MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Cooperative Relay Networks Using Fountain Codes

Andreas Molisch, Neelesh Mehta, Jonathan Yedidia, Jin Zhang

TR2006-092 November 2006

Abstract

We investigate a cooperative communications scheme with N parallel relays, where both the transmissions from the source to the relays and from the relays to the destination use fountain codes. Receiver for codes can accumulate mutual information, while traditional energy collection methods, such as repetition or cooperative space-time codes, only accumulate energy. As a consequence, using fountain codes can reduce the total energy required for transmitting data from the source to the destination. We first analyze the scenario where the source stops transmitting and the relay nodes start transmitting after L relay nodes have successfully decoded the source data. We optimize L, and also give closed-form equations for the energy savings that can be achieved by the use of mutual-information-collection at the receiver instead of the traditional energy-collection methods. We then analyze an alternate scenario where each relay node starts its transmission to the destination as soon as it has decoded the source data, and helps the other relay nodes that are still in reception mode. Doing so further reduces the total transmission time and energy consumption.

IEEE Global Telecommunications Conference (GLOBECOM)

Cooperative relay networks using fountain codes

Andreas F. Molisch, Neelesh B. Mehta, Jonathan S. Yedidia, and Jinyun Zhang

***Abstract*— We investigate a cooperative communications scheme with N parallel relays, where both the transmissions from the source to the relays and from the relays to the destination use fountain codes. Receivers for fountain codes can accumulate mutual information, while traditional energy collection methods, such as repetition or cooperative space-time codes, only accumulate energy. As a consequence, using fountain codes can reduce the total energy required for transmitting data from the source to the destination. We first analyze the scenario where the source stops transmitting and the relay nodes start transmitting after L relay nodes have successfully decoded the source data. We optimize L, and also give closed-form equations for the energy savings that can be achieved by the use of mutual-information-collection at the receiver instead of traditional energy-collection methods. We then analyze an alternate scenario where each relay node starts its transmission to the destination as soon as it has decoded the source data, and helps the other relay nodes that are still in reception mode. Doing so further reduces the total transmission time and energy consumption.**

I. Introduction

Cooperative communications, where different nodes in a network work together in order to transmit information from a source to a destination, decreases energy consumption and improves the outage probability in a wireless network. For this reason, it has drawn great attention in recent years, see [1]–[5] and references therein. Papers in the area can be broadly classified into two categories: (i) analysis of large-scale networks, including routing algorithms and limiting behavior, and (ii) studies of fundamental building blocks, i.e., transmission that involves only a small number of nodes.

One of the building blocks that has been analyzed extensively is the transmission from the source to the destination via several decode-and-forward parallel relays (see Fig. 1). The source broadcasts its information, transmitting it to several or all of the available relay nodes (henceforth called "uplink" phase); the relay nodes then cooperate in transmitting the information to the destination ("downlink" phase). If the relay nodes have channel state information for the downlink, they can perform "virtual beamforming", i.e., adjusting the amplitude and phases of the transmit signal to optimize the receive signal [6], [7]; however, such a scheme requires frequent feedback and may be sensitive to phase noise and variations of the channel impulse response. If the relay nodes do not have channel state information, then the receiver can at best collect the energy from the various relay nodes, e.g., through space-time coding or repetition coding [8], [9]. Refs. [10]–[12] presented a number of algorithms based on relay selection or space-time coding; [13], [14] developed the outage analysis of such relay schemes when the links operate at a given signal-to-noise ratio.

The authors are with the Mitsubishi Electric Research Labs, Cambridge, MA, USA 02139. Emails: { molisch, mehta, yedidia, jzhang } @merl.com.

A. F. Molisch is also at the Dept. of Electroscience, Lund Univ., Sweden.

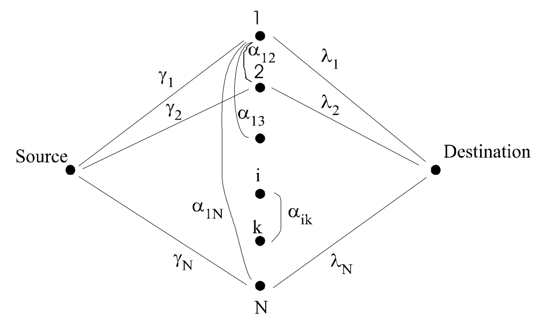

Fig. 1. System setup with source, destination, and N parallel relays. Channel gains between source to the i-th relay, i-th relay and the destination, and i-th and k-th relays are denoted by γ_i, λ_i, and α_{ik}, respectively.

In this paper, we propose a new approach for the relaying of information that is based on the use of fountain codes [15]–[17] (see also [18], [19] for an overview). They encode the source information in an infinitely long codestream. A receiver can recover the original information from unordered subsets of the codestream, as long as the total obtained mutual information marginally exceeds the entropy of the source information. Thus, it is certain that the destination can decode the transmitted signal; only the required transmit energy and the transmission time depend on the channel states. Fountain codes were originally designed for erasure channels, but their performance on AWGN channels has since been studied and shown to be good [20], [21]. They have been suggested for use in wired ethernet-like applications, as well as for point-to-point [22] and broadcast and multicast applications [23] in wireless networks. However, to the best of our knowledge, their use in cooperative relay networks has not been analyzed.

In this paper, we investigate how fountain codes can help the relaying of information through several parallel relay nodes. We investigate both a quasi-synchronous and an asynchronous protocol. In the quasi-synchronous protocol, each relay node informs the other nodes when it has decoded the source information, and is thus ready to forward the information to the destination; after L nodes have decoded the information, the source stops its transmission, and all the relay nodes then transmit the information to the destination. If we use different fountain codes, the destination can accumulate the *mutual information* (and not just the energy) from the different relay nodes. In the asynchronous protocol, each relay node starts to transmit to the destination as soon as it has decoded the source data. Due to the properties of fountain codes, this speeds up transmission as it provides useful information not only to the destination, but also to the relay nodes that have not finished the decoding process yet.[1]

The rest of the paper is organized as follows: in Sec. II, we describe the basic system model and the assumptions underlying our analysis. Section III describes the quasi-synchronous protocol and its performance with different types of receivers, while Sec. IV concentrates on the asynchronous protocol. A summary and conclusions in Sec. V wrap up this paper.

II. System model

The basic system model is shown in Fig. 1. A source needs to transmit a codeword with bandwidth-normalized entropy (size) H_{target}, given in nats/Hz, to the destination via N parallel relays, which employ a decode-and-forward approach. To simplify notation, we assume that the destination cannot obtain information directly from the source, though inclusion of such a direct path in the performance analysis is straightforward. The source, as well as the relays, use fountain codes for encoding the information. We will discuss in Secs. III and IV the details of the transmission protocols, i.e., at which time which node transmits what information. All nodes operate in half-duplex mode, i.e., they can either transmit or receive, but not do both simultaneously.

In the following, we also assume that transmission is done with a direct-sequence spectrum spreading technique. Such an approach is useful for sensor networks, as it allows different information streams to be transmitted in a flexible and decentralized way, and distinguished at the receiver. The transmit power of all nodes is P_{T}. The propagation channels between the different nodes are modeled as frequency-flat, block-fading channels, and the channel gains are independent and exponential-distributed, corresponding to Rayleigh fading of the amplitudes. The channel gain γ_i between the source and the i-th relay node has the probability density function (pdf)

$$f_{\gamma_i}(\gamma_i) = \frac{1}{\overline{\gamma}} \exp\left[-\gamma_i/\overline{\gamma}\right], \quad \gamma_{\mathrm{i}} \geq 0 \tag{1}$$

where $\overline{\gamma}$ is the mean channel gain of the i-th channel. Without loss of generality, we assume that the noise and transmit powers are normalized to unity, so that channel gains and SNR become synonymous. For ease of the evaluation, we assume that $\overline{\gamma}_i = \overline{\gamma}$, for all i; the case of unequal mean channel gains is considered in [25]. Similarly, the channel gains from the i-th relay node to the destination are denoted as λ_i with mean $\overline{\lambda}_i = \overline{\lambda}$, and the channel gains from the i-th to the k-th relay node is written as α_{ik} with mean $\overline{\alpha}_{ki} = \overline{\alpha}$.

We consider two receiver types at the destination: ideal energy collection receivers, which can accumulate the energy of the signals from the different relay nodes, and ideal mutual information accumulating receivers. In a CDMA system, a Rake receiver well approximates an energy accumulating receiver if the signals from the different relay nodes arrive with relative delays that are larger than the chip duration. Alternatively, we could also use space-time codes for transmission. On the other hand, for accumulating mutual information, despreaders and decoders that can separate the different information streams sent using different spreading codes and fountain codes are required.[2]

In order to simplify the following computations and discussions, we make the following assumptions:
(i) The fountain codes are perfect at all desired rates, i.e., the receiver is capable of correctly identifying the transmitted codeword as soon as the transmission time multiplied with the instantaneous channel capacity is equal to the entropy of the codeword.[3]
(ii) The feedback, in which the receiver indicates the successful identification of the codeword, is instantaneous. We furthermore assume that its impact on the energy budget and spectral efficiency is negligible, which is reasonable if the codewords are long.

III. Quasi-synchronous transmission

A. The protocol

In the first step, the source transmits the data stream encoded by a fountain code. The various relay nodes listen to the source data; as soon as they have acquired sufficient energy to decode the data, they transmit an acknowledgment to the source that their reception was successful. Once the source has received L acknowledgments, it ceases transmission. At the same time, the relay nodes switch from reception to transmission. For this second phase, we consider two cases:
1. All relay nodes transmit the source data encoded with the same fountain code, which can be the same as the one used by the source. (Sec. V will discuss the advantages of fountain codes versus conventional capacity-achieving codes for that step.) Given the random locations of the relay nodes, the signals arrive at the destination with slightly different delays. We assume in the following that those delays are larger than the chip duration, but much smaller than the symbol duration.[4] This assumption can be well fulfilled in direct-sequence CDMA systems with large spreading factors. At the destination, a Rake receiver is used to accumulate the energy from the signals transmitted by the different nodes.

[1] Note that this is somewhat reminiscent of the idea of "cognition" [24]. However, in cognition, all nodes have source information they need to transmit, while we are considering a pure relaying scenario.

[2] The problem of transmitting different codes from different nodes, where the nodes can help each other, bears a certain similarity to the problem of coded cooperation, as explored, e.g., in [5]. However, there are some key differences, most notably that (i) the underlying source information is the same for all nodes, and (ii) the nodes start transmission at different times.

[3] This assumption is an oversimplification in two respects:

1) It is impossible to generate "universal" fountain codes that are simultaneously perfect at all possible rates [20]. However, in practice, fountain codes can be found whose overhead compared to perfect codes is bounded and not too large [20].
2) A fountain code is only capable of providing the mutual information between channel input and output, not necessarily the channel capacity [26]. Achieving the capacity requires the knowledge of the channel so that the correct input distribution can be chosen. However, under our assumptions (quasi-static flat-fading channel and fixed transmit power), this is not a problem, since the optimum input distribution is Gaussian for any channel state.

[4] Note that the relay nodes thus need to be synchronous only within one symbol duration. There is no necessity to be chip-synchronous, or to be co-phased (as they would have to be for beamforming transmission).

2. Each relay nodes uses a different fountain code, and a different spreading code for the transmission. In that case, the destination distinguishes the signals from the different relay nodes through their different spreading codes, and accumulates the mutual information. The synchronization between nodes can be relaxed in this case: there is no requirement for the signals from different nodes to arrive at the destination within a symbol duration.

In either case, the destination sends a signal to the relay nodes to stop transmission as soon as it has successfully decoded the source data. The feedback of the bit that indicates successful decoding (from relay to source, as well as from destination to relays) can be done on a separate channel, e.g., on a different time or frequency channel.

Intuitively, the difference between the use of a single and multiple fountain codes can most easily be understood for the simple example of binary signaling using two relays on an erasure channel with erasure probability p_e. If the relays use the same fountain code, then each bit will be erased with probability p_e^2, so $1-p_e^2$ bits are effectively received per relay transmission. On the other hand if two different fountain codes are used, the transmissions are independent, and $2(1-p_e)$ bits per relay transmission are received.

Note that the complexity of the receivers required for energy accumulation and mutual information accumulation does not differ significantly. If the sampling rate of the analog-to-digital converter is identical to the symbol rate, then both receivers require L correlators. In the first case, all correlators form the correlation with the same spreading sequence, and add up the results using maximum-ratio-combining.[5] In the second case, each correlator is used for the detection of the signal from a different relay node. The main complexity difference lies in the actual decoder, which is more complex if multiple fountain codes are used. If the considered CDMA-system is code-limited, the spectral efficiency of this second method is worse because it uses up multiple spreading codes for the transmission of one source codeword, though the improved coding gain partly offsets this effect.

B. Theory

In the following, we compute the energy required for the uplink (source to relays) and the downlink (relay to destination) transmissions. We can compute these two steps separately since the fading of the uplink and downlink channels are assumed to be independent.

1) Cost of transmission until L nodes have received information: We derive the pdf of the time it takes for L nodes to each receive and decode the source data. For this, we first compute the pdf of the time, y_i, required for each of the relay nodes, i, and then derive its order statistics.

[5]Alternatively, a receiver can use only a single correlator, whose output is sampled L times during each symbol duration. This saves some hardware complexity. However, the signals can arrive from the different relay nodes at irregular intervals, and thus necessitate an ability of the ADC to sample at the chip rate. This fast sampling of the ADC increases the energy consumption significantly (more than by a factor of L), and thus might not be desirable for sensor-network applications.

From Shannon's famous capacity equation,

$$y_i = \frac{H_{\text{target}}}{\log\left[1+\gamma_i\right]}, \qquad \text{for } \gamma_i \geq 0 \tag{2}$$

where the distribution of the channel gains γ_i is given in Eq. (1). Using a standard transformation of variables with the Jacobian [27], the pdf of y_i is

$$f_{y_i}(y) = \frac{H_{\text{target}}}{\overline{\gamma} y^2} \exp\left[\frac{1}{\overline{\gamma}} + \frac{H_{\text{target}}}{y} - \frac{e^{H_{\text{target}}/y}}{\overline{\gamma}}\right] \tag{3}$$

for $y \geq 0$, and its cumulative distribution function (cdf) is

$$F_{y_i}(y) = \frac{1}{H_{\text{target}}} \exp\left[\frac{1}{\overline{\gamma}} - \frac{e^{H_{\text{target}}/y}}{\overline{\gamma}}\right], \qquad \text{for y} \geq 0 \tag{4}$$

Let us now consider the ordered times $y_{(i)}$, so that $y_{(1)} < y_{(2)} < .... < y_{(n)} < ... < y_{(N)}$. We want to find out the time that is required for the L-th node to decode the source data, since this is the time when the source stops the transmission. It is well known that [28]

$$f_{y_{(L)}}(y) = \frac{N!}{(L-1)!(N-L)!} f_y(y) F_y(y)^{L-1} \left[1 - F_y(y)\right]^{N-L} \tag{5}$$

Inserting Eqs. (3) and (4), and using binomial expansion of the term $\left[1-F_y(y)\right]^{N-L}$, we obtain, for $y_{(L)} \geq 0$,

$$f_{y_{(L)}}(y) = \frac{H_{\text{target}}}{\overline{\gamma}} \frac{N!}{(L-1)!(N-L)!} \sum_{k=0}^{N-L} \binom{N-L}{k} (-1)^k \frac{e^{H_{\text{target}}/y}}{y^2} \exp\left[\frac{L+k}{\overline{\gamma}}\left(1 - e^{H_{\text{target}}/y}\right)\right]$$

The mean energy expenditure is then given by

$$\frac{H_{\text{target}}}{\overline{\gamma}} \frac{N!}{(L-1)!(N-L)!} \times \sum_{k=0}^{N-L} \binom{N-L}{k} (-1)^k \exp\left[\frac{L+k}{\overline{\gamma}}\right] R_0\left(\frac{L+k}{\overline{\gamma}}\right)$$

where $R_m(x) = \int_1^{\infty} t^m \exp(-xt)/\ln(t) dt$.

2) Cost of transmission from relay nodes to destination – Single fountain code case: We now compute the pdf of the energy required for the downlink transmission when using only a single fountain code. The relay nodes transmit with equal energy, and the receiver accumulates the energy. Assuming equal mean channel gains for all downlink channels, the pdf of the effective channel gain is [29]

$$f_\lambda(\lambda) = \frac{1}{(L-1)!} \frac{\lambda^{L-1}}{\overline{\lambda}^L} \exp\left[-\frac{\lambda}{\overline{\lambda}}\right], \qquad \text{for } \lambda \geq 0 \tag{6}$$

Performing a variable transformation analogous to Eq. (2), the pdf of the downlink transmission time, z, is

$$f_z(z) = \frac{H_{\text{target}}}{(L-1)! \overline{\lambda}^L z^2} \left(e^{H_{\text{target}}/z} - 1\right)^{L-1} \times \exp\left[\frac{1}{\overline{\lambda}}\left(1 - e^{H_{\text{target}}/z}\right) + \frac{H_{\text{target}}}{z}\right]$$

for $z \geq 0$. The mean energy expenditure for the downlink equals $L\overline{z}$, which can be shown to be

$$\frac{LH_{\text{target}}\exp(1/\overline{\lambda})}{(L-1)!\overline{\lambda}^L}\sum_{k=0}^{L-1}\binom{L-1}{k}(-1)^{L-k-1}R_k(1/\overline{\lambda})$$

3) Cost of transmission from relay nodes to destination – Multiple fountain codes case: When the relay nodes use different fountain codes, the receiver accumulates the mutual information of the signals transmitted by the relays, and not their energy. Thus, the total transmission rate is the sum of the rates from the relays. The pdf of the rate from a single node is

$$f_r(r) = \frac{1}{\overline{\lambda}}\exp\left[\frac{1}{\overline{\lambda}} + r - \frac{e^r}{\overline{\lambda}}\right], \quad \text{for } r \geq 0 \tag{7}$$

The sum of the rates is most easily computed via its moment-generating function, which can be shown to be [25]

$$M(j\omega) = \left[\exp(1/\overline{\lambda})\left(1/\overline{\lambda}\right)^{j\omega}\Gamma(1-j\omega, 1/\overline{\lambda})\right]^L \tag{8}$$

where $\Gamma(\alpha, x) = \int_x^\infty e^{-t}t^{\alpha-1}dt$ [30]. From this, we obtain the pdf of the mutual information, and – via a variable transformation – the pdf of the required transmission time

$$f_z(z) = \frac{H_{\text{target}}}{2\pi}\int_{-\infty}^{\infty} M(j\omega)\exp\left[\frac{j\omega H_{\text{target}}}{z}\right]\frac{1}{z^2}d\omega \tag{9}$$

The total mean expended energy can be computed directly from this pdf.

C. Results

Figure 2 shows the mean energy expenditure as a function of used relay nodes, L, for different values of available relay nodes, N. We find that there is a pronounced minimum that depends on the number of available relay nodes. Further analysis (not shown here for space reasons) shows that the energy expenditure for the uplink (source-to-relay) part sharply increases with increasing L. This is intuitive because a larger L means that the information has to be transmitted to nodes with progressively worse uplink channels. On the other hand, we find that the energy expenditure for the downlink drops sharply as L increases from 1 to 5, and saturates thereafter if the receivers accumulate mutual information. For energy-accumulating receivers, even the downlink part by itself shows a clear minimum in the required transmission energy.

Furthermore, it is interesting to investigate the pdf of the total energy expenditure. Figure 3 shows the pdf for $N = 10$, and $L = 2$ and 5. Here we find that – as expected – the amount of concentration around the mean value increases with increasing L. We see that only for mutual-information-collecting receivers, a high diversity order can be achieved without an excessive penalty in the mean expended energy.

IV. Asynchronous transmission

A. The protocol

In the protocol of Sec. III, the relay nodes receive their information *only* from the source node. However, we find that when we use fountain codes, the relay nodes can help each other to receive the information faster, and thus accelerate the information relaying process. The key idea here is that a relay node starts to transmit information to the destination as soon as it has received sufficient information to decode the codeword. This transmission can also be heard by relay nodes that are still in the reception mode. Thus, the relay nodes that are in transmit mode help the nodes that are still receiving.

The protocol uses the following steps:

1. We establish an ensemble of $M \geq N+1$ spreading codes and fountain codes. The source node, and each of the relay nodes, is assigned one of those code pairs.
2. The source node transmits information to all of the relay nodes, using its assigned spreading code and fountain code.
3. All relay nodes constantly receive on all the possible spreading codes and accumulate the mutual information. To avoid unnecessary reception of noise, the protocol signals when the transmission on a given spreading code starts.
4. As soon as a relay node has sufficient information to decide on a codeword, it stops receiving and starts transmitting the information using its assigned spreading and fountain code. The relay nodes that are in reception mode receive and accumulate the mutual information from the source node and

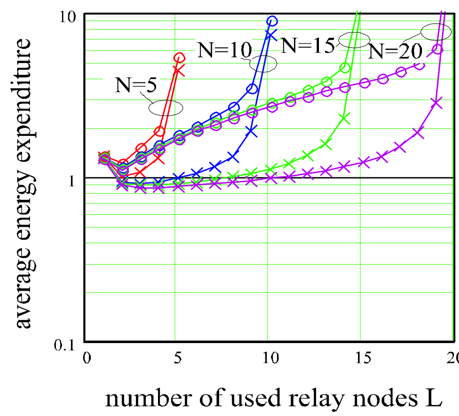

Fig. 2. Mean energy expenditure as a function of the number of active relay nodes L for different numbers of available relay nodes, N. Lines with crosses: multiple fountain codes (mutual-information-accumulating receiver). Lines with circles: single fountain code (energy-accumulating receiver). $\overline{\gamma} = \overline{\lambda} = 10$, $H_{\text{target}} = 1$.

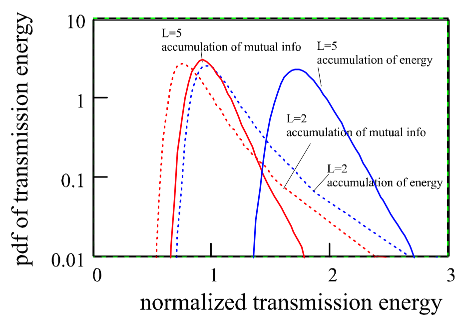

Fig. 3. Pdf of transmission energy for $N = 10$, $L = 2$ and 5, $\overline{\gamma} = \overline{\lambda} = 10$.

all the transmitting relay nodes.
5. The destination node also constantly receives on all possible spreading codes, and, thus, accumulates the information from the various relay nodes. As mentioned, the direct contribution from the source to the destination is neglected in this paper. But, it can be easily incorporated in to the analysis.

B. Theoretical formulation

Let us assume in the following that all channel gains are deterministic. In that case, a closed-form equation for the total transmission time is feasible. We denote as τ_1 the time until one relay node has gathered sufficient information:

$$\tau_1 = \frac{H_{\text{target}}}{\log\left[1 + \gamma_{k_1}\right]} \tag{10}$$

where k_1 is the index of the relay node that finishes the decoding first, i.e., has the highest channel gain to the source node. Next, we determine the time until a second relay node has sufficient information. The mutual information that has arrived at the i-th node by time T_i is

$$H_i = T_i \log\left[1 + \gamma_i\right] + (T_i - \tau_1) \log\left[1 + \alpha_{k_1 i}\right] \tag{11}$$

Thus, the time at which a second node decodes the codeword is

$$\widetilde{\tau}_2 = H_{\text{target}} \min_{i \neq k_1} \frac{1 + \log\left[1 + \alpha_{k_1 i}\right] / \log\left[1 + \gamma_{k_1}\right]}{\log\left[1 + \gamma_i\right] + \log\left[1 + \alpha_{k_1 i}\right]} \tag{12}$$

We denote the index of the node that achieves this minimum as k_2. The time during which *exactly* two nodes (the source node plus the relay node with index k_1) is active, is denoted as $\tau_2 = \widetilde{\tau}_2 - \tau_1$. This procedure can be continued to include more relay nodes. The mutual information that the i-th relay node gets after two relay nodes have switched to transmission mode is $H_i = T_i \log\left[1 + \gamma_i\right] + (T_i - \tau_1) \log\left[1 + \alpha_{k_1 i}\right] + (T_i - \widetilde{\tau}_2) \log\left[1 + \alpha_{k_2 i}\right]$, from which we can compute $\widetilde{\tau}_2$ and τ_2 and so on. Transmission stops at time t when

$$\sum_{i=1}^{N} \mathcal{H}(t - \widetilde{\tau}_i) \log\left[1 + \lambda_{k_i}\right] = H_{\text{target}} \tag{13}$$

where $\mathcal{H}(x)$ is the Heaviside step function. We set $\tau_i = 0$ if the transmission to the source is complete before all relay nodes have decoded the message. The total transmission time can then be computed as $\sum_i \tau_i$, and the total transmission energy is $\sum_{i=1}^{N+1} i\tau_i$, as transmission during time τ_i involves transmission from i relay nodes plus the source node. We assume here that the transmission from the source node will continue until the destination has successfully received the code word; some energy can be saved if the source monitors the relay nodes, and stops transmitting as soon as all N relay nodes are in transmission mode.

Each step in the above formulation requires the repeated use of order statistics, which makes a closed-form evaluation of the statistics of the total required transmission time difficult. Therefore, we resort to Monte Carlo-based averaging over the fading statistics in the next section.

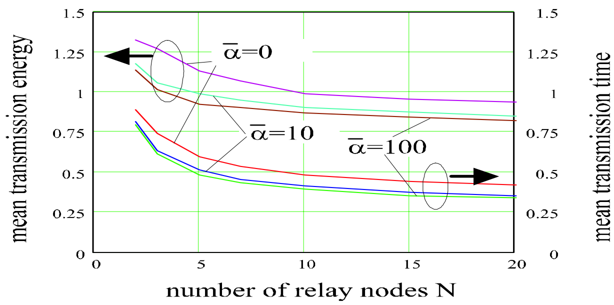

Fig. 4. Mean transmission time and mean expended energy for the asynchronous protocol as a function of the number of available relay nodes. Mean link gain between the relay nodes $\overline{\alpha}$ is 0, 10, and 100. $\overline{\gamma} = \overline{\lambda} = 10$.

C. Simulation results

Figure 4 shows the mean time that is required to transmit the message to the receiver as a function of the number of relay nodes that are involved in the process. We find that most of the benefits can be realized with about five relay nodes. Note that now the results strongly depend on the channel gains *between* the relay nodes. From top to bottom, the three curves show the results for the inter-relay average channel gains being 0 (no inter-relay links), 10 (the same as the average uplink and downlink channel gains), and 100 (strong inter-relay links). As the relay nodes that have already decoded help the other relay nodes that have not yet decoded, the total transmission time decreases as the channel gains between relays increase.

Fig. 4 also shows the mean energy expenditure as a function of the number of available relay nodes. We see that the total consumed energy decreases as the number of available relay nodes increases, but this decrease is relatively smaller than the decrease in the transmission time. This is because the number of transmitting nodes is not fixed, but can even increase as more nodes become available. Further investigations show that the energy expenditure for relaying saturates very quickly as N increases if the channel gains between the relay links are sufficiently strong. That can be explained as follows: due to the strong links between the relays, the relay nodes finish their decoding almost simultaneously – at the time when the *first* relay node gets all the required information. The subsequent part of the protocol then becomes similar to the first protocol with $L = N$.

However, it is noteworthy that in the case of delay-constrained applications, having a large number of nodes has definite advantages. Not only does the mean transmission time decrease with the number of nodes, but also the pdf of the transmission times is more concentrated (less variability) around that mean value. Figure 5 shows the pdf of the transmission energy when the number of available relay nodes is $N = 10$. We see that this pdf is more concentrated than the pdfs for the quasi-synchronous case in Fig. 3.

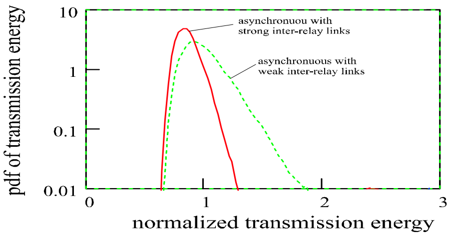

Fig. 5. Pdf of transmission energy with $N = 10$ relay nodes for weak and strong inter-relay links. $\overline{\gamma} = \overline{\lambda} = 10$.

V. Summary and conclusions

In this paper, we proposed and analyzed the use of fountain codes for relaying in sensor networks. We compared a quasi-synchronous and an asynchronous protocol. In the quasi-synchronous case, all relay nodes switch from the receiving to the transmitting mode at the same time, while in the asynchronous case, each relay node switches to the transmission mode as soon as it has decoded the source data. We found that for synchronous transmission, there is a distinct optimum for the number of relay nodes that should transmit the information to minimize total energy consumption; this optimum typically lies between 2 and 4. Furthermore, we found that the asynchronous protocol leads to additional savings in the transmit energy and, especially, reduces the latency of the transmission.

It is worthwhile to discuss whether fountain codes are strictly necessary for the schemes presented in this paper. For the quasi-synchronous scheme, the same performance could be achieved without fountain codes if the transmitters had knowledge of the channel gains, and used capacity-achieving codes specifically designed for the channel gain of the desired link. However, this requires a feedback of all the channel gains to the respective transmitters, which is less spectrally efficient than the single bit we require to inform the relays about the successful decoding. For the asynchronous scheme, fountain codes have a unique advantage as they allow the "better" relay nodes to help the weaker ones. The transmission of ordinary capacity-achieving codes, e.g., LDPC codes, from a relay node would not help the other relay nodes with their decoding until *all* the coded information bits have been received. As the LDPC code used by a relay depends on its channel gain to the destination, and not on the inter-relay links, it is likely that the destination receives and finishes decoding the codeword sent by the relay before other relays do.

Future work involves generalizing the schemes of this paper to multi-hop relay networks. The promising results obtained here motivate a further study especially of asynchronous protocols where all relay nodes can help each other in the decoding and forwarding of the information. At the same time, smart algorithms need to be found that prevent the participation of an excessive number of nodes.

Acknowledgment

We thank Prof. Frank Kschischiang for valuable discussions about fountain codes.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part i: System description," *IEEE Trans. Comm.*, vol. 51, p. 1927 -Ű 1938, 2003.
[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part ii: Implementation aspects and performance analysis," *IEEE Trans. Comm.*, vol. 51, p. 1939 -Ű 1948, 2003.
[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, 2004.
[4] A. Stefanov and E. Erkip, "Cooperative coding for wireless networks," *IEEE Trans. Comm.*, vol. 52, pp. 1470 – 1476, 2004.
[5] T. E. Hunter, s. Sanayei, and A. Nosratinia, "Outage analysis of coded cooperation," *IEEE Trans. Inf. Theory*, vol. 52, pp. 375–391, 2006.
[6] G. Barriac, R. Mudumbai, and U. Madhow, "Distributed beamforming for information transfer in sensor networks," in *3rd Int. Symp. Inf. Proc. Sensor Networks IPSN 2004*, pp. 81–88, 2004.
[7] M. Abdallah and H. Papadopoulos, "Beamforming algorithms for decode-and-forward relaying in wireless networks," in *Proc. Conf. Inform. Sciences & Systems (CISS)*, 2005.
[8] I. Maric and R. D. Yates, "Cooperative multihop broadcast for wireless networks," *IEEE J. Selected Areas Comm.*, vol. 22, pp. 1080–1088, 2004.
[9] I. Maric and R. D. Yates, "Cooperative multicast for maximum network lifetime," *IEEE J. Selected Areas Comm.*, vol. 23, pp. 127–135, 2005.
[10] J. Luo, R. S. Blum, L. J. Cimini, L. J. Greenstein, and A. M. Haimovich, "Link-failure probabilities for practical cooperative relay networks," in *Proc. IEEE VTC 2005-Spring*, pp. 1489–1493, 2005.
[11] A. Jardine, S. McLaughlin, and J. Thompson, "Comparison of space-time cooperative diversity relaying techniques," in *Proc. IEEE VTC 2005-Spring*, pp. 2374–2378, 2005.
[12] R. Madan, N. B. Mehta, A. F. Molisch, and J. Zhang, "Energy-Efficient Cooperative Relaying over Fading Channels with Simple Relay Selection," in *Proc. IEEE Globecom*, in press, 2006.
[13] Y. Zhao, R. Adve, and T. J. Lim, "Outage probability at arbitrary SNR with cooperative diversity," *IEEE Comm. Lett.*, vol. 9, pp. 700–702, 2005.
[14] A. Khisti, U. Erez, and G. Wornell, "Fundamental limits and scaling behavior of cooperative multicasting in wireless networks," *IEEE Trans. Inf. Theory, submitted*, 2006.
[15] M.Luby, "LT codes," in *Proc. 43rd Annual IEEE Symp. Foundations of Computer Science(FOCS)*, pp. 271–282, 2002.
[16] J. W. Byers, M. Luby, and W. Mitzenmacher, "A digital fountain approach to asynchronous reliable multicast," *IEEE J. Selected Areas Comm.*, vol. 20, pp. 1528–1540, 2002.
[17] A. Shokrollahi, "Raptor codes," in *Proc. Int. Symp. Inf. Theory*, p. 36, 2004.
[18] M. Mitzenmacher, "Digital fountains: A survey and look forward," in *Proc. IEEE Inf. Theory Workshop 2004*, pp. 271–276, Oct. 2004.
[19] D. J. C. MacKay, "Fountain codes," *IEE Proc. - Comm.*, vol. 152, pp. 1062–1068, 2005.
[20] O. Etasami, M. Molkaraie, and A. Shokrollahi, "Raptor codes on symmetric channels," in *Proc. Int. Symp. Inf. Theory*, p. 38, 2004.
[21] R. Palanki and J. Yedidia, "Rateless codes on noisy channels," in *Proc. Int. Symp. Inf. Theory*, p. 37, 2004.
[22] J. Castura and Y. Mao, "Rateless coding over fading channels," *IEEE Comm. Lett.*, vol. 10, pp. 46–48, 2006.
[23] H. Jenkac and T. Stockhammer, "Asynchronous media streaming over wireless broadcast channels," in *IEEE Int. Conf. Multimedia 2005*, pp. 1318–1321, 2005.
[24] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, to appear, 2006.
[25] A. F. Molisch, N. B. Mehta, J. Yedidia, and J. Zhang, "Cooperative communications with fountain codes," *IEEE Trans. Wireless Comm., submitted*, 2006.
[26] F. Kschischiang, "private communication," 2006.
[27] A. Papoulis, *Probability, random variables, and stochastic processes*. McGraw Hill, 1965.
[28] S. Kokoska and D. Zwillinger, *Standard probability and statistics tables and formulae*. CRC Press, 2000.
[29] A. F. Molisch, *Wireless communications*. IEEE Press - Wiley, 2005.
[30] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. Dover, 1966.