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A Graph-based Approach to Multi-Cell OFDMA Downlink Resource Allocation

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Abstract—A novel, practical and low-complexity multi-cell OFDMA downlink channel assignment method using a graph-based approach is proposed in this work. The inter-cell interference (ICI) information is obtained through inference from the diversity set of mobile stations (MSs) and presented in the form of an interference graph. The proposed downlink channel assignment method consists of two phases. The task of ICI reduction is mapped to the MAX \( k \)-CUT problem in graph theory and solved in the first phase. Then, channel assignment is conducted by taking into account instantaneous channel conditions in the second phase. State-of-the-art ICI management techniques such as ICI coordination (ICIC) and base station cooperation (BSC) are incorporated in our framework. Heuristic algorithms are proposed to solve both phases of the problem efficiently. Simulation is conducted to demonstrate the effectiveness of the proposed solution, where the SINR improvement can be as high as 4.5 dB. The proposed solution can be used in next generation cellular systems such as 3GPP Long Term Evolution (LTE) and IEEE 802.16m.

I. INTRODUCTION

The radio spectrum is a scarce resource in wireless communications. It is desirable to reuse the same spectrum of all cells for effective deployment of wireless cellular networks (i.e., with the frequency reuse factor equal to one). When mobile stations (MSs) in adjacent cells use the same spectrum, it will inevitably incur inter-cell interference (ICI). Actually, ICI has been shown to be the predominant factor that limits the performance of a wireless cellular network. Thus, it is important to develop a good radio resource allocation scheme that achieves effective spectrum sharing with ICI mitigation.

Resource allocation in an Orthogonal Frequency Division Multiple Access (OFDMA) system has been studied extensively for the single-cell case. Most of existing methods focus on the optimization of power [1] or throughput [2], [3] under the assumption that the same subchannel is used by a single MS to avoid intra-cell interference. Such a problem formulation leads to a combinatorial problem that is proved to be NP-hard. Some suboptimal solutions have been proposed in the literature (e.g., [1]–[3]) to solve this problem.

Another key assumption in single-cell resource allocation is the availability of the channel signal-to-noise ratio (SNR). That is, it can be estimated and fed back to the transmitter. Its counterpart in the multi-cell scenario, called the signal-to-interference-and-noise ratio (SINR), is however more difficult to obtain since the interference comes from multiple cells and depends on the distance, location, and occupied channel status of interferers (e.g., other MSs in the downlink). This results in mutual dependency of ICI and complicates the resource allocation problem. Thus, a multi-cell resource allocation scheme contingent upon global and perfect knowledge of SINR is practically infeasible.

Most existing work on multi-cell OFDMA resource allocation either assumes the availability of SINR or bases its study on explicit expressions of SINR. For example, Li and Liu [4] proposed a two-level resource allocation scheme, where the first level coordinates cells while the second level performs per-cell optimization. The first level is conducted based on perfect and predetermined knowledge of SINR for all MSs on all subchannels, which is difficult to obtain before actual channel assignment due to mutual ICI dependency. A similar approach was adopted in [5] with some special treatment on ICI. An SINR expression was derived in [6] for a two-cell scenario and the performance gain of coordinated subchannel allocation was demonstrated. However, the approach proposed in [6] and its analysis cannot be easily generalized from the two-cell case to the multi-cell case.

In this work, we propose a systematic approach for multi-cell resource allocation with ICI consideration. After a brief background review in Sec. II, we formulate the problem in Sec. III. Then, our algorithm is presented in Sec. IV. Our solution method consists of two phases: 1) a coarse-scale ICI management scheme and 2) a fine-scale channel-aware allocation scheme. In the first phase, ICI management is accomplished using a graph-based framework. In this phase, ICI coordination (ICIC) [6], base station cooperation (BSC) [7] and Space Division Multiple Access (SDMA) [8] techniques are all incorporated, and no precise SINR information is required. In the second phase, channel assignment is conducted by considering instantaneous channel conditions. Finally, the low computational complexity and high SINR performance of the proposed scheme are demonstrated by computer simulation in Sec. V.

II. BACKGROUND REVIEW AND SOLUTION FRAMEWORK

A. System Description

We consider a downlink cellular system with \( L \) base stations (BSs), each with \( N_T \) antennas, and a total of \( M \) MSs, each with \( N_R \) antennas, distributed in \( L \) cells. It is assumed that
there are $N$ subchannels and the frequency reuse factor is equal to one. The downlink signal for MS $m$ is sent with power $P_m$, depending on its proximity to the BS. Specifically, we have

$$P_m = \begin{cases} P_0, & \text{if MS } m \text{ is in cell center}, \\ P_1, & \text{if MS } m \text{ is in cell edge}, \end{cases}$$

and $P_0 < P_1$. The boundary that separates the cell center and the cell edge is a design parameter. The transmitted signal then undergoes slow fading (due to path loss) as well as fast fading (due to the Rayleigh fading) before it reaches the target MS. Let $\varphi_{m}^{(l)}$ be the path loss attenuation factor from BS $l$ to MS $m$, and $\beta_{mn}$ the fast fading channel power in subchannel $n$, from BS $l$ to MS $m$. Thus, the received signal power at MS $m$ from BS $l$ in subchannel $n$ is given by $P_m\beta_{mn}\varphi_{m}^{(l)}$.

Typically, each MS is registered at and communicates with one BS, which is called the anchor (or serving) BS. However, in some scenarios (e.g., the handover process), simultaneous communication with more than one BS may take place. A diversity set has been defined in the 802.16e standard to serve this purpose. It keeps track of the anchor BS and neighboring BSs potentially within the communication range. This information is maintained at the MS as well as the BS. The diversity set of MS $m$ is given by $D_m = A_m \cup B_m$, where $A_m$ is the anchor BS set which has only one element (i.e. anchor BS $A_m$) and $B_m$ is the neighbor BS set that may have zero, one or multiple BSs as its elements depend on the geographic location of MS $m$ in relation to its neighboring BSs.

The signal-to-interference-and-noise ratio (SINR) is used to evaluate the performance of a multi-cell wireless cellular network. It is a more accurate measure than SNR in interference-limited cellular networks. In the downlink scenario, the SINR (in the linear scale) of the received signal at MS $m$ using subchannel $n$ is given by

$$\text{SINR}_{mn} = \frac{P_m\beta_{mn}\varphi_{m}^{(A_m)}\varphi_{m}^{(A_m)}}{\sum_{v \in I_m} P_v\beta_{mn}\varphi_{m}^{(A_v)} + N_0W},$$

where $I_m$ is the set of interfering MSs, $N_0$ is the thermal noise density, and $W$ is the transmission bandwidth.

Since ICI dominates the system performance, proper ICI management is needed. In the following, we consider two possible solutions.

### B. Inter-cell Interference Coordination (ICIC)

Inter-cell interference coordination (ICIC) was proposed in [6], [9] to effectively reduce ICI in cell-edge regions. It is achieved by allocating disjoint channel resources to cell-edge MSs that belong to different cells. Since cell-edge MSs are most prone to high ICI, the overall ICI can be reduced by judicious coordination of channel allocation between cell-edge MSs. Specifically, ICIC reduces the size of $I_m$ and/or the “damage” of each interferer, as reflected by the term $P_v\beta_{mn}\varphi_{m}^{(A_v)}$, in the denominator of (2). The latter can be achieved by, for instance, allocating the same resource to MSs that are geographically further apart so that the interference is mitigated due to the increased path loss.

However, ICIC solely based on cell-edge collision avoidance offers a small amount of performance gain in the downlink scenario since it overlooks interference caused by transmission from the BS to cell-center MSs [9]. This motivates us to consider a holistic channel assignment scheme where all MSs, cell-center and cell-edge alike, are taken into consideration.

### C. Base Station Cooperation (BSC)

Base station cooperation (BSC) proposed in [7] allows multiple BSs to transmit signals to multiple MSs concurrently sharing the same resource. It is specifically used in cell-edge MSs that are within the transmission ranges of multiple BSs. Cooperative BSs can use the Space Division Multiple Access (SDMA) technique to send signals to MSs. The idea of BSC can be well explained by an example as shown in Fig. 1, where we give an illustrative scenario consisting of 2 BSs and 2 MSs. MS 1 and MS 2 are communicating with BS 1 and BS 2, respectively. The downlink signal to MS 1 causes interference at MS 2, and vice versa. With BSC, BS 1 and BS 2 transmit signals jointly to both MSs and the interfering signal becomes part of the useful signal. Thus, BSC has two advantages: provision of more spatial diversity and ICI reduction. If intra-cell SDMA (SDMA for short) is employed, which allows a BS to transmit to multiple MSs that it serves using the same resource by choosing proper precoding matrices, BSC may be integrated with SDMA to result in even higher spectral efficiency.

The SINR expression for the BSC scheme involves an additional term as compared to (2). For simplicity, we do not consider SDMA in the SINR expression. We also assume that the transmitting power of each cooperating base station is equally split among MSs involved in the cooperation, which can be achieved by a proper design of precoding matrices. Let $C_m$ be the set of other MSs that engage in BSC with MS $m$. Then, the received SINR (in the linear scale) at MS $m$ using subchannel $n$ is in the form of

$$\text{SINR}_{mn} = \frac{1}{1+|C_m|} \left( P_m\beta_{mn}\varphi_{m}^{(A_m)} + \sum_{u \in C_m} P_u\beta_{mn}\varphi_{m}^{(A_u)} \right) + N_0W,$$

where $|C_m|$ is the cardinality of the set $C_m$, and $I_m^\prime$ is the set of interfering MSs for MS $m$. More specifically, the downlink transmission from the corresponding serving BS to the MS in the set $I_m^\prime$ will cause interference to MS $m$.

### D. Proposed Solution Framework

The channel assignment problem in cellular and mesh networks has been studied for decades in the context of graph multi-coloring problem (see, e.g., [10], [11]). In the traditional formulation, each node in the graph corresponds to a BS or an access point (AP) to which channels are assigned. The edge connecting two nodes represents the potential co-channel interference in between, which typically corresponds
III. PROBLEM FORMULATION

We decompose the problem into two phases. First, ICI is managed at a coarse scale using a graph-assisted approach. Then, the instantaneous channel information is exploited at a fine scale.

A. First Phase: ICI Reduction

Consider an illustrative example with 3 BSs and 5 MSs as shown in Fig. 2. We can infer the interference intensity from MS’s geographic location and construct a corresponding interference graph in Fig. 3. In this graph, which is denoted by $G = (V, E)$, each node (from set $V$) represents an MS and each edge (from set $E$) contains a “cost” or weight that characterizes the potential interference between two MSs. The weight between node $i$ and node $j$, $i < j$, is denoted by $w_{ij}$. The higher the value of $w_{ij}$, the stronger the potential interference between MSs $i$ and $j$.

There is a close relationship between the ICI-minimizing channel assignment problem and the MAX $k$-CUT problem [12] in graph theory. That is, given $N$ subchannels and $M$ MSs, a good solution to the former can be obtained by considering the solution to the latter.

$P1$: Given graph $G = (V, E)$ with $M$ nodes and edge weight $w_{ab}$ for each edge ($a, b$), find a partition of the graph into $N$ ($N \geq 2$) disjoint clusters $R_i$, $i = 1, \ldots, N$, such that $\bigcup_{i=1}^{N} R_i = V$ and $\sum_{a \in R_i, b \in R_j, i < j} w_{ab}$ is maximized.

Here, each cluster corresponds to a subchannel. Nodes (or MSs) in the same cluster will be assigned the same subchannel. Since the goal of the problem $P1$ aims to maximize the inter-cluster edge weight, the result will tend to separate strong interferers into different subchannels, thus achieving ICI reduction.

B. Second Phase: SNR Maximization

After the first-phase assignment, MSs are grouped into $N$ clusters for subchannel allocation. In the second phase, we should decide which subchannel to allocate to which cluster. Among $N$ possible subchannel assignment choices, the second-phase assignment aims to find the one that best leverages the instantaneous channel quality. The problem is formulated as follows.

$P2$: Let $Y = \{y_{mn}\}$ be the channel assignment matrix where entry $y_{mn}$ is equal to one if subchannel $n$ is assigned to MS $m$ and zero, otherwise. Let $J$ be the set formed by $N$! legitimate subchannel assignment choices after the first phase. Our goal is to find an assignment matrix $Y_{opt}$ such that

$$Y_{opt} = \arg\max_{Y \in J} \sum_{m} \sum_{n} \log_2(1 + \text{SNR}_{mn} y_{mn}),$$

where $\text{SNR}_{mn}$ is the instantaneous channel quality between MS $m$ and its anchor BS on subchannel $n$, which is proportional to $\beta_{mn}^2$. Note that, since ICI is dealt with in the first phase, the second phase considers SNR only, which is much easier to obtain than SINR.

C. Edge Weight Construction for $P1$

In this subsection, we propose a method to construct the interference graph without accurate SINR measurements since the measurement of SINR can be difficult in practice. The
The basic idea is to determine the weight associated with edge \((a,b)\) with the diversity set maintained at MSs \(a\) and \(b\).

The diversity set contains useful geographical information that is related to interference between MSs. To give an example, the diversity set for the scenario in Fig. 2 is given in Table I, where each row indicates the diversity set maintained at the corresponding MS. Each MS has an anchor BS and possibly several neighbor BSs if it is located at the cell edge. We can infer interference intensity between any two MSs from the table as discussed below. First, since MS 2 and MS 4 have the same anchor BS, they are within the same cell and have intra-cell interference to each other, unless they perform SDMA. Second, MS 1 and MS 4 may have ICI with each other, although ICI will exist between MS 3 and MS 4, BSC cannot possibly exist between them. Third, BSs are in each other’s neighbor BS sets, indicating that the interfering (or cooperating) signal can reach each other. Finally, since MS 2 and MS 4 have intra-cell interference to each other, unless they perform BSC.

Table I, where each row indicates the diversity set maintained at MSs, is related to interference between MSs. To give an example, Table I can be used to determine the weight associated with edge \((a,b)\) with the diversity set maintained at MSs \(a\) and \(b\).

### Table I: The Diversity Set of MSs in Fig. 2.

<table>
<thead>
<tr>
<th>Anchor BS, (A_m)</th>
<th>Neighbor BSs, (B_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS 1 (A_1 = {1})</td>
<td>(B_1 = {3})</td>
</tr>
<tr>
<td>MS 2 (A_2 = {3})</td>
<td>(B_2 = \phi)</td>
</tr>
<tr>
<td>MS 3 (A_3 = {2})</td>
<td>(B_3 = {3})</td>
</tr>
<tr>
<td>MS 4 (A_4 = {3})</td>
<td>(B_4 = {1})</td>
</tr>
<tr>
<td>MS 5 (A_5 = {2})</td>
<td>(B_5 = {1,3})</td>
</tr>
</tbody>
</table>

There are seven possible weight values between any two nodes,

\[ w_B, w_S, w_N, w_0, w_1, w_2, w_A, \]

where \(w_B\), \(w_S\), \(w_N\) and \(w_A\) correspond to weights associated with BSC, SDMA, no-interference, and intra-cell interference, respectively, and \(w_0, w_1, w_2\) are ICI weights at various levels depending on the geographical location of the two MSs. That is, the mutual ICI is the weakest if both MSs are in the cell center (denoted by \(w_0\)), medium if one MS is in the cell edge and the other in the cell center (denoted by \(w_1\)), and strongest if both MSs are in the cell edge (denoted by \(w_2\)). Overall, the seven weight values can be ranked as

\[ w_B \approx w_S \ll w_N < w_0 < w_1 < w_2 \ll w_A. \]

Note that \(w_B\) and \(w_S\) are the smallest since they demand that MSs use the same subchannel and \(w_A\) is the largest since we would like to avoid the intra-cell interference. The complete algorithm to determine the edge weight is summarized in Table II.

![Fig. 4: The interference graph for the scenario given in Fig. 2.](image)

SDMA are optional functionality which may be incorporated when this is physically feasible.

### IV. PROPOSED ALGORITHMS

#### A. Heuristic Algorithm for P1

The optimal solution for P1 is computationally prohibitive for large graphs (i.e., a large number of MSs). Thus, a suboptimal heuristic algorithm is presented instead. We adopt the simple heuristic algorithm in [12] to solve Problem P1. It can be proved that it achieves an absolute ratio of \((1 - 1/k)\) for a general MAX \(k\)-CUT problem. That is, the algorithm achieves a clustering in which the inter-cluster weight sum is at least \((1 - 1/k)\) times of the largest possible solution. The idea of the algorithm is to iteratively assign nodes to the cluster such that the increased intra-cluster weight is minimized. The detailed description of the algorithm is given in Table III for a practical scenario with \(M > N\). If \(M \leq N\), the algorithm terminates at Step 2 with the optimal solution. This heuristic algorithm is of complexity \(O(M^2/2 + M/2 + N)\).

#### B. Heuristic Algorithm for P2

Exhaustive search through all \(N!\) choices to solve Problem P2 is also computationally infeasible. We propose a heuristic suboptimal algorithm that iteratively assigns subchannels to
TABLE III: A Heuristic Algorithm to Solve Problem P1.

Initialize: Let $W_i = \sum_{w_i \in R_i} w_{iw}$ be the weight of cluster $R_i$, $i = 1, \ldots, N$, $W_i = 0$, $i = 1, \ldots, N$.

1. Arbitrarily order $M$ nodes.
2. Assign the first $N$ nodes to $N$ clusters, one each cluster.
3. For the rest of node, take one at a time. For node $m$, let $W_m$ be the increased weight to cluster $R_i$ if node $m$ is assigned to cluster $R_i$.
4. Collect $W_i$ for $i = 1, \ldots, N$.
5. Assign node $m$ to cluster $R_i$, where $i^* = \arg \min_i W_i$. If there is more than one minimum, break the tie randomly.
6. Update the weight of cluster $R_i$ to $W_i + W_m$.
7. Repeat Steps 3–5 for all nodes.

TABLE IV: A Heuristic Algorithm to Solve Problem P2.

Initialize: Let $\Phi = \{1, \ldots, N\}$ be the subchannel pool. Order $N$ clusters in size from the smallest to the largest (break the tie arbitrarily).

1. Examine clusters in order, one at a time. For cluster $R_i$, calculate $T_n = \sum_{w_i \in R_i} \log_2(1 + \text{SNR}_{i,n})$ for all $n \in \Phi$.
2. Assign subchannel $n^*$ to cluster $R_i$, where $n^* = \arg \max_n T_n$.
3. Update subchannel pool to exclude $n^*$, i.e., $\Phi = \Phi \setminus \{n^*\}$.
4. Assign node $a_i$ to cluster $R_i^*$ according to the weight.
5. Update the weight of cluster $R_i$ to $T_n + W_{i,n}$.
6. Repeat Steps 1–3 for all clusters.

clusters as described in Table IV. We call this method max-SNR channel assignment and it is of complexity $O(N^2)$.

An alternative method, called random channel assignment, can also be used here to solve the second-phase problem. In this method, one assignment out of $N!$ choices is randomly picked as the solution. The complexity of this random assignment method is $O(1)$.

V. SIMULATION RESULTS

In this section, we study the performance of the proposed schemes by computer simulation. The simulation setup follows closely the suggestion given for the IEEE 802.16m evaluation [13]. It is summarized in Table V.

Five schemes to be tested are shown in Table VI. ICI-blind is the traditional OFDMA scheme where no ICI-aware mechanism is employed; i.e., each cell performs its own channel allocation independently without coordination. The rest are our proposed schemes, which differ in the ICI management mechanism in the first phase (ICIC or ICIC+BSC) and in the second phase (random or max-SNR assignments). SDMA is not employed in all simulation results but can easily be included. The graph edge weights are chosen to be $(w_B, w_N, w_0, w_1, w_2, w_A) = (-10^5, 0, 50, 100, 200, 10^5)$.

It is worthwhile to note that the performance of our proposed graph-based scheme is not sensitive to the chosen weight values, which is another highly desirable feature of this solution approach. Indeed, as revealed by the simulation, a small variation in the weight does not change the final channel assignment decision yielded by the proposed algorithm.

Figs. 5–7 show the cumulative distribution function (CDF) of SINR for five test schemes under different traffic load conditions (with 25, 15 and 5 uniformly distributed MSs per cell, respectively). It is evident that both ICIC and BSC schemes have a remarkable improvement on the SINR performance as compared to the ICI-blind scheme. This demonstrates the effectiveness of proposed ICI-reduction schemes. We also see a higher additional gain of ICIC2 (BSC2) compared to ICIC1 (BSC1) in lower load conditions. This is because interference dominates in higher load conditions, and the channel-aware resource assignment makes a diminishing impact in the second phase. Besides, due to fewer interferers, the average SINR increases for all schemes as the traffic load decreases as shown in Figs. 5–7.

The average SINR gains of proposed schemes with respect to the ICI-blind scheme under various traffic loads are compared in Fig. 8. As discussed previously, we see a more significant gain of ICIC2 and BSC2 in low load situations. Furthermore, the ICIC gain drops significantly in very high load situations. This is because the inevitable channel collision in the presence of a large number of MSs has rendered the ICIC strategy ineffective, if still feasible at all. In contrast, BSC retains the gain of about 4.5 dB as traffic load increases, and experiences only minor degradation in very high load situations. This is explained by the fact that, while a high load creates a high interference environment, it also creates more BSC opportunities. As BSC can only be established among MSs that are “geographically fitting”, a higher load increases the number of MSs that can be engaged in BSC, and consequently the number of actual events of BSC. This
Finally, we compare the complexity of proposed algorithms with an existing solution for multi-cell OFDMA resource allocation given in [4]. The ICI-aware allocation in [4] has a complexity of $O(M \times N \times L)$ while ours is proportional to $O(M^2/2 + M/2 + N)$ (see the discussion in Sec. IV-A). For example, for a heavy load scenario with $M = 19 \times 25$, our scheme is 2 times more efficient than that in [4]. For the light load scenario with $M = 19 \times 5$, our scheme is 10 times more efficient than that in [4]. Note that our proposed ICIC and BSC schemes have exactly the same complexity.

Fig. 5: The SINR distribution for a heavy traffic load (with 25 MSs per cell).

Fig. 6: The SINR distribution for a medium traffic load (with 15 MSs per cell).

Fig. 7: The SINR distribution for a light traffic load (with 5 MSs per cell).

Fig. 8: The average SINR gains with respect to the ICI-blind scheme under different traffic loads.

REFERENCES


