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***Index Terms*—Cyclic prefix (CP), intercarrier interference (ICI), interference cancellation, intersymbol interference (ISI), iterative algorithms, orthogonal frequency-division multiplexing (OFDM).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) transmits data simultaneously on a number of subcarriers, which, together, form OFDM symbols [1]–[4]. The frequencies of the subcarriers are chosen in such a way that the signals on different subcarriers are orthogonal to each other, even though their spectra overlap. This principle causes an OFDM symbol to be much longer than a data symbol in a single-carrier system (for equal data rates) and, thus, makes the system more robust to the delay dispersion of the channel. An efficient implementation of OFDM [with or without a "cyclic prefix" (CP)] can be obtained by a blockwise inverse fast Fourier transform (IFFT) of the input signal. For these reasons, OFDM is particularly suitable for high-data-rate transmission and has received great attention for wireless LANs (IEEE 802.11a and g standard [5]), as well as wireless personal area networks (multiband-OFDM standard ECMA-368 [6]) and fixed wireless access (IEEE 802.16, WiMax [7]). In order to avoid intersymbol interference (ISI) and intercarrier interference (ICI), the CP, i.e., a copy of the last part of the OFDM symbol, must be prepended to the transmitted symbol; alternatively, a zero-suffix can be used [8], [9]. The duration of this prefix has to be at least as large as the maximum excess delay of the channel in order to retain orthogonality between the subcarriers; furthermore, it is required that the channel does not change during the transmission of one OFDM symbol. If those conditions are fulfilled, then the receiver can compensate for the channel distortions by a one-tap equalizer in the frequency domain.

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In many practical systems, the orthogonality between the subcarriers is destroyed by two effects.

1) *Temporal variations of the channel*: Those variations lead to ICI (loss of orthogonality between the subcarriers) due to the Doppler shift associated with the temporal variations. Temporal variations of the channel are unavoidable in wireless systems—even if both the transmitter and the receiver are stationary, moving scatterers lead to Doppler shifts of the incoming waves. An important issue in that context is the correct estimation of the channel at any given time; this issue is discussed, e.g., in [10]–[12] and is considered to be perfectly solved for the remainder of this paper. Similarly, oscillator frequency offset and frequency drift can lead to ICI; also, this problem is assumed to be solved for the purpose of this paper (see, e.g., [13] and [14]).
2) *Insufficient length of the CP*: If the CP is shorter than the maximum excess delay of the channel, ISI occurs, i.e., each OFDM symbol affects the subsequent symbol; this effect can be eliminated by decision feedback [15], [16] and will be disregarded henceforth. Furthermore, the insufficient CP also leads to ICI, as the original symbols cannot be reconstructed by means of a one-tap equalizer alone. An insufficient duration of the CP can arise for various reasons. A system might consciously shorten or omit the CP in order to improve the spectral efficiency. In other cases, a system was originally designed to operate in a certain class of environments (and, thus, a certain range of excess delays) and is later on deployed also in other environments that show a larger excess delay. Finally, for many systems, the length of the CP is a compromise between the desire to eliminate ISI and to retain spectral efficiency—in other words, a CP should not be chosen to cope with the worst-case channel situation, as this would decrease the spectral efficiency in the typical-case situation.

Irrespective of the reasons, ICI (and ISI) can lead to a considerable performance degradation in many systems and must be combated. A large range of techniques has been developed in recent years and can be classified as follows.

1) *Optimum choice of the carrier spacing and OFDM symbol length*: By trading off ICI caused by temporal variations and by delay dispersion, the bit error rate (BER) can be minimized without requiring any structural changes in either the transmitter or receiver [17]. In a related approach, the basic pulseshape for the signaling of the subcarriers can also be modified [18], [19].
2) *Self-interference-cancellation techniques*: In this approach, the information is modulated not just onto a single subcarrier but onto a group of them, which leads to a strong reduction in the self-interference. The technique was suggested in [20] and later extended in [21]–[23]. This technique is very effective for the mitigation of ICI but leads to a reduction of the spectral efficiency of the system.
3) *Temporal equalizers*: In these techniques, the equalization is done in the time domain, before the FFT at the receiver (with a possible feedback that undergoes back-and-forth transformations) [24], [25]. Reference [26] found the coefficients for a temporal filter that maximizes the SINR for both conventional OFDM and multiple-input–multiple-output (MIMO)-OFDM. An efficient scheme called RISIC, based on tail cancellation and cyclic reconstruction, was first suggested in the study in [15] and improved in the study in [27]. In a similar approach, channel shortening can be used to make the effective impulse response shorter than the CP [28].
4) *Techniques that depend on the presence of unused subcarriers (null carriers)*: In this approach, redundancy is placed in the frequency domain by having more carriers than is strictly necessary. Similarly to the principle of the CP, which allows elimination of the ISI by redundancy in the time domain, the redundancy in the frequency domain allows elimination of the ICI [29]–[31].
5) *Forward-error correction* [32]: The capability of any forward-error-correcting code can be used to eliminate the errors caused by the ICI, and correlative coding [33] can improve the ICI as well.

An area of particular interest has been linear equalization in the frequency domain. The impact of the channel can be described by a transfer-function matrix, which is inverted explicitly or implicitly. Equalization of ICI due to temporal variations of the channels has been studied in [11], [12], and [34]–[39]; most of the equalization strategies suggested in these papers rely on the special structure of the transfer-function matrix in the case of pure Doppler-induced ICI (i.e., with sufficient guard interval). Reference [38] also provides a matched-filter bound for time-varying frequency-selective channels for the case that the CP is sufficiently long. Reference [40] showed that the elimination of ISI due to time variations is dual to the temporal equalization in single-carrier systems. Equalization of ICI due to insufficient guard intervals, possibly in combination with decision-feedback equalization (DFE), was studied in [16]; moreover, various algorithms have been considered for the matrix inversion. Direct inversion of the channel matrix [either according to a zero-forcing or an minimum mean-squared error (MMSE) criterion] is computationally too expensive. For wireline communications, Bogucka and Wesolowski [41] advocated the use of the LMS algorithm to find the coefficients in the frequency domain; however, this approach is not feasible in time-variant wireless channels.

In this paper, we therefore suggest an iterative inversion technique for the general case of time-varying delay-dispersive channels with insufficient CP. We also present a serial-interference-cancellation (SIC), as well as a parallel-interference-cancellation (PIC) scheme that shows rapid convergence and lower error floor for some ranges of delay spread. Since 2000, when we first suggested these algorithms in our conference papers [42], [43], interest in this approach has greatly increased and various related schemes have been published. Reference [44] uses a Gauss–Seidel iteration for the inversion of the channel matrix; [45] and [46] suggested SIC schemes and analyzed their performance.

The remainder of this paper is organized the following way: in Section II, we describe the system model as well as the channel characteristics determining the system behavior. Section III is dedicated to the "operator-perturbation technique" (OPT) for the efficient inversion of matrices. Next, we investigate interference-cancellation techniques, covering both PIC and SIC. Section V gives numerical results for both the OPT and the interference cancellation. A summary and the conclusions wrap up this paper.

II. System Model

This section briefly summarizes the model we use for OFDM transceiver and channel (further details can be found in [2]–[4]). OFDM divides the data stream into blocks of length N. The subcarrier modulation is done by means of an inverse discrete Fourier transform (IDFT) of the data blocks (typical length $N = 64 \ldots 1024$ carriers, but up to 8000 carriers are possible for digital video broadcasting (DVB). Let the block $X_k^{(i)}$ be the ith block of the transmission, with k indexing the N complex modulation symbols. The transmit signal in the time-domain $x^{(i)}[n]$ can then be written as

$$x^{(i)}[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_k^{(i)} e^{j\frac{2\pi}{N}nk}, \qquad -G \leq n < N \tag{1}$$

where G is the length of the CP; for the case of the CP being absent, we simply set $G = 0$. The channel performs the convolution with the impulse response of the channel $h[n, l]$, which consists of L multipath components $0 \leq l < L$ and is time-variant, as reflected by the double indexing. The received signal $y^{(i)}[n]$ (without noise added by the channel) is then

$$y^{(i)}[n] = \left(h * x^{(i)}\right)[n]. \tag{2}$$

As the CP is discarded by the receiver, we henceforth consider only $0 \leq n < N$. The first $L - G - 1$ samples of $y^{(i)}[n]$ are distorted by ISI from the previous symbol $x^{(i-1)}$; thus, $y^{(i)}$ consists of two parts:

$$\begin{aligned} y^{(i)}[n] &= \sum_{l=0}^{L-1} h[n,l]x^{(i)}[n-l]\sigma[n-l+G] \\ &\quad + \sum_{l=G+1}^{L-1} h[n,l]x^{(i-1)}[n-l+G+N](1-\sigma[n-l+G]) \\ &= y^{(i,i)}[n] + y^{(i,i-1)}[n] \end{aligned} \tag{3}$$

where σ is the Heaviside function, and $y^{(i,i)}[n]$ and $y^{(i,i-1)}[n]$ are the contributions to $y^{(i)}[n]$ stemming from the ith and $i-1$th block, respectively. To reconstruct the sent symbols, the receiver performs a DFT on $y^{(i)}[n]$, $Y_k^{(i)} = \mathrm{DFT}\{y^{(i)}[n]\}_k$, where k again is the subcarrier index. Throughout this paper, we assume perfect synchronization of carriers and blocks, and further, if not otherwise stated, absence of noise.

We now write the DFT of (3) in the receiver [15] as (4), shown at the bottom of the page, where $\tilde{N} = N + G$.

$Y_k^{(i,i)}$ and $Y_k^{(i,i-1)}$ are, again, the parts of $Y_k^{(i)}$ that originate from the own symbol i and the previous symbol $i-1$, respectively. If we use (1), these parts can then be written as

$$Y_k^{(i,i)} = \sum_{m=0}^{N-1} X_m^{(i)} H_{k,m}^{(i,i)} \tag{5}$$

$$Y_k^{(i,i-1)} = \sum_{m=0}^{N-1} X_m^{(i-1)} H_{k,m}^{(i,i-1)} \tag{6}$$

where

$$H_{k,m}^{(i,i)} = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h[n+i\tilde{N},l]e^{j\frac{2\pi}{N}(nm-lm-nk)}\sigma[n-l+G] \tag{7}$$

and

$$H_{k,m}^{(i,i-1)} = \frac{1}{N}\sum_{n=0}^{N-1}\sum_{l=G+1}^{L-1} h[n+i\tilde{N},l]e^{j\frac{2\pi}{N}(nm-lm-nk)} \times \{1-\sigma[n-l+G]\}. \tag{8}$$

This can be written in a compact form as a vector–matrix-product

$$\mathbf{Y}^{(i)} = \mathbf{Y}^{(i,i)} + \mathbf{Y}^{(i,i-1)} = \mathbf{H}^{(i,i)} \cdot \mathbf{X}^{(i)} + \mathbf{H}^{(i,i-1)} \cdot \mathbf{X}^{(i-1)} \tag{9}$$

where $\mathbf{Y}^{(i,i-1)}$ is the ISI term, and $\mathbf{Y}^{(i,i)}$ contains the desired data disturbed by ICI. Note that if $\mathbf{X}^{(i-1)}$ was detected successfully (and the channel is completely known, as we always assume in this paper), then $\mathbf{Y}^{(i,i-1)}$ can be computed and subtracted from $\mathbf{Y}^{(i)}$. This procedure can either be done in the frequency domain, as we showed here, or in the time domain, as in [15]. Due to this reason, ISI is not considered in the remainder of this paper. For simplicity, we henceforth drop the index of the block number superscript $^{(i)}$.

The above formulation is general in the sense that it includes the ICI due to the time variations and the insufficient guard interval.

III. Operator-Perturbation Technique (OPT)

After ISI cancellation, the data vector still contains interference from its own symbol, the ICI. To equalize this interference, we have to compute the data vector from (in the absence of noise)

$$\mathbf{Y} = \mathbf{H}\mathbf{X}. \tag{10}$$

A straightforward solution is the inversion of this equation, namely

$$\Rightarrow \mathbf{X}_i = \mathbf{H}^{-1} \cdot \mathbf{Y}_i. \tag{11}$$

A matrix inversion require $\mathcal{O}(n^3)$ operations, where n is the size of the square matrix. In the next section, we will introduce a technique (OPT) that only needs $\mathcal{O}(n^2)$ operations per iteration.

A. Standard OPT

The OPT is an iterative method to efficiently approximate and invert linear or nonlinear operators. It was originally introduced by Cannon in [47] and is well known in the astrophysics community. For matrices, it is also known as Jacobi-Iteration

$$\begin{aligned} Y_k^{(i)} &= \sum_{n=0}^{N-1}\left(y^{(i,i)}[n] + y_i^{(i,i-1)}[n]\right)e^{-j\frac{2\pi}{N}nk} \\ &= \underbrace{\sum_{n=0}^{N-1}\sum_{l=0}^{L-1} h[n+i\tilde{N},l]x^{(i)}[n-l]\sigma[n-l+G]e^{-j\frac{2\pi}{N}nk}}_{Y_k^{(i,i)}} \\ &\quad + \underbrace{\sum_{n=0}^{N-1}\sum_{l=G+1}^{L-1} h[n+i\tilde{N},l]x^{(i-1)}[n-l+G+N]\left(1-\sigma[n-l+G]\right)e^{-j\frac{2\pi}{N}nk}}_{Y_k^{(i,i-1)}} \end{aligned} \tag{4}$$

(see [48, ch. 10.1]), which has also been used in the context of interference cancellation in code-division multiple access (CDMA) [49]–[51]. We write (10) as

$$\mathbf{Y} = \widehat{\mathbf{H}}\mathbf{X} - \epsilon \tag{12}$$

where $\widehat{\mathbf{H}}$ is the approximate operator whose inverse is easy to compute, and ϵ is the deviation from the exact solution. For example, $\widehat{\mathbf{H}}$ could consist of the diagonal elements of $\mathbf{H}$ and zero off-diagonal elements and would thus be trivial to invert. Alternatively, $\widehat{\mathbf{H}}$ can be taken as a banded matrix, i.e., a matrix that contains only the main diagonal and a few off-diagonals of the matrix $\mathbf{H}$. The solution of (11) is found by the following iteration:

$$\mathbf{X}_{(0)} = \widehat{\mathbf{H}}^{-1}\mathbf{Y} \tag{13}$$

$$\epsilon_{(0)} = (\widehat{\mathbf{H}} - \mathbf{H})\mathbf{X}_{(0)} \tag{14}$$

$$\mathbf{X}_{(i+1)} = \widehat{\mathbf{H}}^{-1}\left(\epsilon_{(i)} + \mathbf{Y}\right) \tag{15}$$

$$\epsilon_{(i+1)} = (\widehat{\mathbf{H}} - \mathbf{H})\mathbf{X}_{(i+1)} \tag{16}$$

where subscript (i) indexes the ith iteration. After initialization [steps (13) and (14)], steps (15) and (16) are repeated until a criterion is reached. Substituting step (15) in step (16) gives

$$\mathbf{X}_{(i+1)} = \mathbf{X}_{(i)} + \underbrace{\widehat{\mathbf{H}}^{-1}\left(\mathbf{Y} - \mathbf{H}\mathbf{X}_{(i)}\right)}_{\mathbf{e}\left(\mathbf{x}_{(i)}\right)} \tag{17}$$

where $\mathbf{e}_{(\mathbf{X}_{(i)})}$ is the error of the old solution $\mathbf{X}_{(i)}$. If $\mathbf{X}_{(i)}$ converges to $\mathbf{X}_{(\infty)}$, then

$$\begin{aligned}
\mathbf{X}_{(\infty)} &= \widehat{\mathbf{H}}^{-1}\left(\epsilon_{(\infty)} + \mathbf{Y}\right)\\
\epsilon_{(\infty)} &= (\widehat{\mathbf{H}} - \mathbf{H})\mathbf{X}_{(\infty)}\\
\mathbf{X}_{(\infty)} &= \widehat{\mathbf{H}}^{-1}\left((\widehat{\mathbf{H}} - \mathbf{H})\mathbf{X}_{(\infty)} + \mathbf{Y}\right)\\
&= \mathbf{X}_{(\infty)} + \widehat{\mathbf{H}}^{-1}\left(\mathbf{Y} - \mathbf{H}\mathbf{X}_{(\infty)}\right)\\
\Rightarrow 0 &= \widehat{\mathbf{H}}^{-1}\left(\mathbf{Y} - \mathbf{H}\mathbf{X}_{(\infty)}\right).
\end{aligned} \tag{18}$$

This means that if $\det\{\widehat{\mathbf{H}}^{-1}\} \neq 0$, then we get the solution

$$\mathbf{Y} - \mathbf{H}\mathbf{X}_{(\infty)} = 0 \tag{19}$$

and therefore, $\mathbf{X}_{(\infty)} = \mathbf{X}$.

Step (16) of the iteration procedure requires a vector–matrix-product that needs $\mathcal{O}(n^2)$ operations. This step determines the overall performance. The question of how many iteration loops are necessary for a sufficient accuracy will be treated in the next section.

B. Acceleration of Convergence

Auer [52] and Ng [53] developed methods to improve the convergence speed of linear iterative schemes. These techniques have been invented in astrophysics and chemical physics but, astonishingly, do not seem to be widely known in mobile radio. Clearly, linear iterative schemes are only linearly convergent. An improvement of convergent speed can be achieved, when we use more information than the last solution X_{i-1} to calculate the new X_i:

$$\mathbf{X}_{(i)} = \alpha_0\mathbf{X}_{(i-1)} + \sum_{m=1}^{M}\alpha_m\mathbf{X}_{(i-1-m)}. \tag{20}$$

$\alpha_0 = 1 - \sum\alpha_m$ is a normalization factor. The coefficients α_m are determined as the solutions of the following set of linear equations [53]:

$$\mathbf{M}^{\mathrm{acc}}\alpha = \mathbf{b}^{\mathrm{acc}} \tag{21}$$

where

$$\begin{aligned}
M_{i,j}^{\mathrm{acc}} = \sum_l \frac{1}{|X_{N-1,l}|}&[\Delta X_{n,l} - \Delta X_{n-i,l}]\\
&\times[\Delta X_{n,l} - \Delta X_{n-j,l}]
\end{aligned} \tag{22}$$

$$b_i^{\mathrm{acc}} = \sum_l \frac{1}{|X_{N-1,l}|}\Delta X_{n,l}[\Delta X_{n,l} - \Delta X_{n-i,l}] \tag{23}$$

where $\Delta X_{n,l}$ is the lth component of the vector $\mathbf{X}_{(n)} - \mathbf{X}_{(n-1)}$. For a Mth order acceleration, we must supply $M+2$ successive estimates of the solution. Auer [52] gives a pseudocode for a second-order acceleration. Unfortunately, there is no clear optimal choice of M; the results of the study in [52] indicate that the utilization of higher orders than $M = 2$ usually do not significantly improve the acceleration. This was also validated by our own investigations; therefore, we implemented the second-order acceleration only ($M = 2$).

According to the study in [52], the choice for $M = 2$ requires the last four successive samples of the sequence of approximations of the solution. Following this algorithm, after every fourth iteration in the approximation of $\mathbf{Y} = \widehat{\mathbf{H}}\mathbf{X}$, one acceleration step is inserted. Then, the next four standard iterations are done followed by another acceleration, etc. In principle, this acceleration scheme is a very general method that can be applied to every linear iterative approximation algorithm.

C. Further Modifications

We can further exploit the finite-alphabet properties of the modulation: As we know that the solution of $\mathbf{X} = \mathbf{H}^{-1}\mathbf{Y}$ must lie in the set $\{-1, +1\}$ for BPSK, we can include a BPSK slicer after step (15) of the OPT algorithm. This slicer decides on the components of $\mathbf{X}_{i+1}$ and forces them to $+1$ or -1. Such an "OPT with decision" technique can also be interpreted as a PIC technique, which we will discuss in the next section.

IV. ICI Cancellation

In the last 15 years, efficient techniques have been developed for the suppression of cochannel interferers in CDMA systems [54]. Closer inspection of (5) and (7) reveals that the ICI is mathematically equivalent to cochannel interference (even though it is stemming from other subcarriers). Thus, the techniques developed for CDMA can be reused for our application.

The basic idea behind OFDM interference cancellation is that the decisions of the symbols on the subcarriers are improved iteratively. These improvements are due to the determination and subtraction of the interference of all the other subcarriers, based on the decisions of the previous iteration. A proper initialization has to be done before entering the iteration loop. We distinguish between PIC and SIC, depending on the order in which the decisions of the subcarrier symbols are done.

A. Parallel Interference Cancellation (PIC)

For initialization, we again calculate a first estimate $\mathbf{X}_{(0)} = (\mathrm{diag}\mathbf{H})^{-1} \cdot \mathbf{Y}$, similar to the OPT technique. Here, $\mathrm{diag}\mathbf{H}$ is the diagonal of $\mathbf{H}$ only and can be inverted with computational effort $\mathcal{O}(n)$. The interference $\mathbf{I}_i$ in step i is

$$\mathbf{I}_{(i)} = \mathbf{H}\mathbf{X}_{(i-1)} - \mathrm{diag}\mathbf{H}\,\mathbf{X}_{(i-1)}. \tag{24}$$

The updated interference-canceled received symbol vector $\mathbf{Y}_{(i)}$ is simplified to

$$\mathbf{Y}_{(i)} = \mathbf{Y} - \mathbf{I}_{(i)}. \tag{25}$$

Hence, the updated estimate of the transmitted symbol on subcarrier k is

$$(X_k)_{(i)} = (Y_k)_{(i)}/H_{kk} \tag{26}$$

where we equalize the interference-canceled received symbol by the channel response on the frequency of subcarrier k. This algorithm is called "parallel," because (25) and (26) can be applied to all channels $k = 0 \ldots n-1$ simultaneously by a vector–matrix calculus. Due to the vector–matrix multiplication, the computational complexity of the algorithm is $\mathcal{O}(n^2)$. These steps are identical to those of the OPT technique, with the special assumption of $\widehat{\mathbf{H}} = \mathrm{diag}\mathbf{H}$.

Before entering the next round of iterations, the estimated symbols can be processed in a nonlinear fashion. In the most simple case, they could be forced to the nearest symbol of the used alphabet. More generally

$$(\hat{X}_k)_{(i)} = f\left((X_k)_{(i)}\right) \tag{27}$$

where $f(\cdot)$ is the nonlinear decision function. A good choice for BPSK is the hyperbolic tangent function [55] $(\hat{X}_k)_{(i)} = \tanh(c \cdot (X_k)_{(i)})$ because far-off estimates are forced to ± 1 and thus reduced in their impact on the next cancellation stage, while estimates with small amplitudes are more or less unchanged. The factor c in the hyperbolic tangent function controls the slope near zero. Small c (e.g., $c < 1$) are appropriate if the estimates are not yet reliable. Large c ($c > 10$) are appropriate for accurate estimates, e.g., if the signal-to-interference ratio (SIR), which is defined in Section IV-B, is large. That is the case for good channels with small delay spread.

Performance can be further improved if c is increased from one iteration step to the next. In the beginning, decisions are still unreliable so that small c is appropriate. At the end of the iteration, the decision function approximates the signum function, and all decisions are in the set $\{\pm 1\}$. This technique stems from an optimization technique called "simulated annealing" and avoids the convergence of the MSE to a local minimum [55].

The error vector and the MSE in the receiver are defined as

$$\begin{aligned} \epsilon &= \mathbf{Y} - \mathbf{H} \cdot \hat{\mathbf{X}}_i \\ \mathrm{MSE} &= \|\boldsymbol{\epsilon}\|. \end{aligned} \tag{28}$$

The iteration stops after the MSE falls below a predefined threshold or the number of iterations reaches a predefined maximum number.

The most computationally expensive part of the algorithm is the calculation of the interference (24)—it is a matrix–vector product and is the reason for the $\mathcal{O}(n^2)$-complexity of this method. For each subcarrier, the interference of all other subcarriers is computed. Due to the fact that the influence of two subcarriers decreases with increasing frequency distance, we can restrict the interference calculation to the neighboring carriers. It is also noteworthy that the optimum PIC scheme makes use of the log-likelihood ratio from the MAP decoder [56].

B. Serial Interference Cancellation (SIC)

The main difference between SIC and PIC is that the update steps (25) and (26) are not done in parallel for all subcarriers but sequentially one channel after the other. In contrast to PIC, an initialization is not required. After each estimate and decision, the new interference is determined. This technique, suggested by us in [43], was also used in [45], which proposed simultaneous cancellation of several symbols in order to reduce processing delays.

Let us first define a SIR for the subchannel k as

$$\mathrm{SIR}_k = \frac{|H_{kk}|^2}{\sum\limits_{\substack{l=0 \\ l \neq k}}^{N-1} |H_{kl}|^2}. \tag{29}$$

We then order the subchannels by their SIR. This ensures that high reliable channels (i.e., with high SIR) are processed before the weak channels. In each iteration step (outer loop), the subchannels are processed in the order of their corresponding SIR (inner loop) that was calculated beforehand for the actual channel situation. For each subchannel k of iteration step i, the interference

$$(I_k)_{(i)} = \sum_{\substack{l \in X_l \text{decided in} \\ \text{previous iteration} \\ \text{and } l \neq k}} H_{kl}(\hat{X}_l)_{(i-1)} + \sum_{\substack{l \in X_l \text{decided in} \\ \text{current iteration}}} H_{kl}(\hat{X}_l)_{(i)} \tag{30}$$

is computed and subtracted. The computational effort is again $\mathcal{O}(n^2)$ due to the necessity to sum up the off-diagonal elements of the channel matrix and the computation of the interference terms. Note that the interference is calculated from the actual estimate $\hat{\mathbf{X}}_{(i)}$ compared to Section IV-A, where it is calculated from the previous estimate $\mathbf{X}_{(i-1)}$. $\hat{\mathbf{X}}_{(i)}$ contains all the decisions of subchannels that were previously made because they have higher SIR. The other channels were decided during the last iteration step. The rest of the algorithm is similar to PIC. The major advantage of SIC compared to PIC is that, within a single iteration step, the impact of estimated symbols on low-SIR carriers is considered immediately.

V. Simulation Results

In the following, we present numerical results to demonstrate the effectiveness of our proposed techniques. We investigate 1) the OPT, without any nonlinear processing (decisions) and using Auer's acceleration of convergence; 2) the PIC technique with progressively increasing values of the parameter c, and 3) SIC. These cases will be simply called OPT, PIC, and SIC, henceforth. For the OPT and the SIC (outer loop), we use a predetermined number of iterations, namely ten. For the PIC, we iterate until the MSE between subsequent iteration rounds falls below a threshold of 10^{-3}, or the number of iterations becomes 30.[1] For the SIC and the PIC, we use the tanh mapping function; we set $c = 0.5$ in the first iteration. For the PIC, c increases by 1.0 in each iteration step, while for the SIC, it increases by 2.45. Furthermore, we use a depth of five, i.e., four off-diagonals, for the OPT.

We show the BER of an uncoded OFDM system for various propagation channels. The channels are characterized by their scattering functions $P_s(\nu, \tau)$, which is defined via the relationship

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\left\{h^*(t,\tau)h(t',\tau')\right\}\exp\left[2\pi j(\nu t - \nu' t')\right] dt dt'$$
$$= P_s(\nu,\tau)\delta(\nu-\nu')\delta(\tau-\tau') \qquad (31)$$

assuming that the wide-sense stationary uncorrelated-scattering assumption [57] is fulfilled. Here, ν is the Doppler frequency. We furthermore assume that the statistics of the amplitudes are zero-mean complex Gaussian, resulting in the familiar Rayleigh fading. For most of the examples, we analyze a static channel so that the channel is characterized by its power-delay profile (PDP) only

$$\mathrm{PDP}(\tau) = E\left\{|h(\tau)|^2\right\}. \qquad (32)$$

We first analyze the impact of the shape of the PDP on the effect of the bit-error probability (BER) for the different equalization techniques. It is well known that, for unequalized single-carrier systems, the BER is proportional to the squared magnitude of the rms delay spread [58], [59]. For OFDM systems with CP, however, we need to consider only the part of the PDP that exceeds the length of the CP. We can thus expect

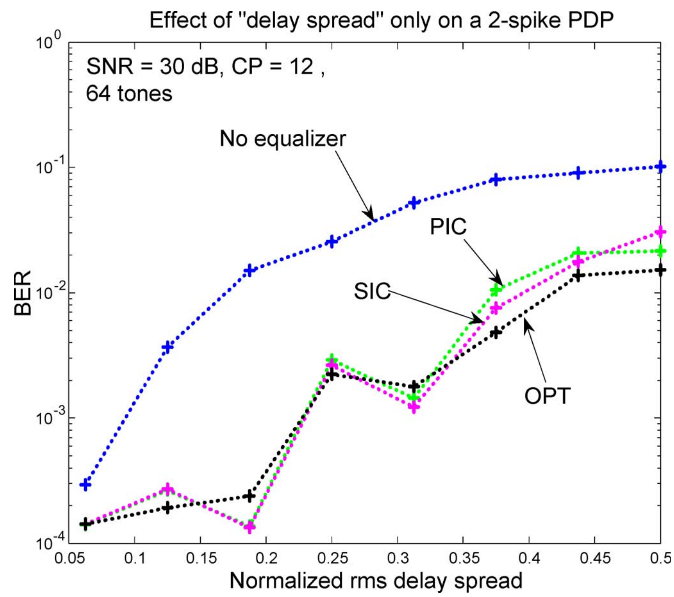

Fig. 1. BER as a function of the rms delay spread for a two-spike profile. OFDM system with 64 tones and CP with 12 samples length.

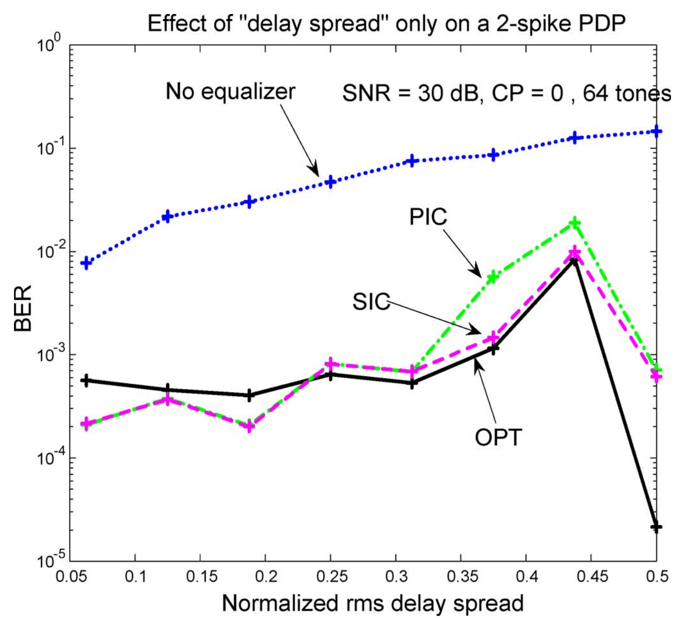

Fig. 2. BER as a function of the rms delay spread for a two-spike profile. OFDM system with 64 tones and no CP.

a different dependence of the BER for different PDPs. Figs. 1 and 2 show the BER for a two-spike profile

$$\mathrm{PDP}(\tau) = \delta(\tau) + \delta(\tau - 2\tau_{\mathrm{rms}}) \qquad (33)$$

where τ_{rms} is the rms delay spread normalized to the OFDM symbol duration; the simulations use 100 channel realizations, in each of which 100 OFDM symbols are transmitted (sample simulations with 500 channel realizations gave the same results). We find that the BER is lower in an equalized system without CP than in a conventional system with CP, even if the CP is large enough to eliminate all ICI. This fact, which is astonishing at first glance, can be explained by the frequency

[1] In the simulations that follows, the MSE threshold was not reached, so that the number of iterations was always 30.

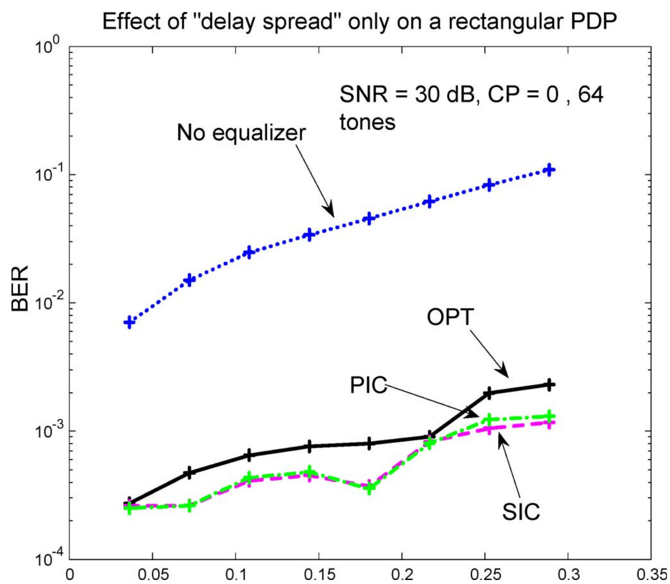

Fig. 3. BER as a function of the rms delay spread for a rectangular PDP. OFDM system with 64 tones and no CP.

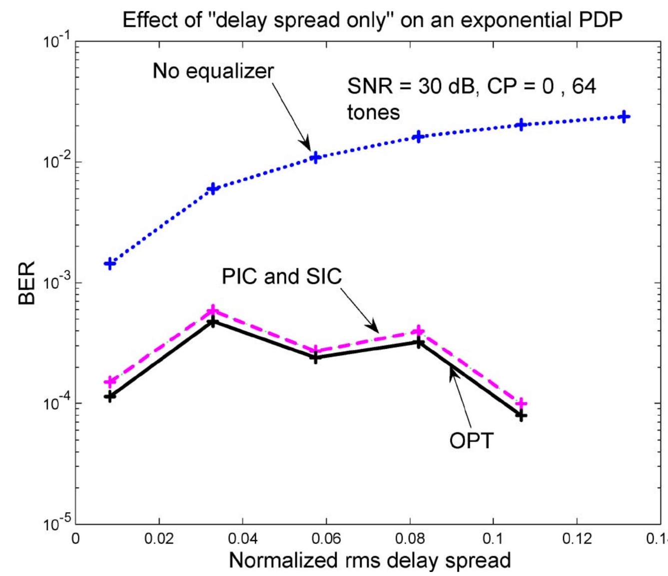

Fig. 4. BER as a function of the rms delay spread for an exponential PDP. OFDM system with 64 tones and no CP.

diversity inherent in delay dispersion. A conventional system with CP discards all information "hidden" in the first G received samples. For an OFDM system without equalizer, this information is detrimental, since it leads to ICI and, thus, interference on the different tones. However, a system with one of our equalizers can actually make good use of this information—essentially, it carries information about each bit on several tones simultaneously and is, thus, less sensitive to fading. We also observe that, for very large delay spreads, all of the equalization techniques start to break down. Finally, we note that the SIC and PIC perform slightly better than the OPT. The "choppiness" (nonmonotonicity) of the curves stems from the idiosyncrasies for the channel models, i.e., for a given channel model, some delay spreads can be equalized more easily than others (this was confirmed by runs with more channel realizations). Finally, we find that, if there is a CP and no equalizer, the impact of ICI is noticeable for all rms delay spreads larger than 0.094 $(0.5{*}12/64)$ for the choice of parameters in Fig. 1. Figs. 3 and 4 show the BER for rectangular and exponential PDPs, respectively. Note that, for those profiles, the maximum rms delay spread considered here is smaller: We investigate only cases where the maximum excess delay is smaller or equal to one symbol duration (as the exponential PDP extends to infinity, we require that the "significant" part of the PDP lasts only one symbol duration).

We also investigate the impact of using different approximate operators $\widehat{\mathbf{H}}$ in the OPT. Fig. 5 shows the BER as a function of the delay spread of a rectangular PDP when $\widehat{\mathbf{H}}$ contains two, eight, 32, and 64 off-diagonals of $\mathbf{H}$. We see that there is hardly any difference in the resulting BER, which shows that good results can be achieved with a $\widehat{\mathbf{H}}$ that requires little numerical effort for its inversion. We also investigate the impact of the acceleration procedure for the OPT. Fig. 6 shows how the acceleration steps improve of the MSE.

We analyze next the performance in a scenario that does not show delay dispersion (single-tap channel) but rather only time

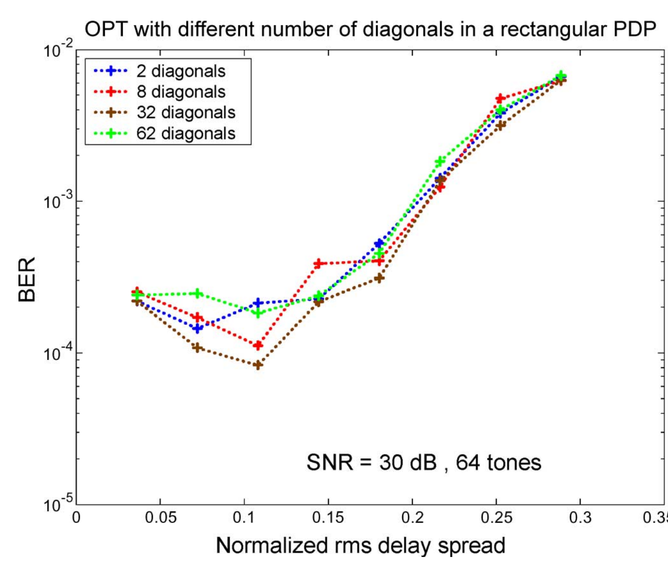

Fig. 5. BER as a function of delay spread for OPT with different numbers of diagonals accounted for in $\widehat{\mathbf{H}}$.

variance of the channel, following a "classical" Jakes Doppler spectrum. We plot the BER versus the rms Doppler spread (normalized to the inverse of the OFDM symbol duration) for the case of a classical Jakes Doppler spectrum

$$S_{\mathrm{D}}(\nu) \propto \frac{1}{\sqrt{\nu_{\max}^2 - \nu^2}} \tag{34}$$

at an SNR of 30 dB; see Fig. 7. Once again, the OFDM symbol length being considered here is 64 samples. It is noteworthy that the OPT performs better than the interference-cancellation technique in this case. Fig. 8 shows the impact of using different numbers of off-diagonal elements in the iterations. We also note that, in the Doppler case, the simple but effective approach of [60] can be used.

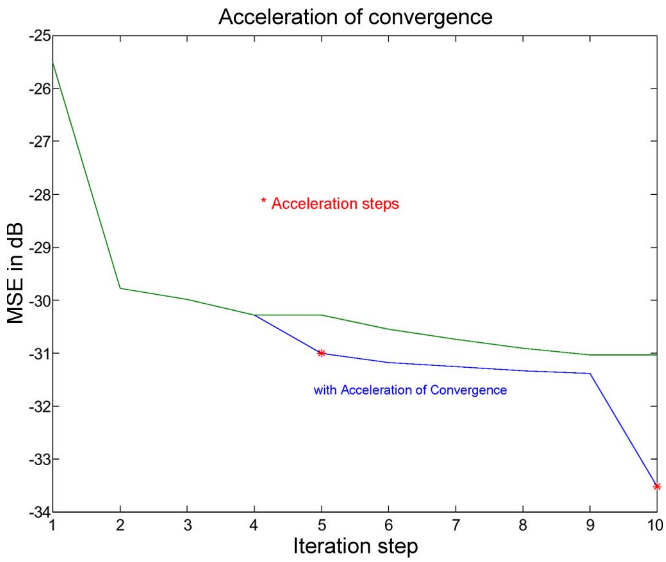

Fig. 6. Impact of the acceleration of convergence onto the MSE. Red stars: Acceleration steps. Blue line: With acceleration. Black line: Without acceleration. Simulation parameters: OFDM block size: 16 samples; two-spike PDP, maximum excess delay: One sample; mean spike powers: [1,0.1]; no CP.

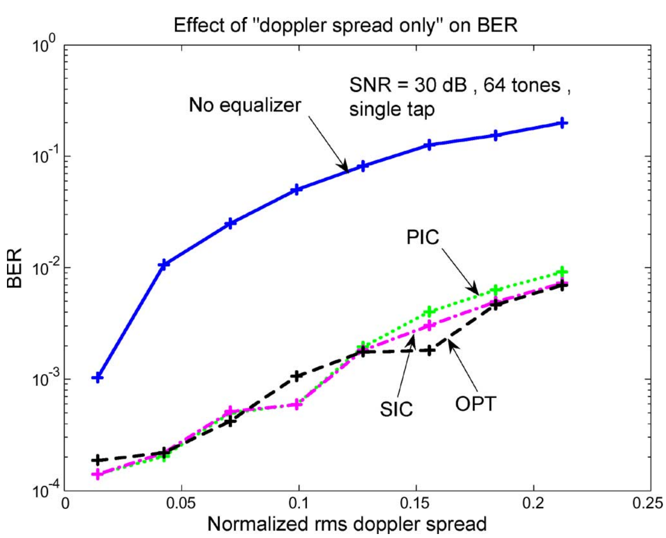

Fig. 7. BER as a function of the normalized Doppler spread for Jakes Doppler spectrum. Single-tap PDP, 64 carriers.

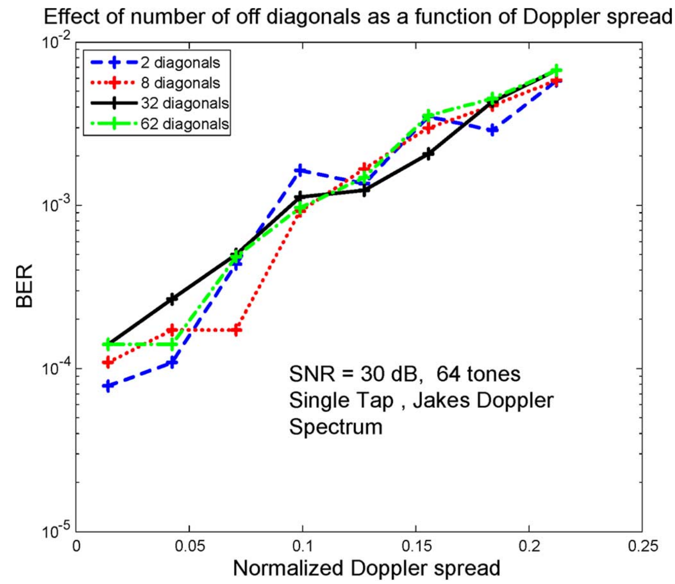

Fig. 8. BER as a function of Doppler spread for OPT with different numbers of diagonals accounted for in $\widehat{\mathbf{H}}$.

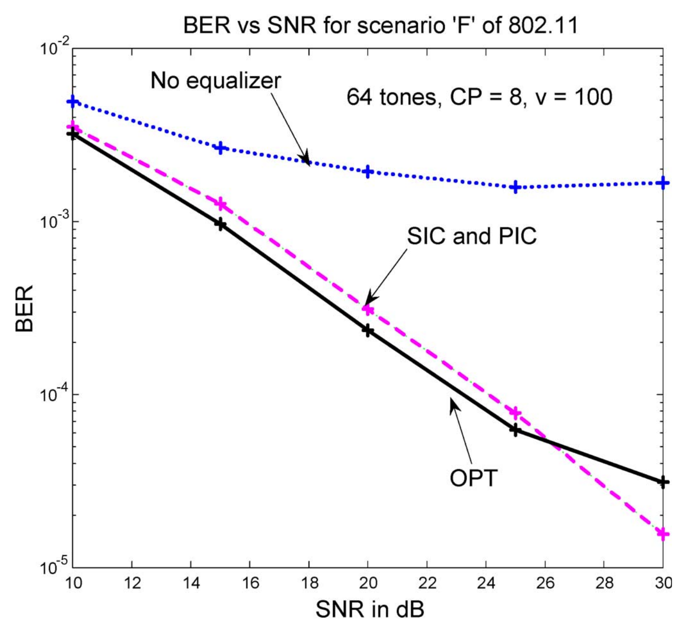

Fig. 9. BER as a function of SNR for an 802.11-like OFDM system with 64 carriers and eight samples CP. Performance is analyzed in a channel model F of the 802.11n channel models with 100-m/s velocity.

In order to analyze the usefulness of our algorithms in comparison to an OFDM system without any equalizer, we performed simulations of a system similar to the IEEE 802.11 standard. The system bandwidth was taken to be 20 MHz, and the carrier frequency was taken to be 5 GHz; we used the IEEE 802.11n channel models [61] which define both the delay and angular dispersion; from this, the scattering function can be derived. We approximate the standardized system by taking a 64-carrier OFDM system (the actual 802.11a system has 48 data carriers as well as pilots and null carriers); the velocity was taken to be 100 m/s (which is much higher than can usually be anticipated for 802.11a systems), making this case suffer from ICI due to Doppler, as well as delay spread. If the CP is 16 samples long, as defined in the 802.11a standard, then ICI is negligible (simulation results not shown here for space reasons). However, if the CP is only eight samples long, which was recently suggested as an option for the 802.11n standard, ICI becomes noticeable. Fig. 9 plots the BER for various interference-cancellation techniques versus SNR in comparison with a case where no equalizer is present in the OFDM receiver. We find that, while the unequalized receiver shows an error floor of 10^{-3}, the OPT, as well as the PIC and SIC, does not exhibit a floor at SNRs up to 30 dB.

VI. Summary and Conclusion

In this paper, we have presented several algorithms for the equalization of OFDM systems that suffer from ICI. We considered all sources of ICI, namely, an insufficient duration of the guard interval, as well as temporal variations of the channel. ISI can be eliminated by decision-feedback algorithms. We first established a general system model to represent all ICI contributions as off-diagonal elements of a channel matrix that multiplies the transmit symbols. We then investigated various schemes for the efficient inversion of this process.

We first considered a linear-equalization approach, where the inversion of the channel matrix is done iteratively by means of OPT. This approach can be improved by adding an "acceleration of convergence scheme." Furthermore, introducing decisions between the iteration steps leads to a PIC scheme, where the interference from neighboring tones (based on preliminary decisions) is subtracted in several stages. Alternatively, a SIC scheme first decides about the tone with the strongest SINR, subtracts its interference from the neighboring tones, decides for the next-best tone, and so on.

Simulations demonstrated that all three schemes lead to a drastic reduction of the error floor created by ICI and can even lead to performance that is better than with long CP, while, at the same time improving spectral efficiency. Our simulation results show that the low-complexity equalization techniques can be effectively used in practical OFDM systems and give guidelines for their implementation.

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