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Ritesh Madan, Neelesh Mehta, Andreas Molisch, Jin Zhang

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## I. Introduction

Cooperative communication schemes, in which wireless nodes cooperate with each other in transmitting information, promise significant gains in overall throughput and energy efficiency [1]–[3]. In this paper, we concentrate on data transmission where a set of relay nodes forward data in parallel from a source to a destination, using a decode-and-forward approach. Our goal is to minimize the total energy consumption of the transmission process, while keeping the outage probability in fading channels below a specified level to guarantee a certain reliability in delivering data.

The broadcast nature of the wireless channel can be exploited to save energy by transmitting (broadcasting) to multiple relay nodes simultaneously; some or all of which decode the signal. Cooperative beamforming algorithms have been proposed in [5]–[10] to save energy in transmitting data from the relays to the destination. In this approach, the relays, armed with the required channel state information (CSI), linearly weight their transmit signals so that they add up coherently at the destination. The optimal weights to minimize energy consumption can be computed easily (see, for example, [5]).

While [4]–[9] considered this approach, they did not factor in the cost of obtaining CSI for relay cooperation. However, for fading channels, it is often energy-intensive to feed back CSI reliably to all the relays. Not factoring in the overhead for obtaining CSI leads to the trivial result that all the available relays should beamform information to the destination. In this paper, we explicitly model and factor in the cost of acquiring CSI. In our setup, the source broadcasts data to the relays, some or all of which then decode the signal and cooperatively beamform to forward their received data to the destination. The system is designed to satisfy an outage constraint. The relays obtain the CSI necessary to enable energy efficient cooperation by means of a training process, where the destination first obtains the CSI, and feeds it back to the relays.

When we minimize the total energy consumption for data transmission and CSI acquisition, there is a tradeoff between decreasing the energy consumption for data transmission by using more relays and decreasing overhead for CSI acquisition by using less relays. This naturally raises the problem of computing an optimal relay selection rule. The relay selection rules in [10]–[15], are restrictive in the sense that they either always use all the available relays or always use just one relay. Of the four simple relay selection criteria described in [10], two criteria select a single relay based on mean channel gains, while the other two select all the relays. A single relay node is selected based on average CSI, e.g., distance or path loss, in [11], [14], [15], and on the instantaneous fading states of the various links in [12].

For cooperative beamforming, we analyze the total energy cost of data transmission and we optimize over a more general class of relay selection rules than those considered in the literature. In particular, we analyze a general, yet simple, class of relay selection rules that select the number of relays based on the set of nodes that decode data from the source, but not on their instantaneous fading states. Both single relay selection and selection of all relays are clearly special cases of this rule. Note that the actual set of relays used does depend on the instantaneous channel states; the relays with the best instantaneous channels to the destination are chosen. Conditioned on the number of relays that decode, the class we consider also achieves the same diversity order as the instantaneous state-based rules (see, for example, [16]).

## II. System Model

Figure 1 shows a schematic of the relay network. It contains one source node, one destination node, and $N$ relays. The channels from the source to the relays as well as from the relays to the destination are frequency non-selective channels that undergo independent Rayleigh fading. Thus, the channel power gains from source to relays (S-R), denoted by $h_i$, and from relays to destination (R-D), denoted by $g_i$, are independent, exponentially distributed random variables with means $\bar{h}_i$ and $\bar{g}_i$, respectively, where $i = 1, \ldots, N$.

At all nodes, the additive white Gaussian noise has a power spectral density of $N_0$. All the transmissions use a bandwidth

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of $B$ Hz and occur with a fixed (bandwidth-normalized) rate of $r$ bits/symbol and use Nyquist pulses. Fixing $r$ simplifies the design of the relays as they do not need to remodulate their transmissions using a different signal constellation. We assume that a node can decode data only if the received signal to noise ratio (SNR) exceeds a threshold, $\gamma_t$, which is a function of the rate, $r$. For purposes of illustration, we use the Shannon capacity formula to determine the following relationship between the threshold and $r$: $\gamma_t = (2^r - 1)$.[1]

### A. Cooperative Communication Scheme

A single message gets sent from the source to the destination, via the cooperative relay nodes, via the following steps.

*Broadcast*: The source, which does not know the relay channel gains *a priori*, has no choice but to broadcast data to the relays using a fixed transmission power, $P_S$, at a predetermined fixed rate, $r$ bits/symbol, for $T_d$ symbol durations. The received power at relay $i$ is $h_i P_S$. This node can decode the data correctly only if the received signal to noise ratio (SNR) exceeds the threshold $(2^r - 1)$. Thus, depending on the channel states, only a subset, $\mathcal{M} \subseteq \{1, \ldots, N\}$, of the relays successfully receives the data from the source. We use $M$ to denote the size of $\mathcal{M}$. Note that we assume the channel from the source to the destination is weak enough such that we can neglect the signal received by the destination directly from the source. With a minor modification, the same analysis framework can be used to analyze this extension, as well.

*Training*: Only the $M$ relay nodes that receive data successfully from the source send training sequences, at a rate $r$ bits per symbol and power $P_t$, to the destination. This enables the destination to estimate the instantaneous R-D channel gains, $\{g_i, i \in \mathcal{M}\}$. $P_t$ is taken to be sufficient for the destination to accurately estimate the gains of channels. Also, we assume that training transmission duration is $T_t = M$ symbol durations, which is the minimum possible value.[2] Each relay node only transmits for time $T_t/M$.

*Feedback of CSI*: Based on the channel gains, $\{g_i, i \in \mathcal{M}\}$, the destination either declares an outage with probability $P_{\text{out}}(\mathcal{M})$ or it selects a subset of $\mathcal{M}$, consisting of $K(\mathcal{M})$ relays with the best channel gains to the destination, and feeds back to them the required CSI. The CSI requirements are discussed in the next step.

*Simple Relay Selection Rules*: As mentioned above, the *number of relays*, $K(\mathcal{M})$, selected for data transmission (when outage is not declared) is only based on $\mathcal{M}$ and not on the instantaneous channel states, $\{g_i, i \in \mathcal{M}\}$. However, the actual set of relays (of size $K(\mathcal{M})$) used at each step does depend on the instantaneous channel gains. Similarly, outage is declared by the destination with a probability $P_{\text{out}}(\mathcal{M})$ that is a function of the set $\mathcal{M}$ and is independent of the channel gains.

[1]Similar threshold formulas exist for MFSK and MQAM and can also take into account the impact of error correction codes [17].

[2]Several mechanisms can be used to enable uncoordinated training among relay nodes. One approach, which is easily implementable, but is time inefficient, is to pre-assign a training slot for each relay. However, only relays that decode transmit a training sequence. MAC-based training mechanisms can be used to reduce this time-inefficiency.

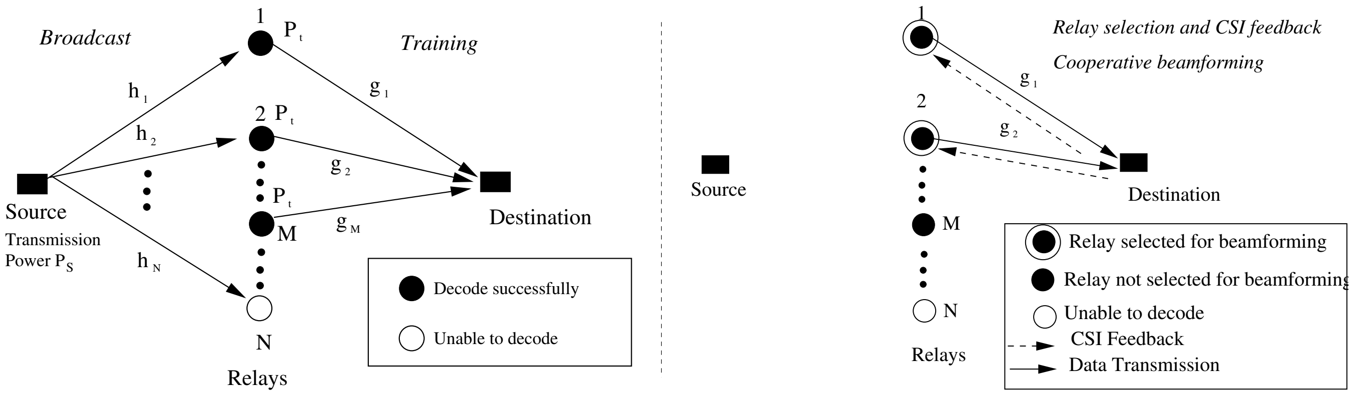
As shown in the next step, it is sufficient for the destination to feedback the sum of channel gains, $\sum_{i=1}^{K(\mathcal{M})} g_{[i]}$, to all the selected $K(\mathcal{M})$ nodes, and the channel power gain, $g_{[i]}$, and channel phase to only the corresponding selected relay $[i]$, where $g[1] > \ldots > g[M]$ and $[i]$ denotes the index of the relay with the ith largest gain. The feedback, at a rate of $r$, takes $T_f$ symbol durations. If $c$ symbols are required to feedback each channel gain and phase, then $T_f(K(\mathcal{M})) = c(1 + K(\mathcal{M}))$. Using the SNR threshold formula based on Shannon capacity, the minimum feedback power required to reach relay $i$ is $N_0 B(2^r - 1)/g_{[i]}$ and the minimum feedback power to broadcast the sum of channel gains to all the $K(\mathcal{M})$ relays is determined by node $[K(\mathcal{M})]$ (with the worst channel) and is $N_0 B(2^r - 1)/g_{[K(\mathcal{M})]}$.

The $M = 1$ case (when one relay decodes the data) needs special attention because the minimum power at which the relay needs to transmit to reach the destination is proportional to the inverse of the channel power gain. As is well known, infinite average power is necessary for channel inversion with zero outage over a Rayleigh fading channel [18]. Therefore, for this special case, the node is allowed to transmit only if its channel power gain exceeds a threshold. Thus, it does *not* transmit, with a probability of $\delta$, even if the destination has not declared an outage. We will assume that $\delta$ is a fixed system parameter. To summarize, when $M \geq 2$, the relays cannot transmit if the destination declares an outage, while for $M = 1$, the relay cannot transmit when the destination declares an outage or when the gain is below a threshold. Another minor difference is that when the destination does allow the single relay to transmit, the destination has to feedback to this relay only the CSI for the channel from relay to the destination, which takes $c$, not $2c$, symbol durations.

*Cooperative Beamforming*: Given the knowledge of the CSI, the optimal transmission power at each selected relay $i$ can be shown to be [5] $\frac{g_{[i]}}{\left(\sum_{j=1}^{K(\mathcal{M})} g_{[j]}\right)^2} N_0 B(2^r - 1)$. The $K(\mathcal{M})$ nodes cooperate, i.e., transmit coherently, to send data at a rate $r$ bits/symbol to the destination for $T_D$ symbol durations.

$N + cN + T_d$ must be less than the coherence time of the channel. We model only the energy required for radio transmission and not the energy consumed for receiving. This is justifiable as the radio transmission is the dominant component of energy consumption for long range transmissions [17]. Feedback quantization is taken to be sufficiently fine to not affect beamforming performance.

For cooperative beamforming to work, the relay transmissions need to be coherent and synchronized. Preliminary mechanisms for ensuring synchronization among simple distributed nodes for cooperative beamforming were proposed in [7], which also showed that cooperative beamforming reduces energy consumption even with imperfect synchronization. Imperfect synchronization can also be overcome by employing a more sophisticated Rake receiver at the destination. A detailed analysis of the impact of quantization and imperfect synchronization is beyond the scope of this paper.




(a) Broadcast and training

(b) CSI feedback and data transmission

Fig. 1. Cooperative communication scheme

## III. Analysis: Homogeneous Fading Channels

In this paper, we analyze the case where all S-R (and R-D) channels have the same mean channel gain, i.e., $\bar{h}_i = \bar{h}$, and $\bar{g}_i = \bar{g}$, for $i = 1, \ldots, N$. This is an interesting case because the analysis simplifies a lot leading to simple algorithms for computing the optimal transmission scheme. Also, the tradeoffs become much clearer, which helps us gain good intuition for system design. The general case in which the mean channels gains are non-identical is analyzed is [21], and not shown here for want of space.

It can be shown, using symmetry arguments, that there exists an optimal transmission strategy for which $P_{\text{out}}(\mathcal{M})$ is the same for all sets $\mathcal{M}$ of the same cardinality. Hence, in this section, without loss of generality, we will restrict ourselves to relay selection rules $K(M)$ and outage rules $P_{\text{out}}(M)$ that depend only on $M$, the cardinality of $\mathcal{M}$.

Let $p(M, P_S)$ denote the probability that exactly $M$ relays successfully decode the data broadcast by the source, when the source broadcast power is $P_S$. For a given relay selection rule $K$, let $P_f(K(M), M)$ denote the average power consumed in feeding back the CSI to the selected relays, and $P_d(K(M), M)$ denote the average power consumed by the relays to coherently transmit data. Both are conditioned on the events that $M$ relays decode data and that the destination does not declare outage. Here, all the averages are with respect to the joint distribution of the $h_i$'s and $g_i$'s. Note that for $M = 1$, these quantities also take into account the possibility that the single relay is not allowed to transmit because its relay gain to the destination is below a threshold.

Data does not reach the destination when no relays receive data (with probability $p(0, P_S)$), or when only one relay receives data and outage is declared (with probability $P_{\text{out}}(1)p(1, P_S) + \delta p(1, P_S)(1 - P_{\text{out}}(1))$), or when $M > 1$ relays receive data and outage is declared by the destination (with probability $p(M, P_S)P_{\text{out}}(M)$). In the above discussion, recall that when a single relay node, $i$, receives data, it does not transmit if at least one of the following two independent events occurs: (1) the channel gain from the relay to the destination is too low, or (2) the destination does not allow relay to transmit with probability $P_{\text{out}}(1)$ (independent of the channel gain). Therefore, the constraint that the destination receives data from the source with a probability greater than or equal to $(1 - P_{\text{fail}})$ can be written as

$$
\begin{aligned}
P_{\text{fail}} \geq p(0, P_S) + \sum_{M=1}^{N} p(M, P_S) P_{\text{out}}(M) \\
+ \delta p(1, P_S)(1 - P_{\text{out}}(1)).
\end{aligned}
\tag{1}
$$

The energy consumed in broadcasting a message from the source to the relays is $T_d P_S$. The $M$ relays, which receive the data, transmit training sequences to the destination. This consumes energy $M P_t$. If the destination does not declare outage, it feedbacks CSI to the relays. This consumes an average energy of $T_f(K(M))P_f(K(M), M)$. The relays beamform to transmit the message to the destination, which consumes an average energy of $T_d P_d(K(M), M)$. Thus, the total average energy consumption, $E(P_{\text{out}}, K, P_S)$, is given by

$$
\begin{aligned}
E(P_{\text{out}}, K, P_S) = T_d P_S + \sum_{M=1}^{N} p(M, P_S) M P_t + \\
\sum_{M=1}^{N} p(M, P_S)(1 - P_{\text{out}}(M)) T_f(K(M)) P_f(K(M), M) + \\
\sum_{M=1}^{N} p(M, P_S)(1 - P_{\text{out}}(M)) T_d P_d(K(M), M).
\end{aligned}
\tag{2}
$$

### A. Feedback and Data Power Consumption

*$M > 1$ case:* The statistics of $g_i$ are independent of $M$ because all the channel gains are independent of each other. For notational simplicity, we relabel the relay nodes such that nodes $1, \ldots, M$ successfully decode the data broadcast by the source. Sort the channel gains in the descending order $g_{[1]} > \ldots > g_{[M]}$. The $K(M)$ best relays with indices $[1], \ldots, [K(M)]$ are chosen. The destination broadcasts $g_{\text{sum}} = \sum_{j=1}^{[K(\mathcal{M})]} g_{[j]}$ to all the selected relays, and the individual channel gains and phases only to the corresponding relays. Hence, the average power consumption for feedback of CSI is given by

$$
P_f(K(M), M) = \frac{N_0 B (2^r - 1)}{K(M) + 1} \mathbb{E}\left[ \frac{1}{g_{[K(M)]}} + \sum_{i=1}^{K(M)} \frac{1}{g_{[i]}} \right]. \tag{3}
$$

The average power consumed by the relays to cooperatively beamform and transmit data is

$$P_d(K(M), M) = N_0 B\left(2^r - 1\right)\mathbb{E}\frac{1}{g_{\text{sum}}}. \tag{4}$$

$M = 1$ *case:* Let $i$ denote the single relay that decodes the data from the source. This case is different because the relay also does not transmit if the instantaneous gain, $g_i$, is too low. When outage is not declared, the node inverts the channel to transmit data to the destination at rate $r$. The average power consumed to feedback CSI is given by

$$P_f(K(1), 1) = -\frac{N_0 B(2^r - 1)}{\bar{g}}\text{Ei}\left(\frac{-\alpha}{\bar{g}}\right), \tag{5}$$

where $\alpha = -\bar{g}\log_e(1-\delta)$ and Ei is the standard exponential integral function [19] given by $\text{Ei}(u) = \int_{-\infty}^{u}\frac{e^x}{x}dx$. Using similar arguments, $P_d(K(1), 1) = -\frac{N_0 B(2^r-1)}{\bar{g}}\text{Ei}\left(\frac{-\alpha}{\bar{g}}\right)$.

### B. Expressions for Average Energy Consumption

Let $h_{[i]} = \sum_{n=i}^{N}\frac{W_n}{n}$, $i = 1, \ldots, N$, and $g_{[i]} = \sum_{n=i}^{M}\frac{V_n}{n}$, $i = 1, \ldots, M$. It can be shown that for Rayleigh fading, $V_i$ are i. i. d. random variables and have an exponential distribution with mean $\bar{g}$. Similarly, $W_i$ are also i. i. d. and have an exponential distribution with mean $\bar{h}$ [20].

The probability that $M$ nodes successfully decode the data transmitted by the source can then be shown to be

$$p(M, P_S) = \frac{N!}{M!}\sum_{j=M+1}^{N}\frac{e^{-\frac{\gamma_r M}{h}} - e^{-\frac{\gamma_r j}{h}}}{(j-M)\prod_{l\geq M+1, l\neq j}(l-j)}. \tag{6}$$

Using the above change of variables, we can show that

$$\mathbb{E}\frac{1}{g_{[i]}} = \int_0^\infty \frac{1}{x}\frac{M!}{\bar{g}(i-1)!}\sum_{j=i}^{M}\frac{e^{-jx/\bar{g}}}{\prod_{l=i,l\neq j}^{M}(l-j)}dx,$$

$$\mathbb{E}\frac{1}{g_{\text{sum}}} = \int_0^\infty \frac{1}{x}\frac{N!}{\bar{g}K(M)K(M)!} \times \sum_{j=K(M)+1}^{N}\left(1 - \frac{j}{K(M)}\right)^{-K(M)} e^{-\frac{jx}{K(M)\bar{g}}}dx.$$

Using the first $k$ terms of a Taylor series expansion and $\eta < \min(1, \bar{g}/N)$, the above expressions can be approximated in terms of the Ei function as follows:

$$\mathbb{E}\frac{1}{g_{[i]}} = \frac{M!}{\bar{g}(i-1)!}\sum_{j=i}^{M}\frac{\left(\sum_{l=1}^{k}(-1)^l\frac{(j\eta)^l}{(l-1)!\bar{g}^l} + \text{Ei}\left(\frac{-j\eta}{\bar{g}}\right)\right)}{\prod_{l=i,l\neq j}^{M}(l-j)} + o(\eta^{k+1}),$$

$$\mathbb{E}\frac{1}{g_{\text{sum}}} = c_1\left(\sum_{l=1}^{k}(-1)^l\frac{(\eta)^l}{(l-1)!\bar{g}} + \text{Ei}\left(\frac{-\eta}{\bar{g}}\right)\right) + \sum_{j=2}^{K(M)}c_j(\bar{g})^{j-1}(j-2)! + \sum_{j=K(M)+1}^{M}c_j\text{Ei}\left(\frac{-j\eta}{\bar{g}K(M)}\right) + \sum_{j=K(M)+1}^{M}c_j\left(\sum_{l=1}^{k}(-1)^l\frac{(j\eta)^l}{(l-1)!(\bar{g}K(M))^l}\right) + o(\eta^{k+1}),$$

where, for $1 \leq j \leq K(M)$,

$$c_j = \frac{M!/K(M)!}{((K(M)-j)!\bar{g}^{K(M)}}\frac{d^{K(M)-j}}{ds^{K(M)-j}}\prod_{i=K(M)+1}^{M}\frac{1}{K(M)s+i}\Bigg|_{s=-1},$$

and, for $K(M)+1 \leq j \leq M$,

$$c_j = \frac{M!/K(M)!}{\bar{g}K(M)\left(1 - \frac{j}{K(M)}\right)^{K(M)}\prod_{i=K(M)+1,i\neq j}^{M}(i-j)}.$$

### C. Optimal Minimum Energy Transmission Strategy

Note that $P_f(K(M), M)$ and $P_d(K(M), M)$ do not depend on $K(M')$ for $M' \neq M$. Thus,

$$K^*(M) = \underset{1\leq K(M)\leq M}{\arg\min}\Big[T_f(K(M))P_f(K(M), M) + T_d P_d(K(M), M)\Big].$$

The optimal outage strategy that minimizes total energy consumption can be shown to have a simple structure if, for $M \geq 2$,

$$\frac{1}{(1-\delta)}\left(cP_f(K^*(1), 1) + T_d P_d(K^*(1), 1)\right) \geq c(1 + K^*(M))P_f(K^*(M), M) + T_d P_d(K^*(M), M), \tag{7}$$

*i.e.*, the optimal feedback and data power consumption conditioned on $M \geq 2$ (nodes can beamform to transmit with zero outage) is less than or equal to $\frac{1}{1-\delta}$ times that conditioned on $M = 1$. The following lemma then follows.

*Lemma 3.1:* The optimal outage strategy has the following structure. For some $0 < M^* < N$,

$$\begin{aligned} P^*_{\text{out}}(M) &= 1, \text{ for } M < M^*, \\ 0 \leq P^*_{\text{out}}&(M^*) \leq 1, \\ P^*_{\text{out}}(M) &= 0, \text{ for } M > M^*. \end{aligned} \tag{8}$$

Moreover, $p(0, P_S) + \sum_{M=1}^{N}p(M, P_S)P^*_{\text{out}}(M) + \delta p(1, P_S)(1 - P^*_{\text{out}}(1)) = P_{\text{fail}}$.

*Proof:* Let $\hat{K}$ be another relay selection rule such that $\hat{K}(M) = K^*(M)$ and $\hat{K}(L) = K^*(M)$, when $L > M > 1$. Then, we can prove that

$$T_f(K^*(M))P_f(K^*(M), M) + T_d P_d(K^*(M), M) \geq T_f(K^*(L))P_f(K^*(L), L) + T_d P_d(K^*(L), L). \tag{9}$$

Using (7) and (9), the optimal outage policy, $P^*_{\text{out}}$, in (8) can be shown to be a solution to the following linear program:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{M}p(i, P_S)(1 - P_{\text{out}}(i))c_i \\ \text{subject to} \quad & \sum_{i=1}^{M}\frac{p(i, P_S)}{\beta_i}P_{\text{out}}(i) \leq b, \\ & 0 \leq P_{\text{out}}(i) \leq 1, \quad \text{for all } i = 1, \ldots, M, \end{aligned}$$

where $\beta_1 = 1/(1-\delta)$ and $\beta_i = 1$ for $i > 1$, $c_i = T_f(K^*(i))P_f(K^*(i), i) + T_d P_d(K^*(i), i)$ and $b = P_{\text{fail}} - p(0, P_S)$. ■

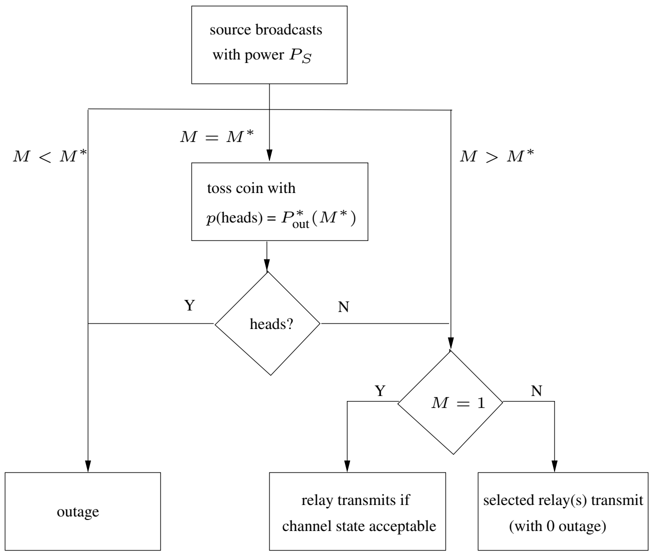



Fig. 2. Structure of optimal minimum energy transmission policy

The structure of the optimal transmission strategy is illustrated in Fig. 2. If the number of relays, $M$, that decode the data successfully is less than the threshold, $M^*$, then the destination declares outage. If $M = M^*$, the destination randomly declares outage with probability $P^*_{\text{out}}(M^*)$. If $M > M^*$, the destination selects $K^*(M)$ relays and never declares outage unless $M = 1$, in which case the destination allows the relay to transmit only if its channel gain exceeds a threshold determined by $\delta$. Thus, for $M = 1$, the probability that the relay transmits is smaller, $(1 - P_{\text{out}}(1))(1 - \delta)$. In the event that the destination allows the selected relay(s) to transmit, it feedbacks CSI to the relay(s), which then transmit data with sufficient power to the destination.

The following lemma about convexity gives a sufficient condition that allows an efficient bisection search, of complexity $O(\log_2(M))$, to be used to determine $K^*(M)$ for each $M$. Its proof is omitted due to space constraints.

*Lemma 3.2:* For $M \geq 3$, if for all $v = 2, \ldots, M-1$, $\mathbb{E}\left[3\left(\frac{1}{g_{[v+1]}} - \frac{1}{g_{[v]}}\right) - \left(\frac{1}{g_{[v+1]}} - \frac{1}{g_{[v-1]}}\right)\right] \geq 0$, then there exists a convex function $z : \mathbb{R} \to \mathbb{R}$ such that $z(v) = T_f(v)P_f(v, M) + T_d P_d(v, M)$, for all $v \in \{2, \ldots, M\}$.

### D. Computational Algorithms

To optimize the transmission scheme, the optimal outage probabilities, $P^*_{\text{out}}(M)$, broadcast power, $P^*_S$, and relay selection rule, $K^*(M)$, need to be computed. There are two main computations. First, from Lemma 3.2, we see that the search for $K^*(M)$ can be done efficiently for each $M$ under certain conditions; in general the search has worst case linear complexity in $M$. Note that $K^*(M)$ is independent of $P_S$. Once, $K^*(M)$ has been computed, $P^*_{\text{out}}(M)$ (or equivalently $M^*$) can be found using Lemma 3.1. Second, to find an optimal $P_S$, we need to search over the range of $P_S$. For each value of $P_S$, the optimal outage rule can be computed efficiently using Lemma 3.1 and using expressions derived in this Sec. III-C.

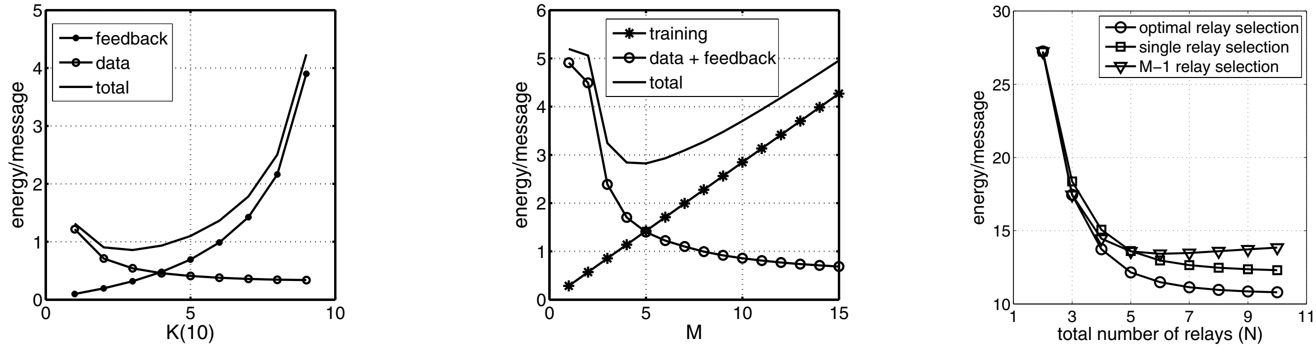
## IV. Numerical Results

Consider a cooperative relay network with $N = 10$ relays, rate, $r = 2$ bits/symbol (QPSK), $T_d$ =100 symbol durations, and $P_{\text{fail}} = 0.01$. Unless otherwise mentioned, the mean channel gains are $\bar{h} = \bar{g} = 1$. Assuming that 8 bits are required to feedback each channel gain and phase, we have $c = 4$. When one relay decodes the data broadcast by the source, it does not transmit with probability $\delta = 0.005$ due to a bad channel state. For the sake of illustration, we assume that the training power, $P_t$, is such that it equals the power needed for transmitting from a relay to the destination at rate $r$ and with an outage of 0.1 (which is higher than $P_{\text{fail}}$). This is justifiable because transmit diversity enables us to use a relay only when its channel to the destination is good. If the training sequence received by the destination has low power, it means that the channel is bad and, hence, will not be used for data transmission. All the computed energy values are normalized with respect to $N_0 B$. The computational results were verified by Monte-Carlo simulations of the system using $10^8$ samples.

Figure 3(a) shows the variation with $K(M)$ of the energy for feedback of CSI and energy for data transmission from the relays to the destination, when $M = 10$ relays receive the data broadcast from the source. As the training power does not change when $M$ is fixed, it is not shown. In this case, $K(10)$ denotes the number of relays selected by the destination. We see that as $K(10)$ increases, the energy consumption for CSI feedback increases because the destination has to feedback to more relays with progressively worse channels. At the same time, the energy consumption for data transmission decreases because more relays now beamform. Also, the total feedback and data power consumption as a function of $K$ can be fitted to a convex function, as in Lemma 4.3.

Figure 3(b) shows the variation of the energy consumed, as a function of $M$, for training and for cooperative beamforming and feedback of CSI for an optimal relay selection rule. As more relays decode the data from source, the power consumption for feedback and data transmission decreases due to greater diversity in the system. However, this also increases the training overhead. The optimal relay selection rule (not shown in the figure) turns out to be the following: for $M \leq 2$, $K^*(M)$ equals 1, which is the conventional single relay selection. However, single relay selection is sub-optimal for larger $M$ as for $3 \leq M \leq 6$, $K^*(M) = 2$, and for $7 \leq M \leq 15$, $K^*(M) = 3$. Hence, only a small subset of the relays – that changes depending on the fading on the relay links – is active at any given time. Therefore, while the energy cost of acquiring CSI limits the number of relays that cooperate at any instant, it is still beneficial to cooperate.

In Fig. 3(c), we show the energy consumption per message as a function of $M$ for the following three relay selection rules: *optimal relay selection*, in which the best $K^*(M)$ relays are chosen; *single relay selection*, in which only one relay with the highest R-D gain is chosen; and $(M-1)$ *relay selection*, in




(a) Effect of relay selection rule on energy for CSI feedback and data transmission when $M = 10$ relays decode data broadcast by source

(b) Energy for feedback and data transmission for optimal relay selection rule as a function of the number of relays that decode data broadcast by source

(c) Energy consumption as a function of number of relays for different relay selection rules

Fig. 3. Numerical Results: Energy consumed by various steps and their combinations

which the best $(M-1)$ relays are always chosen. Selecting all the $M$ relays is impractical as its the average energy consumed for CSI feedback will be infinity for Rayleigh fading R-D links. Figure 3(c) plots the energy consumption per message when the S-R channels have mean channel gains of 6.0, while the R-D channels have mean channel gains of 0.3. This corresponds to the case where the relays are closer to the source. We can see that the incremental energy savings due to an additional relay are high when a small number of relays are present; the incremental savings decrease as the number of relays increases. Also, relay selection has an impact on the total energy consumption. The optimal relay selection rule consumes approximately $14\%$ less total energy than the other two selection rules when 10 relays are present. Thus, it is beneficial to vary the number of relays as a function of system parameters and the number of relays that decode.

## V. Conclusions

We analyzed the total energy consumption for a general class of cooperative transmission schemes. The overhead energy consumption for obtaining the CSI was explicitly modeled. The class of cooperation schemes was more general than many considered in the literature, but at the same time was amenable to analysis and optimization. For the homogeneous case, the optimal transmission scheme has a very simple structure and can be computed efficiently in real-time even for a large number of relay nodes. The optimal strategy is for a varying subset (and number) of relay nodes to cooperatively beamform at any given time. The numerical results illustrated the tradeoff between energy consumption for data transmission and the CSI acquisition energy overhead. They also showed that the cooperative communication scheme considered in this paper offers considerable energy savings compared to non-cooperative schemes and cooperative schemes that use either a single relay or all available relays.

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