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TR2007-091 August 2008

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IEEE Global Telecommunications Conference

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Asymmetric Cooperation Among Relays with Linear Precoding

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Abstract—Fixed and mobile relays are used, among other applications, in the downlink of cellular communications systems. Cooperation between relays can greatly increase their benefits in terms of extended coverage, increased reliability, and improved spectral efficiency. In this paper, we introduce the fundamental notion of *asymmetric cooperation*. For this, we consider a two-phase transmission protocol where, in the first phase, the base station (BS) sends several available messages to the relays over wireless links. But, depending on the channel state and the duration of the BS transmission, not all relays decode all messages. In a second phase, the relays, which may now have asymmetric message knowledge, use cooperative linear precoding for the transmission to the mobile stations. We show that for many channel configurations, asymmetric cooperation, although (slightly) sub-optimum for the second phase, is optimum from a total-throughput point of view, as it requires less time and energy in the first phase. We give analytical formulations for the optimum operating parameters and the achievable throughput, and show that under typical circumstances, 20-30% throughput enhancement can be achieved over conventional systems.

I. INTRODUCTION AND MOTIVATION

Recent literature and standards such as IEEE 802.16j [1] propose augmenting cellular networks with fixed or mobile wireless relays for extending cell coverage, boosting transmission rates, improving spectral efficiency, and achieving all this at much lower costs than building more full-fledged base stations [2]–[6]. Traditionally, relays are used to forward information in a sequence of “hops”, where each hop is a single-link transmission between two nodes. More recently, the cooperative nature of relays has been extended to more general multi-terminal cooperative networks [7]–[10].

A fundamental scenario that allows the study of collaborative wireless relays in a cellular context is downlink communication between a single base station and multiple mobiles via wireless relays, using a two-hop strategy. Transmission from the base station (BS) to the mobile stations (MSs) thus takes place in two phases: In the first phase, the message(s) travel from the base station to the relays. In the second phase, the relays *cooperate* in transmitting the received message(s) to the mobiles. In this paper, we introduce an important, and hitherto ignored, fundamental notion of asymmetric cooperation that inevitably arises in such scenarios. This asymmetry arises from the fact that in

the first phase, the relays receive unequal amounts of data from the BS; the extent of this inequality depends on the differences in the fading states of the BS-to-relay channels and the duration and rate of BS transmission. An example of an asymmetric scenario is one in which one of the two available relays in a cell has to transmit multiple messages for multiple mobiles, while the other relay has to transmit to only one mobile.

In this paper, we explicitly and jointly optimize the *total* throughput over both phases. We thus account for the time (and energy) required to transfer the various messages to the relays. We will demonstrate, through analysis and simulation, that the cases where the relays have asymmetric message knowledge are relevant and arise often when optimizing throughput and reliability. The extent to which this asymmetry arises depends on the optimization criterion. We therefore optimize for two diametrically opposed throughput criteria – *maximum throughput*, in which the sum throughput to all the MSs is maximized, and *extreme fairness*, in which each MS is provided the same throughput. Due to space constraints, and in order to focus on the fundamental issues, we restrict the exposition of this paper to the case of two relays and two mobile stations, and consider only linear precoding [11] for the cooperation between the relays. As we shall see, even this problem is theoretically rich and difficult.

While numerous papers have studied two-hop downlink cellular systems, the schemes considered ignore either the asymmetry of relay cooperation or the cooperation between relays. Most of the existing literature deals with single-relay links. Given the large body of literature, we refer the interested reader to [3], [7], [12] and references therein for further details. Reference [13] considers an adaptive downlink system that uses either direct transmission to the MS or a two-hop transmission with relays, but does not investigate relay cooperation. In [14], the authors propose a centralized downlink scheduling scheme in a cellular network with a small number of relays, but do not consider cooperation between relay nodes. Most papers on cooperative two-hop relays (see, e.g., [15], [16]) consider the transmission of only a single message via multiple, cooperating relays [17], [18] or multiple messages via only a single relay [19]. Modeling multiple relays as a single relay with multiple antennas, as in [19], precludes asymmetry. Thus, to the best of our knowledge, the communication of multiple messages via multiple asymmetric relays is yet to be treated.

The remainder of this paper is organized as follows:

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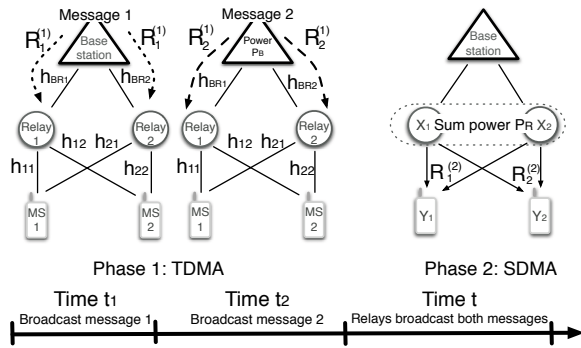


Fig. 1. Transmission takes place in two phases: in phase 1 the BS broadcasts messages in TDMA fashion: message W_1 for t_1 time units, then message W_2 for t_2 time units. During phase 2 the relays simultaneously transmit all received messages to the mobiles. Illustrated are the channel gains, power constraints, rates, and input-output variables.

Section II formulates the relay cooperation model and the optimization problem. Section III optimizes the classical symmetric cooperation cases and the novel asymmetric cooperation cases for both the max throughput and the extreme fairness optimization criteria. Section IV numerically compares the performance of different cooperation scenarios. We conclude in Section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider downlink communication between a single BS, two relays, and two mobiles, MS_1 and MS_2 , in additive white Gaussian noise and fading channels, see Fig. 1. The gains of the channels between the BS and relay 1 and relay 2 are denoted as h_{BR_1} and $h_{BR_2} \in \mathbb{C}$, respectively. The channels between the two relays and the two mobiles, MS_1 and MS_2 are given by $\mathbf{h}_1 = [h_{11}, h_{21}]$ and $\mathbf{h}_2 = [h_{12}, h_{22}] \in \mathbb{C}^2$, respectively. These channel gains are assumed to be known to all nodes, and are quasi-static for the duration of transmission [20].

A. Two phase communication

Transmission from the BS to the MSs takes place in two phases: during phase 1 the BS broadcasts the messages W_1 and W_2 sequentially in a TDMA fashion, as shown in Fig. 1. This involves the BS broadcasting message 1, of n_1 bits, at a rate $R_1^{(1)}$ for time $t_1 = n_1/R_1^{(1)}$, and the message 2, of n_2 bits, at a rate $R_2^{(1)}$ for a (possibly different) time $t_2 = n_2/R_2^{(1)}$. Message W_i is encoded by the message symbol U_i , and the average BS power cannot exceed P_B . This TDMA structure is both simple as well as optimal, in terms of maximizing throughput, in the single antenna scenario [21]. It also exploits the broadcast nature of the wireless channel, as both relays can possibly overhear the two messages. The times t_1 and t_2 and rates $R_1^{(1)}$ and $R_2^{(1)}$ determine how many bits can be delivered in the messages to the relays. The relays are assumed to be of the decode-and-forward type, and can either decode a message (W_1 or W_2 or both) in its entirety or not at all.

In phase 2, both relays simultaneously transmit to the mobiles the messages they have received. The transmitted signal vector is denoted by $\mathbf{X} = [X_1 \ X_2]'$, where relay 1 transmits the symbol X_1 and relay 2 transmits X_2 , and \mathbf{A}' denotes the transpose of a matrix \mathbf{A} . The signal X is given by $\mathbf{X} = \mathbf{B}\mathbf{U}$, where $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \in \mathbb{C}^{2 \times 2}$ is the linear precoding matrix used by the relays, and $\mathbf{U} = [U_1 \ U_2]'$ is the vector of message symbols broadcast by the BS in phase 1. The signals $\mathbf{Y} = [Y_1 \ Y_2]'$ received at the mobiles MS_1 and MS_2 , respectively, are given by

$$\mathbf{Y} = \mathbf{H}\mathbf{B}\mathbf{U} + \mathbf{N},$$

where $\mathbf{N} = [N_1 \ N_2]'$ corresponds to realizations of additive white Gaussian noise, which we assume without loss of generality (WLOG) to be of zero-mean and unit-variance. The transmissions by the relays are subject to a total relay power sum constraint of P_R . Assuming, WLOG, that $E[\mathbf{U}\mathbf{U}'] = \mathbf{I}_2$, the 2×2 identity matrix, the sum-power constraint on the signals transmitted by the two relays, becomes $|b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2 \leq P_R$. \mathbf{H} is assumed to be invertible, which happens with probability 1 when its elements are random.

B. How asymmetry arises

The above setup differs from conventional linear precoding and space division multiple access (SDMA) in one critical manner. Depending on the channel gains h_{BR_1} and h_{BR_2} and the phase 1 transmission parameters, both relays may not have decoded both messages. Phase 1 thus imposes constraints on some elements of \mathbf{B} , because a relay cannot transmit a message that it does not know. WLOG, assume that $|h_{BR_1}| > |h_{BR_2}|$. This means that if relay 2 can decode the message from the BS, then relay 1 can as well.

The four scenarios that arise are illustrated in Fig. 2. In Case 1, in which only one relay has decoded both the messages, while the other has not decoded any, phase 2 corresponds to the classical broadcast problem and two elements of \mathbf{B} are forced to be 0. Case 4 corresponds to the classical SDMA problem in which the two relays jointly transmit two messages to the two MSs. However, in Cases 2 and 3, one of the elements of \mathbf{B} is forced to be 0 as relay 2 has not decoded one of the two messages. For example, in Case 2, when relay 1 has both messages W_1 and W_2 and relay 2 only has message W_1 , the signal transmitted by relay 2, X_2 , cannot contain W_2 's encoding, U_2 . Thus, b_{22} is forced to 0. Similarly, in Case 3, relay 1 has both messages W_1 and W_2 while relay 2 has only W_2 , which forces $b_{21} = 0$. We shall refer to the Cases 1 and 4 as *symmetric* and Cases 2 and 3 as *asymmetric*. The constraint that \mathbf{B} must be triangular for the asymmetric cases changes the space of linear precoding matrices over which SDMA is optimized.

C. Receiver model

We assume the receivers decode their respective desired signals by treating the undesired signals as noise; no in-

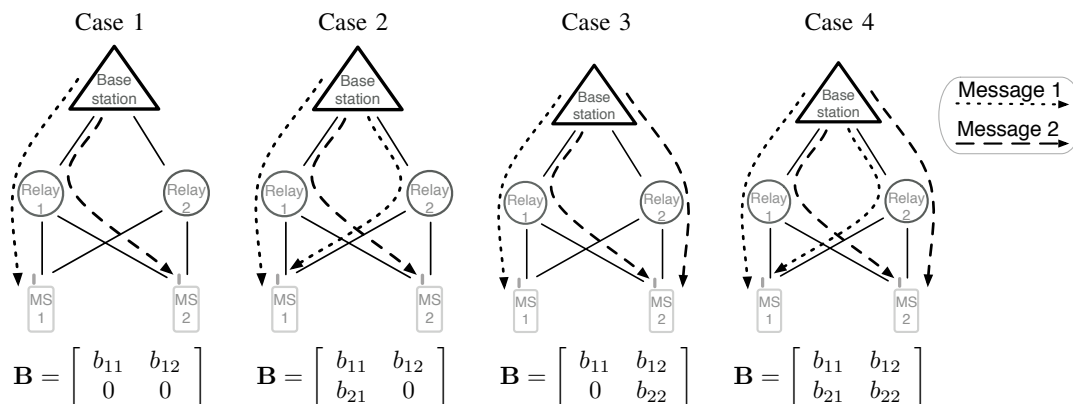


Fig. 2. Four message knowledge scenarios when $|h_{BR_1}| \geq |h_{BR_2}|$, and the corresponding linear precoding matrices \mathbf{B} .

interference cancellation is assumed.¹ The SINRs at the two receivers, γ_1 and γ_2 , and the corresponding information-theoretic phase 2 rates for the Gaussian noise channels², $R_1^{(2)}$ and $R_2^{(2)}$ are then given by:

$$\gamma_1 = \frac{|h_{11}b_{11} + h_{21}b_{21}|^2}{|h_{11}b_{12} + h_{21}b_{22}|^2 + 1}, \quad R_1^{(2)} = \log_2(1 + \gamma_1)$$

$$\gamma_2 = \frac{|h_{12}b_{12} + h_{22}b_{22}|^2}{|h_{12}b_{11} + h_{22}b_{21}|^2 + 1}, \quad R_2^{(2)} = \log_2(1 + \gamma_2).$$

D. System throughput optimization

The question we address is how to *best* transmit messages W_1 and W_2 (of lengths which may be optimizable parameters) to MS₁ and MS₂, respectively. The aim is to maximize system throughput subject to imposed fairness constraints. The overall throughput is the ratio of the total number of bits $n_1 + n_2$ to the total time (over both phases) taken to transmit them, as shown in (1)–(7). The problem therefore involves determining the optimal rates $R_1^{(1)}$ and $R_2^{(1)}$, the linear pre-coding matrix \mathbf{B} , as well as the number of bits n_1 and n_2 , subject to the constraints in equations (4)–(7). Various combinations of the constraints in equations (4)–(7) lead to the four cases in Fig. 2. Given the unavoidable combinatorial nature of the constraints, the overall maximum is obtained by optimizing each of the four cases separately and choosing the one with the highest throughput.

$$\max_{\mathbf{B}, n_1, n_2, R_1^{(1)}, R_2^{(1)}} \frac{n_1 + n_2}{\frac{n_1}{R_1^{(1)}} + \frac{n_2}{R_2^{(1)}} + \max\left(\frac{n_1}{\log_2(1+\gamma_1)}, \frac{n_2}{\log_2(1+\gamma_2)}\right)} \quad (1)$$

$$\text{s.t.} \quad n_1, n_2 \geq 0 \quad (2)$$

$$|b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2 \leq P_R \quad (3)$$

¹The rates achieved are pessimistic, but practically achievable.

²We drop the usual factor of 1/2 seen in the classical Shannon formula $\frac{1}{2} \log_2(1 + \text{SINR})$ as it can be absorbed into n_1 and n_2 .

The requirement that a relay can only transmit messages it has decoded leads to the following additional constraints.

$$\text{If } R_1^{(1)} \geq \log_2(1 + |h_{BR_1}|^2 P_B) \text{ then } b_{11} = 0 \quad (4)$$

$$\text{If } R_2^{(1)} \geq \log_2(1 + |h_{BR_1}|^2 P_B) \text{ then } b_{12} = 0 \quad (5)$$

$$\text{If } R_1^{(1)} \geq \log_2(1 + |h_{BR_2}|^2 P_B) \text{ then } b_{21} = 0 \quad (6)$$

$$\text{If } R_2^{(1)} \geq \log_2(1 + |h_{BR_2}|^2 P_B) \text{ then } b_{22} = 0 \quad (7)$$

E. Fairness metric

The previous section considered the classical *maximum throughput* metric, in which n_1 and n_2 are unconstrained. While the maximum throughput optimization criterion, in a multi-user setting, is useful and well-established, it sacrifices fairness. We therefore also consider the *extreme fairness* criterion, which lies at the other end of the “fairness” spectrum. Under this criterion, the same number of bits ($n_1 = n_2$) are transmitted to each MS, and the optimization problem is to determine the transmission parameters that achieve this in the minimal amount of time.

III. OPTIMIZING EACH SYMMETRIC AND ASYMMETRIC CASE INDIVIDUALLY

A. Symmetric (Conventional) Cases 1 and 4

In these two cases, the phase 2 problem of transmitting the two messages to two non-cooperating receivers reduces to classical well-studied problems: Case 1 corresponds to the standard single transmit antenna information theoretic broadcast channel, and Case 4 corresponds to the two transmit antenna MIMO broadcast channel [21]. Linear pre-coding for the MIMO broadcast channel to maximize the sum-rate is a non-convex problem whose closed-form solution remains an open problem. However, progress can be made along the lines of [11], [22]. Given space constraints, we defer solutions of these conventional cases to [23].

B. Asymmetric Cases 2 and 3

For brevity, we describe only the asymmetric Case 3 in which relay 1 has both W_1 and W_2 , while relay 2 has only W_2 ($b_{21} = 0$).³ Let $x = n_1/n_2$, for $n_2 > 0$ and $b_{11} > 0$ (the cases $n_2 = 0$ and $b_{11} = 0$ are easier, see [23]), and let

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \mathbf{H} \begin{bmatrix} b_{12}/b_{11} \\ b_{22}/b_{11} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} g_{11} & g_{12}/2 \\ g_{12}/2 & g_{22} \end{bmatrix} = (\mathbf{H}\mathbf{H}^\dagger)^{-1},$$

where \mathbf{A}^\dagger denotes the conjugate transpose of the matrix \mathbf{A} . The overall optimization may thus be expressed in terms of the new variables x , α , and β and the old variable b_{11} , as in (8)–(10) (see next page), where $\theta_G = \angle g_{12}$ and $\theta = \angle \alpha\beta^*$.

C. Max throughput

The optimization problem in (8) (see next page) corresponds to the max throughput criterion if $x \in [0, \infty)$ is unconstrained. As we show below, the optimization for Case 3 may be systematically reduced from an optimization over the 8 variables n_1, n_2, b_{11}, b_{12} and b_{22} (recall that the b_{ij} are complex), to one over only 2 variables in (11)–(12). Notice that $R_1^{(1)}$ and $R_2^{(1)}$ are such that Case 3 occurs. The reader is referred to [23] for the proofs of the lemmas.

Lemma 1: The optimal values of $|\alpha|$ and $|\beta|$ must lie on an ellipse, (10), whose axes are determined by b_{11} and θ . They can thus be parameterized by the variables $t, \theta \in [0, 2\pi]$ and $|b_{11}| \leq \sqrt{P_R}$ as

$$|\alpha(t, b_{11}, \theta)| = a' \cos(\phi) \cos(t) + b' \sin(\phi) \sin(t) \quad (13)$$

$$|\beta(t, b_{11}, \theta)| = -a' \sin(\phi) \cos(t) + b' \cos(\phi) \sin(t), \quad (14)$$

where $\phi = \frac{1}{2} \cot^{-1} \left(\frac{g_2 - g_1}{|g_{12}| \cos(\theta_G + \theta)} \right)$, and a' and b' are

$$(a')^2 = \frac{2(P_R/|b_{11}|^2 - 1)(g_1 g_2 - \frac{g_{12}^2}{4})}{(\frac{g_{12}^2}{4} - g_1 g_2)((g_2 - g_1)\sqrt{1 + \frac{g_{12}^2}{(g_1 - g_2)^2}} - g_2 - g_1)}$$

$$(b')^2 = \frac{2(P_R/|b_{11}|^2 - 1)(g_1 g_2 - \frac{g_{12}^2}{4})}{(\frac{g_{12}^2}{4} - g_1 g_2)((g_1 - g_2)\sqrt{1 + \frac{g_{12}^2}{(g_1 - g_2)^2}} - g_2 - g_1)}$$

Lemma 2: For any set of variables b_{11} , α , and β , the optimal solution for x can take only three values: 0, ∞ , or

$$x^* = \frac{\log_2(1 + |h_{11}|^2/(|\alpha|^2 + 1/|b_{11}|^2))}{\log_2(1 + |\beta|^2/(|h_{12}|^2 + 1/|b_{11}|^2))}.$$

Lemma 3: The optimal solution is independent of the absolute angle of b_{11} and the absolute angle of α . Thus, we may WLOG assume b_{11} to be real $\in [0, \sqrt{P_R}]$, and $\angle \alpha = 0$.

Lemma 4: The optimal θ^* satisfies $\theta_G + \theta^* = 0$ or π , and may WLOG be taken to equal 0.

The optimization over the reduced two variable set, t and b_{11} , then simplifies to (11) (see next page) and is performed numerically.

³Case 2 can be obtained easily by an appropriate permutation of indices.

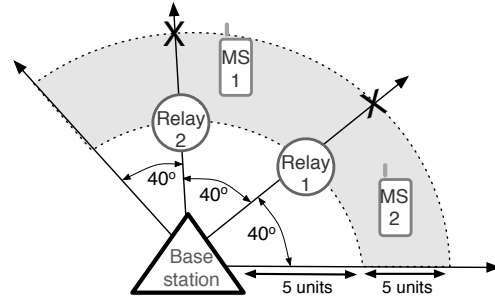


Fig. 3. Geometric distribution: relays at equal distance, spaced at angles 40° on arc of radius 5 units, mobiles random in shaded sector for the random MS placement, or at the points marked X for the fixed MS placement.

D. Extreme Fairness

It can be shown that Lemmas 1, 3, and 4 apply as well for the extreme fairness criterion. However, since $x = 1$, Lemma 2 is no longer applicable. The following Lemma narrows down the solution further for this case:

Lemma 5: If $f_1(t), f_2(t)$ are two continuous, differentiable functions over a compact set T , then the $t^* \in T$ that minimizes $\max(1/f_1(t), 1/f_2(t))$ lies either:

- 1) At the boundary of T .
- 2) At point(s) t_X where $f_1(t_X) = f_2(t_X)$, if such point(s) exist.
- 3) At a local minima of either $1/f_1(t)$ or $1/f_2(t)$.

The significantly reduced optimization problem in two variables t and b_{11} can now be stated in the form (15)–(16) (see next page).

IV. SIMULATION RESULTS

We evaluate the four linear precoding schemes under both the extreme fairness and maximum throughput optimization criteria for an ensemble of random channel realizations. The channels are assumed to obey a power law with exponent of -2 for the dependence of the mean pathloss on the distance, and suffer from Rayleigh fading of mean 1. The relays are always placed along the arc of radius 5 units, separated by 40° , as shown in Fig. 3. In Figs. 4–6 random MS placement is used: the mobiles are randomly placed in the shaded slice between distance 5 and 10 from the BS (thus further from the BS than the relays). In Figs. 7–9 fixed MS placement is used: the mobiles at the X-marks, on the arc of 10 units. The only randomness in this latter model comes from the fading. Relays are numbered such that $|h_{BR_1}| > |h_{BR_2}|$. Considering such a large set of cases helps us get an in-depth understanding of the overall behavior of the system. For each of 2000 sets of geometric positions and fades, the throughputs for all four message knowledge cases are obtained by numerically solving the simplified optimization problem.

Max throughput optimization:

$$\max_{x, b_{11}, \alpha, \beta, \theta} \frac{x+1}{x/R_1^{(1)} + 1/R_2^{(1)} + \max\left(x/\log_2\left(1 + \frac{|h_{11}|^2}{|\alpha|^2+1/|b_{11}|^2}\right), 1/\log_2\left(1 + \frac{|\beta|^2}{|h_{12}|^2+1/|b_{11}|^2}\right)\right)} \quad (8)$$

$$\text{s.t. } x \geq 0 \quad |b_{11}|^2 \leq P_R \quad (9)$$

$$g_1|\alpha|^2 + 2|g_{12}||\alpha||\beta|\cos(\theta_G + \theta) + g_2|\beta|^2 \leq P_R/|b_{11}|^2 - 1 \quad (10)$$

Simplified max throughput optimization:

$$\max_{t, |b_{11}|} \frac{x+1}{x/R_1^{(1)} + 1/R_2^{(1)} + \max\left(x/\log_2\left(1 + \frac{|h_{11}|^2}{|\alpha(t, b_{11}, \theta^*=0)|^2+1/|b_{11}|^2}\right), 1/\log_2\left(1 + \frac{|\beta(t, b_{11}, \theta^*=0)|^2}{|h_{12}|^2+1/|b_{11}|^2}\right)\right)} \quad (11)$$

$$\text{s.t. } x \in \{0, x^*, \infty\}, \quad |b_{11}|^2 \leq P_R, \quad t \in [0, 2\pi] \quad (12)$$

Simplified extreme fairness optimization:

$$\min_{t, |b_{11}|} \max \left(1/\log_2 \left(1 + \frac{|h_{11}|^2}{|\alpha(t, b_{11}, \theta^*=0)|^2 + 1/|b_{11}|^2} \right), 1/\log_2 \left(1 + \frac{|\beta(t, b_{11}, \theta^*=0)|^2}{|h_{12}|^2 + 1/|b_{11}|^2} \right) \right) \quad (15)$$

$$\text{s.t. } |b_{11}|^2 \leq P_R, \quad t \in [0, 2\pi] \quad (16)$$

A. Fractions of time the 4 cases are chosen

Random MS placement: Figure 4 demonstrates the fractions of the time each of the four cases of Fig. 2 are optimal under the max throughput (black) and extreme fairness (grey) constraints assuming *random MS* placement. We can see that under the max throughput scenario, symmetric Case 1 is selected about 45% of the time, while the asymmetric Cases 2 and 3 are optimal roughly 20% of the time each, and the fully symmetric Case 4 is optimal 15% of the time. Under the max throughput criterion, all 4 cases allow for a single message to be sent. Interestingly, 61% of the time sending only a single message is max throughput optimal. Furthermore, it turns out that every time Case 1 is chosen, it is used to send a single message. Thus, when it is optimal to transmit 2 messages, the asymmetric scenarios are often optimal. The grey bars in Fig. 4 correspond to the extreme fairness criterion. There, Cases 1, 2, 3, and 4 are optimal about 35%, 7%, 7%, and 50% of the time. Thus, full cooperation is desirable when two equal length messages must be transmitted.

Fixed MS placement: Figure 7 demonstrates the fraction of time the 4 cases are chosen under *fixed MS* placement. On account of the geometry of the layout, where relay 1 is aligned with mobile 1 and relay 2 is aligned with mobile 2, the asymmetric Case 3 is optimal roughly 50% of the time under the max throughput criterion, in contrast to the 20% for Case 2. Cases 1 and 4 are optimal 20% and 10% of the time, respectively. Sending a single message is optimal only 33% of the time, and again accounts for all the occasions in which Case 1 is chosen. Under the extreme fairness criterion, Case 1, 3 and 4 are optimal 25%, 5% and 70% of the time, respectively.

B. Sum-throughput of cooperation versus two non-cooperative baselines

The plots in Figs. 5, 6, 8, and 9 show the cumulative distribution functions (CDFs) of the throughput of the cooperation proposed here and compare them to two non-cooperative baselines. Baseline 1 is “**round-robin with relay**”, in which the BS (in a round robin fashion) alternates between transmitting to each mobile with the help of the relay with the best relay-mobile channel. Baseline 2 is “**best 2-hop**”, in which the 2 hop ($BS \rightarrow \text{relay } j \rightarrow MS_i$) path which takes the minimal time to transmit one unit of data is chosen. For the extreme fairness, one message is sent to *each* mobile along the best 2-hop path to that mobile, while for the maximum throughput criteria, only a *single* message is sent along the best 2-hop path.

As expected, the cooperative schemes yield higher sum-throughputs than the non-cooperative baselines. In these baselines, the mobile stations maximum ratio combine [20] the signals from the BS and relays. Despite not using any form of combining, the cooperative schemes still perform better due to the spatial diversity offered by both symmetric as well as asymmetric forms of cooperation. Based on our simulation results, the throughputs of the cooperative schemes are typically 20-30% higher than those of the baselines.

V. CONCLUSION

In this work we motivate the study of asymmetric cooperation as a possible optimal transmission strategy in the downlink of cellular systems employing cooperating relays. We provide an analytical framework, outline solutions, and demonstrate for two diametrically opposite optimization criteria that the asymmetric cases are often optimal. The percentage of time that asymmetric cooperation outperforms

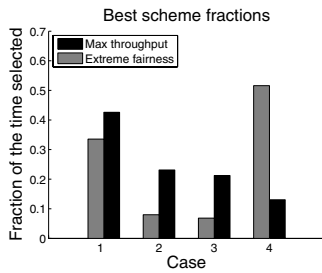


Fig. 4. Percentage of time the 4 cases are chosen, *random MS* placement. $P_R = P_B = 1000$, radius=10 units.

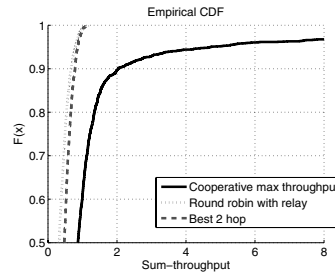


Fig. 5. CDF of sum throughput, max throughput criterion, *random MS* placement.

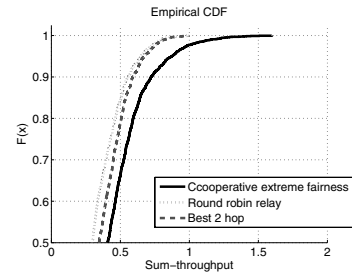


Fig. 6. CDF of sum throughput, the extreme fairness criterion, *random MS* placement.

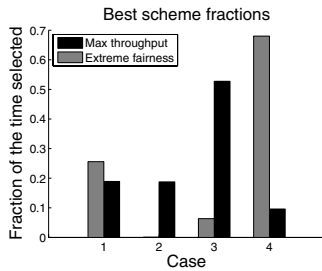


Fig. 7. Percentage of time the 4 cases are chosen, *fixed MS* placement. $P_R = P_B = 1000$, radius=10 units.

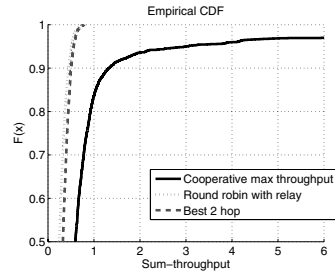


Fig. 8. CDF of sum throughput, max throughput criterion, *fixed MS* placement.

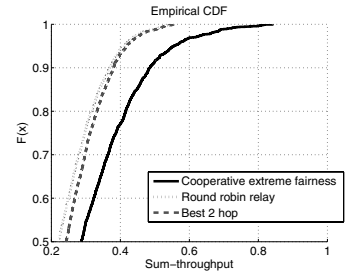


Fig. 9. CDF of sum throughput, extreme fairness criterion, *fixed MS* placement.

symmetric cooperation depends on the optimization criteria and channel conditions. Therefore, the goal of this paper is to highlight a new form of cooperation that should not be neglected and encourage others to consider it when designing standards or analytical frameworks involving cooperation. Future work includes the extension to different downlink coding and decoding techniques including, for example, the interference mitigating dirty-paper coding, considering asymmetry in multiple relay and/or mobile scenarios, and models in which only the channel fading statistics are known.

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