot{V} = -2K_{pv} z^2 \leq 0$$

Whereas, for the resetting dynamics of (5) :

$$\Delta V = V(x_{cl}^+) - V(x_{cl}) = -K_{iv} z_c^2 \leq 0$$

Using Theorem 1 this proves Lyapunov stability of the closed loop system (5). Furthermore, $(z = 0, z_c = 0)$ is the only invariant set within the union of the sets $\{x \notin S_r, \dot{V} = 0\}$ and $\{x \in S_r, \Delta V = 0\}$ and thus $z \to 0$ asymptotically and thus $e \to 0$ asymptotically if assumption 2.2 is satisfied.

Note that for $n = 1$ the resetting set S_r is defined by:

$$z z_c = e \int e\, dt \leq 0$$

Therefore, for $n = 1$ the integral resetting is identical to that of a Clegg integrator and the overall controller is PI controller, with feedforward.

Whereas, for $n = 2$ the resetting condition is given by:

$$z z_c = (\dot{e} + K_{pp} e) \int (\dot{e} + K_{pp} e)\, dt \leq 0$$

The above controller is simply a PID controller, with feedforward, in a cascade (series) realization with integrator resetting. Note that the resetting condition differs from that commonly used with integral resetting control. In fact, if the above PID controller where represented in a parallel realization and a Clegg integrator is used, as commonly done, see Equation (6), then the response will be very different, this will be demonstrated later.

$$\begin{aligned} u &= -K_P e + K_D \dot{e} + K_I e_I \\ \dot{e}_I &= e, \quad \text{if } e\, e_I > 0 \\ e_I^+ &= 0, \quad \text{if } e\, e_I \leq 0 \end{aligned} \tag{6}$$

Note that in order to verify the non-zeno behavior of assumption 2.1, by using the temporal regularization method [14] the resetting controller would be as follows :

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a\dot{z}_r - f(y,\ldots,y^{(n-1)}) \\ & \left.\begin{array}{l} \dot{z}_c = z \\ \dot{\tau} = 1 \end{array}\right\} \quad \text{if } z z_c > 0 \text{ or } \tau < \tau_{min} \\ & \left.\begin{array}{l} z_c^+ = 0 \\ \tau^+ = 0 \end{array}\right\} \quad \text{if } z z_c \leq 0 \text{ and } \tau \geq \tau_{min} \end{aligned}$$

Where τ_{min} is a chosen lower bound on resetting period. Since the stability of the system has been verified independent of the resetting speed or condition, then the temporal regularization or any other equivalent method can be added without any concern or need to re-analyze the system stability. In fact, a statement similar to that of Theorem 1 (i) still applies to the above system, see [13], [6].

A. Remarks

- Note that the assumption that both y and $\dot{y}$ are available is a prerequisite to PID control, even if only y is measured as $\dot{y}$ is usually obtained through some type of filtered differentiation in practice.
- The assumption that $f(y,\ldots,y^{(n-1)})$ and parameter a are known can be relaxed by applying standard parameter adaptive control if f is linearly parameterized, see [11], however, this is not the focus of the paper.

B. Constant Disturbance Rejection

Observe that if the system is given by

$$a\, y^{(n)} = f(y,\ldots,y^{(n-1)}) + u + d \tag{7}$$

Where d is a constant disturbance. It is well known that integral control can achieve zero steady-state rejection of constant disturbances. This is the case as the closed loop system with plant (7) and PID controller (4) without resetting admits the fixed point $(z, z_c) = (0, d/K_{iv})$. Moreover, this fixed point is stable and $z \to 0$ can be shown using standard invariance principle arguments.

Whereas, with resetting, the closed loop system with plant (7) and resetting controller (4) no longer admits such a fixed point, and a solution with $z = 0$ is no longer an equilibrium. Therefore, zero steady-state tracking in the presence of constant disturbances is no longer guaranteed if resetting persists. Possible solutions to this problem are discussed next.

1) Using a Resetting Offset b_d: This can be dealt with by resetting to a nonzero value using the offset term b_d with the following controller:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a\dot{z}_r - f \\ \dot{z}_c &= z, \quad \text{if } z z_c > 0 \\ z_c^+ &= b_d, \quad \text{if } z z_c \leq 0 \end{aligned} \tag{8}$$

Note that if $b_d = d/K_{iv}$ is chosen then the fixed point for the overall hybrid system, plant given by Equation (7) and controller given by Equation (8), is $(z, z_c) = (0, d/K_{iv})$. Stability of this fixed point can be shown using the modified Lyapunov function:

$$V = a\, z^2 + K_{iv}(z_c - d/K_{iv})^2$$

Computing $\dot{V}$ for the continuous-time part of system yields:

$$\dot{V} = -2K_{pv} z^2 \leq 0$$

Whereas, for the resetting dynamics :

$$\Delta V = V(x_{cl}^+) - V(x_{cl}) = -K_{iv} z_c^2 \leq 0$$

This proves Lyapunov stability of the closed loop system for the plant given by Equation (7) and controller given by Equation (8), see Theorem 1, and if assumption 2.2 is satisfied the system converges asymptotically to the unique invariant set within the union of the sets $\{x \notin S_r, \dot{V} = 0\}$ and $\{x \in S_r, \Delta V = 0\}$, which the fixed-point $(z, z_c) = (0, d/K_{iv})$, and thus the tracking error $e \to 0$ asymptotically. Thus zero steady-state tracking in the presence of the constant disturbance d is possible if $b_d = d/K_{iv}$.

However, it is not always possible to obtain an accurate estimate of the disturbance d, and thus a more robust method is needed. Therefore, another possibility is to combine the resetting pole or integrator with a standard nonresetting integrator, as done in [1], which is discussed next in the context of the generalized integrator resetting.

2) Combining a Resetting and a non-resetting Integrators: Consider the following control law:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv1} z_{c1} - K_{iv2} z_{c2} + a\dot{z}_r - f \\ \dot{z}_{c1} &= z, \quad \text{if } z z_{c1} > 0 \\ z_{c1}^+ &= 0, \quad \text{if } z z_{c1} \leq 0 \\ \dot{z}_{c2} &= z \end{aligned} \tag{9}$$

Where $K_{iv1} > 0$ is the resetting integrator gain corresponding to state z_{c1} and $K_{iv2} > 0$ is the nonresetting integrator gain corresponding to state z_{c2}.

Stability of this system can be analyzed with the following Lyapunov function:

$$V = a\, z^2 + K_{iv1} z_{c1}^2 + K_{iv2}(z_{c2} - d/K_{iv2})^2$$

Computing $\dot{V}$ for the continuous-time part of system yields:

$$\dot{V} = -2K_{pv} z^2 \leq 0$$

Whereas, for the resetting dynamics :

$$\Delta V = V(x_{cl}^+) - V(x_{cl}) = -K_{iv1} z_{c1}^2 \leq 0$$

This proves Lyapunov stability of the closed loop system for the plant given by Equation (7) and controller given by Equation (9), with the combined integrators method. Furthermore, if assumption 2.2 is satisfied the system converges asymptotically to the unique invariant set within the union of the sets $\{x \notin S_r, \dot{V} = 0\}$ and $\{x \in S_r, \Delta V = 0\}$, which is the fixed-point $(z, z_{c1}, z_{c2}) = (0, 0, d/K_{iv2})$, and thus the tracking error $e \rightarrow 0$ asymptotically, see Theorem 1. Therefore, zero steady-state rejection of constant disturbances is achieved. Note that if some partial information about the disturbance is known then using an offset term b_d to cancel part of the disturbance is possible in addition to using a nonresetting integrator. This approach is based on that in [1], [2] but utilizes the more general resetting structure developed in this paper.

3) Turning Off the Resetting: Another possible and simple strategy is to turn off the resetting towards the end of the command, when the system is settling to it's desired steady-state value. This can be expressed by:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a \dot{z}_r - f \quad (10) \\ \dot{z}_c &= z, \quad \text{if } z z_c > 0 \text{ or } t > t^* \\ z_c^+ &= 0, \quad \text{if } z z_c \leq 0 \text{ and } t \leq t^* \end{aligned}$$

Where $t*$ is a chosen time based on the reference trajectory and the expected system response time. This can be repeated with every subcommand in repetitive processes. This simple switching strategy allows for benefiting from transient improvements due to resetting while preserving standard integrator's steady-state disturbance rejection capability. A design trade-off in the choice of t^* is expected as turning the resetting off too early means less gains will be made out of using resetting, while delaying it too much degrades steady-state tracking. The stability of the system with this switching is easily verified by redefining the controller as:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a \dot{z}_r - f \quad (11) \\ \dot{z}_c &= z, \quad \text{if } z z_c > 0 \\ z_c^+ &= a_d(t) z_c, \quad \text{if } z z_c \leq 0 \end{aligned}$$

Where $a_d(t) \leq 1$ uniformly and is piecewise constant, which allows for infinitely countable distinct switches with well defined dwell time between switches to be performed. The resetting dynamics with this term now satisfies:

$$\Delta V = V(x_{cl}^+) - V(x_{cl}) = -K_{iv1}(a_d^2 - 1) z_c^2 \leq 0$$

Since $a_d(t) \leq 1$ uniformly. Note that this is no longer a state dependent autonomous system given by Equation (2) but Lyapunov stability as in Theorem 1 (i) is still preserved for linear hybrid systems of this form, see [13], [6]. It follows that this switching does not introduce any destabilizing effects, and the system remains Lyapunov stable, a more elaborate convergence statement for this case is not considered here.

Note that in the above case with a single switch we have:

$$a_d(t) = \begin{cases} 1 & \text{if } t > t^* \\ 0 & \text{if } t \leq t^* \end{cases}$$

Alternatively, turning the resetting off may be defined by the tracking error $|e| \leq \epsilon$ satisfying some bound, as long as some sampling is introduced to prevent chattering.

III. Hybrid Resetting Lag Compensators

In this , the resetting PI compensator used solely for $n = 1$ and within a PID for $n = 2$ of Section II will be modified to yield resetting lag compensators.

A. A Resetting Lag Controller

A resetting Lag controller can be achieved by simply replacing the resetting integrator of Section II with a FORE as shown next. However, a an additional modification to the FORE's resetting condition will be introduced. Consider the following control law:

$$\begin{aligned} u &= -K_{pv}\, z - K_{iv} z_c + a \dot{z}_r - f \quad (12) \\ \dot{z}_c &= z - a_c z_c, \quad \text{if } (z - a_c z_c) z_c > 0 \\ z_c^+ &= 0, \quad \text{if } (z - a_c z_c) z_c \leq 0 \end{aligned}$$

Where the controller pole $a_c \leq 0$. Note that the resetting condition depends on the FORE' pole a_c unlike that commonly used [3], [10] and reduces to the same on used for an integrator. Note that the same resetting logic used for the integrator $z z_c \leq 0$, which reduces to the standard FORE' reset logic when $n = 1$, may also be used. Stability of the system can be shown using the same plant of Equation (3) and the following Lyapunov function:

$$V = a\, z^2 + K_{iv} z_c^2$$

Computing $\dot{V}$ for the continuous-time part of system yields:

$$\dot{V} = -2K_{pv} z^2 - 2K_{iv} a_c z_c^2 < 0$$

Whereas, for the resetting dynamics :

$$\Delta V = V(x_{cl}^+) - V(x_{cl}) = -K_{iv} z_c^2 \leq 0$$

Following Theorem 1, this proves exponential stability of the closed loop hybrid system consisting of plant given by Equation (3) and controller given by Equation (12) since $\dot{V} \leq -\alpha V$ for some $\alpha > 0$.

Note that the modified resetting logic can be related to energy based reset control, which is a general methodology proposed by [12] for stabilization using controller state resetting. In [12], the basic idea is to reset the controller to values that cause it's emulated energy V_c to vanish whenever the controller's energy is about to decrease, i.e., will be sent back to the plant. Although the precise resetting condition logic is different in [12] and it is not specified for any particular controller, the same energy based interpretation can be used. Let $V_c = z_c^2$ then the proposed resetting condition corresponds to $\dot{V}_c = (z - a_c z_c) z_c \leq 0$. Note that the standard Clegg integrator falls under this methodology but not the FORE.

B. Higher Order Resetting Controller

For the plant given by Equation (3), consider the following control law:

$$
\begin{aligned}
u &= -K_{pv}\, z - \sum_{i}^{N} K_i z_{ci} + a \dot{z}_r - f \qquad (13) \\
\dot{z}_{ci} &= z - a_{ci} z_{ci}, \quad \text{if } (z - a_{ci} z_{ci}) z_{ci} > 0 \\
z_{ci}^{+} &= a_{di} z_{ci} + b_{di}, \quad \text{if } (z - a_{ci} z_{ci}) z_{ci} \leq 0
\end{aligned}
$$

Where the scalars $a_{ci} \leq 0$ and $|a_{di}| \leq 1$ for N controller states. Where $a_{ci} = 0$ corresponds to an integrator and $a_{ci} > 0$ corresponds to a FORE. Note that the case of $a_{dj} = 1$ and $b_{dj} = 0$ for the j^{th} controller state means this is a nonreset pole. The default for a_{di} and b_{di} is zero but nonzero values may be useful in some cases, for example for improved disturbance rejection, as shown in Section II.

Let $b_{di} = 0$, which can be added in a manner similar to that of Section III with specific disturbances, and consider the following Lyapunov function:

$$V = a\, z^2 + \sum_{i}^{N} K_i z_{ci}^2$$

Computing $\dot{V}$ for the continuous-time part of the system yields:

$$\dot{V} = -2K_{pv} z^2 - 2 \sum_{i}^{N} K_i a_{ci} z_{ci}^2 \leq 0$$

Whereas, for the resetting dynamics :

$$\Delta V = V(x_{cl}^{+}) - V(x_{cl}) = \sum_{i}^{N} K_i (a_{di}^2 - 1) z_{ci}^2 \leq 0$$

This proves Lyapunov stability of the closed loop system consisting of plant given by Equation (3) and controller given by Equation (13) as well as $e \rightarrow 0$ by invariant set arguments in Theorem 1 if assumption 2.2 is satisfied. Whereas, when $a_{ci} > 0\ \forall i$ then exponential stability of the hybrid system is concluded, see Theorem 1.

IV. Application to Motion Control

Next, the proposed resetting designs will be evaluated via simulations and experiments on a servo driven motion control system, see [15], [12] for instance for applications of FORE's and Clegg integrators to motion control problems.

A. Simulations

In this section a case study simulation will be used to demonstrate the developed controllers. Consider the following plant transfer function:

$$\frac{y(s)}{u(s)} = \frac{1e-4\left((\frac{s}{2\pi 60})^2 + \frac{0.12s}{2\pi 60} + 1 \right)}{s^2 \left((\frac{s}{2\pi 100})^2 + \frac{0.2s}{2\pi 100} + 1 \right) \left(\frac{s}{2\pi 8000} + 1 \right)}$$

This is a typical model for many motion control systems such as that shown in Figure 5 from which this model has been identified. The plant used for simulations is of order 5 and relative degree 3 with lightly damped poles and zeros. However, the system is a 2^{nd} order dominant system and thus the designs of Section II with $n = 2$ and $f = 0$ will be used, i.e., the plant is treated as a double integrator plant. In all the simulations shown, the system is required to follow a filtered step input, with a 400 Hz reference filter. Note that a feedforward gain $0.7a$ is used instead of exactly a for all these simulations, as it is more realistic. Note that ode15s was the chosen solver in SIMULINK as it is better suited to these discontinuous systems.

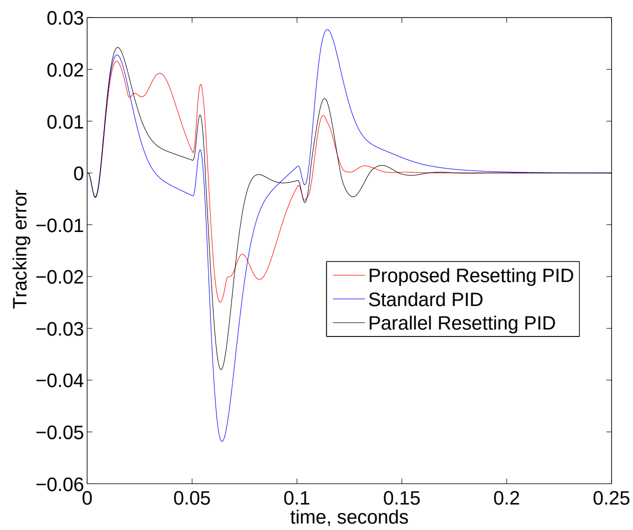

Fig. 1. Tracking error for series and parallel reset and non reset PID controllers.

Figure 1, a standard PID controller is compared to the proposed series resetting PID controller and a parallel resetting PID controller. Both resetting controllers seems to improve transients and settling time compared to the linear controller but the proposed resetting PID outperforms the parallel resetting PID, specially in terms of settling time.

In Figure 2with the addition of a constant input disturbance, steady state error is seen with reset PID although it improves transients. An offset used to reset to a nonzero value close to d/K_{iv} maintains zero steady-sate tracking error same while improving transients due to resetting. Another discussed method is to turn off the resetting at a prescribed time t^* is shown in Figure 2. Whereas, 2 integrator with and without resetting allows for achieving zero steady state tracking while trading off the level of improved transients with steady-state tracking, see Figure 4. All these methods shown for steady-state tracking improvement, as discussed in Section II impose different design trade-offs between retaining good steady-state tracking and maximizing the improvement obtainable from an ideal resetting controller

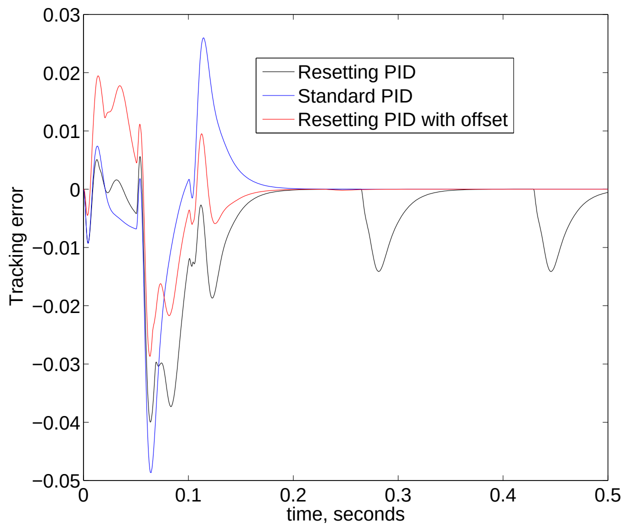

Fig. 2. Effect of constant disturbance on tracking error for different types of reset and non reset PID controllers; Resetting Offset.

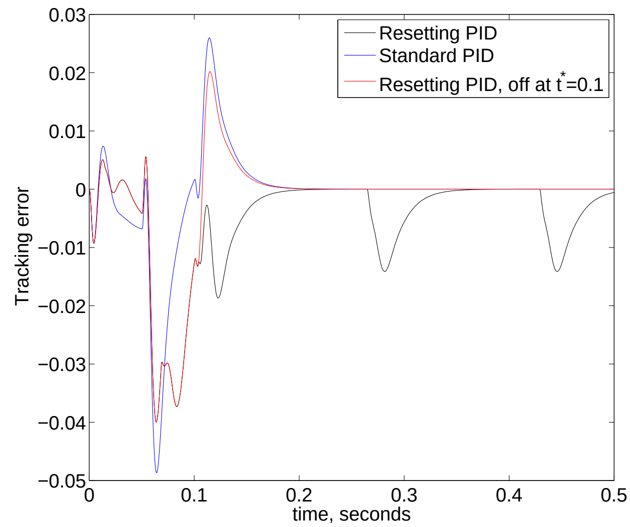

Fig. 3. Effect of constant disturbance on tracking error for different types of reset and non reset PID controllers; Turning the resetting off.

when no disturbances are present.

B. Experiments

In this section, the proposed resetting PID controller for $n = 2$ is tested on a typical motion control system using a servo motor, Kollmorgen AKM $21 - C$ [4], with collocated motor encoder feedback for positioning the carriage on the stage, by Thompson [4], se Figure 5. dSpace board DS 1104 is used for controller implementation. Only one axis positioning is used for this $X - Y$ table, where the bottom long stage is driven and the top stage and the carriage acts as a load to be positioned. Note that velocity feedback is used for the PID by filtered differentiation of the encoder position feedback as commonly done in practice.

The system is commanded to make $12mm$ moves at $0.5g$ acceleration. First consider the case where the PID controller is not properly tuned for the system, e.g. due to inertia uncertainty, leading to excessive vibration, see Figure 6. An interesting observation is the robustness of resetting in compensating for this poor PID tuning and suppressing the system oscillations.

Due to friction, one of the methods proposed in Section II should be used to maintain optimal steady-state tracking. The combined resetting and nonresetting integrators is used instead of only a reset integrator in Figure. In the experiments the nominal linear PID controller is given by $K_{iv2} = 3, K_{pv} = 0.03, K_{pp} = 100$ and $a = 3e-5$ are used. Figure shows positioning tracking error for the system, motor positioning error converted to micrometers using the lead of stage, using a combined PID with a resetting and a nonresetting integrator $K_{iv1} = K_{iv2} = 3$. This is compared with 2 linear PID's one with $K_{iv} = K_{iv1} = 3$, i.e., the resetting integrator is simply removed, and the other with $K_{iv} = K_{iv1} + K_{iv2} = 6$, i.e., resetting is turned off and the larger overall integral gain is retained. The improved settling is evident with resetting.

An alternative to using 2 integrators is to switch off the resetting towards the end of the move. In Figure 8, the resetting PID controller with gains $K_{iv} = 3, K_{pv} = 0.03, K_{pp} = 100$ is used which yields a steady-state offset of about 2 microns. In contrast turning off the resetting at $t* = 0.1$ seconds, retains most of the transient improvement benefits of resetting while eliminating the steady state offset error due to disturbances. The behavior of the integrator in these 2 cases is shown in Figure 8 where the integrator continues to be reset to zero preventing the system for perfect steady-state tracking. Whereas, when switching the resetting off the integrator follows the value or response needed to overcome friction. Of course different combinations of these methods, e.g. 2 integrators and turning off the resetting may also be used with satisfactory results. In summary, the experiments not only demonstrated interesting and promising performance of these hybrid controllers but also verified the predicted comparative behavior of different versions and the design trade-offs discussed earlier.

V. Conclusions

In this paper, new designs for hybrid resetting PID and lag controllers are shown. Lyapunov stability analysis of the system for first and second order plants are presented. The resetting designs reduce to that of a Clegg integrator for first order systems with integral control but differ from existing methods for second order plants and for non-integral lag controllers such as FORE's. Special attention is given to different modifications in order to retain a linear integrator's steady-state disturbance rejection are discussed.

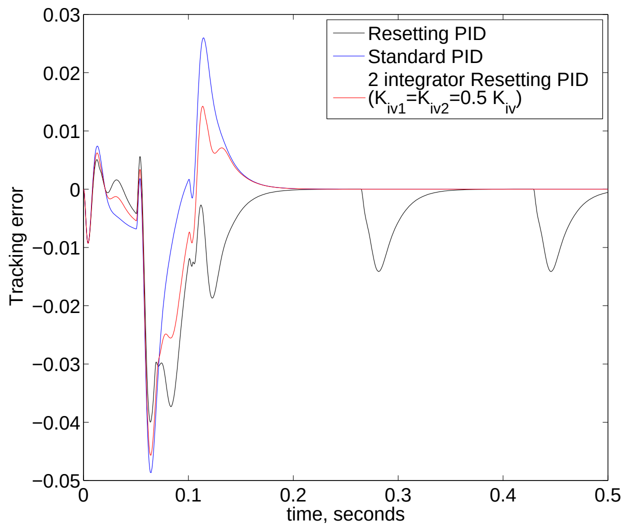

Fig. 4. Effect of constant disturbance on tracking error for different integral gain ratios for 2 integrator reset PID controller.

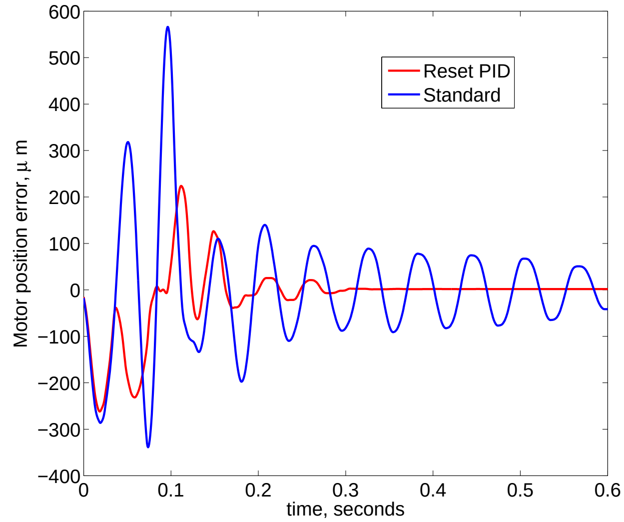

Fig. 6. Experimental positioning tracking error for an ill-tuned PID controller and it's resetting counterpart.

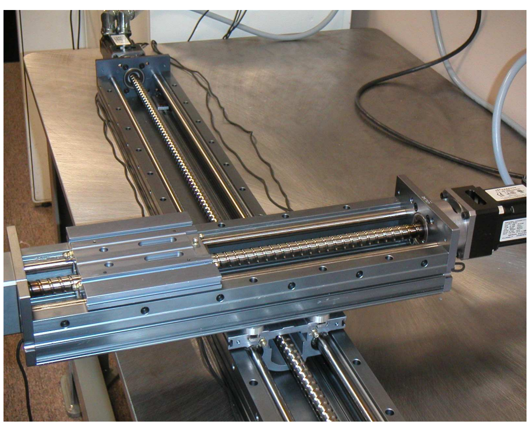

Fig. 5. Picture of the experimental setup.

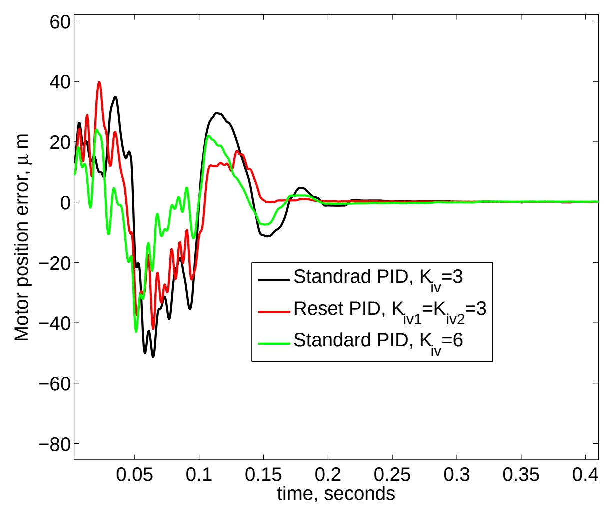

Fig. 7. Experimental positioning tracking error for 2 linear PID controllers and a resetting PID with 2 integrators.

Simulations and experiments on a motion control stage show the effectiveness of these hybrid controllers and verify the comparative behavior predicted by the analysis for different versions. Future work will focus on more general classes of systems and reset control algorithms.

References

[1] Banos, A., and A. Vidal. Design of PI+CI Reset Compensators for second order plants.*IEEE International Symposium on Industrial Electronics* , Vigo, Spain, 2007.

[2] Banos, A., and A. Vidal. Definition and tuning of a PI+CI reset controller.*Proc. European Control Conference* , Greece, 2007.

[3] Beker, O., Hollot, C., Chait, Y., and H. Han. Fundamental Properties Of Reset Control Systems, *IFAC World Congress*, Barcelona, Spain, 2002.

[4] *www.danahermotion.com.*

[5] Clegg,J. A nonlinear integrator for servomechanism,*Trans. AIEE*, 77, 41-42, 1958.

[6] Haddad, W.,Chellaboina, V.,and S. Nersesov. Impulsive and Hybrid Dynamical Systems: Stability, Dissipativity, and Control, *Princeton U Press*, 2006.

[7] Haddad, W.,Chellaboina, V., Hui, Q., and S. Nersesov. Energy and Entropy-Based Stabilization for Lossless Dynamical Syste,s via Hybrid Controllers. *IEEE Transactions On Automatic Control*, 52 (9), 1604-1614, 2007.

[8] Horowitz, I. and P. Rosenbaum. Nonlinear design for cost of feedback reduction in systems with large parameter uncertainty,*International Journal of Control*, 21, 977-1001, 1975.

[9] Krishman, K. and I. Horowitz. Synthesis of a nonlinear feedback system with significant plant-ignorance for prescribed system tolerance,*nternational Journal of Control*, 19, 689-706, 1974.

[10] Nesic, D., Teel, A.R., and L. Zaccarian. On necessary and sufficient conditions for exponential and L2 stability of planar reset

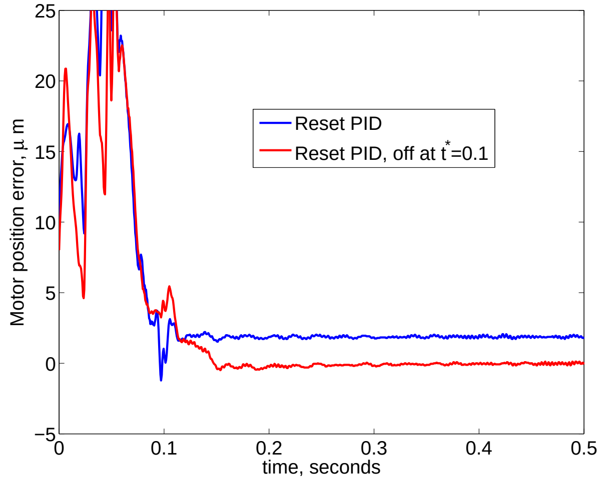

Fig. 8. Experimental positioning tracking error for a a resetting PID controller with and without resetting turned off at 0.1 seconds.

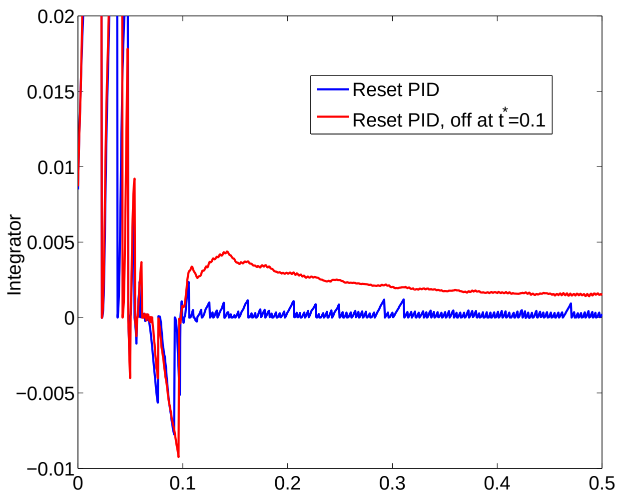

Fig. 9. Experimental integrator state for a resetting PID controller with and without resetting turned off at 0.1 seconds.

systems.*American Control Conference*, Seattle, WA, 2008.

[11] Slotine, J.J.E., and Li, W. Applied Nonlinear Control, *Prentice-Hall*, 1991.

[12] Wu, D., Guo, G., and Wang, Y. Reset Integral-Derivative Control for HDD servo systems. *IEEE Transactions on Control Systems Technology*, 15 (1), 161-167, 2007.

[13] Yang, T. Impulsive Systems and Control: Theory and Applications, *Nova Science*, 2001.

[14] Zaccarian, L., Nesic, D. and A. R. Teel. First order reset elements and the Clegg integrator revisited. *American Control Conference*, Portland, Oregon, 2005.

[15] Zheng, Y., Chait, Y., Hollot, C. V., Steinbuch, M., and Norg, M. Experimental demonstration of reset control design. *Control Engineering Practice*,8(2),113-120, 2000.