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Abstract

The high time resolution of ultra-wideband (UWB) signals results in a large number of multipath components (MPCs) arriving at the receiver, which presents a source of diversity. In addition to this multipath diversity, there is also repetition diversity inherent in impulse radio (IR) UWB systems, since a number of pulses are transmitted for each information symbol. In order to make optimal use of both multipath and repetition diversity, the receiver needs to consider the optimal conbination of contributions from both different frames and different MPCs. In this overview paper, the optimal linear receiver for a given user in frequency-selective multiuser environment, which combines all the samples from the received signal according to the minimum mean square error (MMSE), criterion is studied. Due to the complexity of this optimal receiver, two suboptimal receivers with lower complexity are considered, optimal frame combining (OFC) and optimal multipath combining (OMC) receivers, which reduce computational complexity by suboptimal combining in the multipath diversity and repetition domains, respectively. Finally, a two-step MMSE algorithm which reduces complexity by performing MMSE combining in two steps is presented, and its optimality properties are discussed. Simulations are performed to compare the performance of different receivers.

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Optimal and Suboptimal Linear Receivers for Impulse Radio UWB Systems[0]

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Invited Paper

***Abstract*— The high time resolution of ultra-wideband (UWB) signals results in a large number of multipath components (MPCs) arriving at the receiver, which presents a source of diversity. In addition to this multipath diversity, there is also repetition diversity inherent in impulse radio (IR) UWB systems, since a number of pulses are transmitted for each information symbol. In order to make optimal use of both multipath and repetition diversity, the receiver needs to consider the optimal combination of contributions from both different frames and different MPCs. In this overview paper, the optimal linear receiver for a given user in a frequency-selective multiuser environment, which combines all the samples from the received signal according to the minimum mean square error (MMSE), criterion is studied. Due to the complexity of this optimal receiver, two suboptimal receivers with lower complexity are considered, optimal frame combining (OFC) and optimal multipath combining (OMC) receivers, which reduce computational complexity by suboptimal combining in the multipath diversity and repetition diversity domains, respectively. Finally, a two-step MMSE algorithm which reduces complexity by performing MMSE combining in two steps is presented, and its optimality properties are discussed. Simulations are performed to compare the performance of different receivers.**

***Index Terms*— Ultra-wideband (UWB), impulse radio (IR), diversity, Rake receiver, MMSE combining.**

I. Introduction

Recently impulse radio (IR) ultra wideband (UWB) systems ([1]-[6]) have drawn considerable attention due to their suitability for short-range high-speed data transmission and precise location estimation. In an IR-UWB system, very short pulses with a low duty cycle are transmitted, and each information symbol is represented by positions or amplitudes of a number of pulses. Each pulse resides in an interval called a "frame", and the positions of pulses within frames are determined by time-hopping (TH) sequences specific to each user, which prevent catastrophic collisions among pulses of different users [1].

In an IR-UWB system, N_f pulses/frames are transmitted per information symbol, which results in repetition diversity. In a single user system over an additive white Gaussian noise (AWGN) channel, the received signal consists of N_f pulses in N_f frames. After matched-filtering/correlation and sampling operations, the contributions from N_f different frames are added with equal weight to form a decision variable [1]. In considering a multiuser environment, the contributions from different frames can have different signal-to-interference-plus-noise ratios (SINRs) depending on the TH sequences of the users. Therefore, equally-weighted contributions from different frames no longer form an optimal decision variable [7]. Also, in a frequency-selective environment, there can be self-interference, also called inter-frame interference (IFI), due to multipath propagation, which affects the optimal combining of the frame components at the receiver.

In addition to contributions from N_f frames, there is also diversity due to the frequency-selective environment. In other words, combination of different MPCs should also be taken into account [8]-[11]. Therefore, we need to consider the optimal combination of contributions from both N_f different frames and different MPCs in order to maximize system performance. The optimal linear scheme combines both the frame and multipath components according to the MMSE criterion [12]. However, this optimal scheme is computation-intensive when N_f and/or the number of MPCs combined at the receiver are large. Therefore, suboptimal schemes with lower complexity are desired. In order to reduce computational complexity, frame or multipath components can be combined suboptimally first, and then combined samples can be combined by a linear MMSE scheme, resulting in optimal multipath combining (OMC) and optimal frame combining (OFC) receivers, respectively [12]. These receivers may be computationally feasible, but are not optimal in the sense that they do not make optimal use of all possible MPCs or N_f different pulses per information symbol. Therefore, the performance degradation can be significant in some cases, and also there is no flexibility in the performance-complexity trade-off. A multi-stage approach is employed in [13], where D stages are needed in order to reduce the size of a matrix to be inverted to $D \times D$. Also, chip-rate samples are considered for the proposed receiver structure, which may not be very practical for UWB systems due to cost and power consumption constraints.

A low-complexity linear UWB receiver structure with frame-rate sampling is the two-step MMSE combining receiver, in which received signal samples are combined in two

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steps in order to reduce computational complexity [14]. The samples are divided into disjoint groups and in each group, combining based on the MMSE criterion is employed for optimal performance. Then, the combined samples are used to obtain decision statistics according to the MMSE criterion. The scheme is suboptimal due to the multi-step nature, which also reduces the complexity significantly.

In this overview paper, we study low-complexity linear receiver structures for IR-UWB systems. We start with the optimal linear receiver structure, and then study the OFC and OMC receivers to present practical algorithms with low computational complexity. Then, we consider the two-step MMSE receiver, and state the necessary and sufficient conditions for its optimality. Finally, we compare performance of different optimal and suboptimal algorithms and comment on the trade-off between complexity and performance.

The remainder of the paper is organized as follows. Section II describes the signal model for an IR-UWB system and presents a discrete-time representation of the received signal. Section III investigates the optimal linear receiver that combines all the components of the received signal according to the MMSE criterion. The OFC and OMC receivers are presented in Section IV and Section V, respectively, which is followed by the two-step MMSE receiver in Section VI. After simulation results are presented in Section VII, some conclusions and extensions are stated in Section VIII.

II. Signal Model

We consider a synchronous IR-UWB system with K users, in which the transmitted signal from user k is represented by [12]:

$$s_{\rm tx}^{(k)}(t)=\sqrt{\frac{E_k}{N_f}}\sum_{j=-\infty}^{\infty}d_j^{(k)}\,b_{\lfloor j/N_f\rfloor}^{(k)}p_{\rm tx}(t-jT_f-c_j^{(k)}T_c), \tag{1}$$

where $p_{\rm tx}(t)$ is the transmitted UWB pulse, E_k is the symbol energy of user k, T_f is the frame time, N_f is the number of pulses representing one information symbol, and $b_{\lfloor j/N_f\rfloor}^{(k)}\in\{+1,-1\}$ is the binary information symbol transmitted by user k. In order to allow the channel to be shared by many users and avoid catastrophic collisions, a TH sequence $\{c_j^{(k)}\}$, where $c_j^{(k)}\in\{0,1,...,N_c-1\}$, is assigned to each user. This TH sequence provides an additional time shift of $c_j^{(k)}T_c$ seconds to the jth pulse of the kth user where T_c is the chip interval and is chosen to satisfy $T_c\le T_f/N_c$ in order to prevent the pulses from overlapping. The random polarity codes $d_j^{(k)}$ are binary random variables taking values ±1 with equal probability ([15]-[17]).

Assuming a tapped-delay-line channel model with multipath resolution T_c, the discrete channel model $\boldsymbol{\alpha}^{(k)}=[\alpha_1^{(k)}\cdots\alpha_L^{(k)}]$ is adopted for user k, where $(L-1)T_c$ is set, without loss of generality, equal to or greater than the maximum possible delay, excess of the delay experienced by the corresponding direct-path component, of any MPC of the signal received from any user. Then, the received signal can be expressed as

$$r(t)=\sum_{k=1}^{K}\sqrt{\frac{E_k}{N_f}}\sum_{j=-\infty}^{\infty}\sum_{l=1}^{L}\alpha_l^{(k)}d_j^{(k)}\,b_{\lfloor j/N_f\rfloor}^{(k)}$$
$$\times\,p_{\rm rx}(t-jT_f-c_j^{(k)}T_c-(l-1)T_c)+\sigma_n n(t), \tag{2}$$

where $p_{\rm rx}(t)$ is the received unit-energy UWB pulse, which is usually modelled as the derivative of $p_{\rm tx}(t)$ due to the effects of the antenna, and $n(t)$ is zero mean white Gaussian noise with unit spectral density.

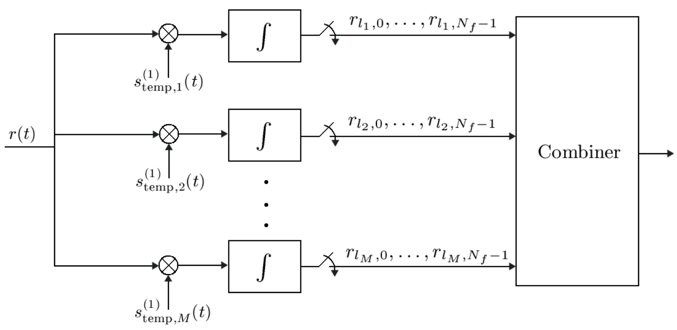

Fig. 1. Rake receiver with M branches. Frame-rate sampling is employed at each branch.

Consider a Rake receiver with M correlator branches as shown in Figure 1. At each branch, the template signal matches the UWB pulse $p_{\rm rx}(t)$ and the TH and polarity randomization codes of the desired user, say user 1, and samples are taken at instants when the path $l\in\mathcal{L}$ arrives in different frames, where $\mathcal{L}=\{l_1,\ldots,l_M\}$ with $M\le L$. Due to possible collisions, the actual number N of distinct samples per information symbol can be smaller than N_fM.

The discrete signal at the lth path of the jth frame can be expressed, for the ith information bit, as [12]

$$r_{l,j}=\mathbf{s}_{l,j}^T\mathbf{A}\mathbf{b}_i+n_{l,j}, \tag{3}$$

for $l\in\mathcal{L}$ and $j\in\mathcal{F}_i=\{iN_f,\ldots,(i+1)N_f-1\}$, where $\mathbf{A}={\rm diag}\{\sqrt{E_1/N_f},\ldots,\sqrt{E_K/N_f}\}$, $\mathbf{b}_i=[b_i^{(1)}\cdots b_i^{(K)}]^T$ and $n_{l,j}\sim\mathcal{N}(0,\sigma_n^2)$. $\mathbf{s}_{l,j}$ is a $K\times1$ signature vector, which can be expressed as the sum of the desired signal part (SP), IFI and multiple-access interference (MAI) terms [12]:

$$\mathbf{s}_{l,j}=\mathbf{s}_{l,j}^{\rm (SP)}+\mathbf{s}_{l,j}^{\rm (IFI)}+\mathbf{s}_{l,j}^{\rm (MAI)}. \tag{4}$$

For simplicity of the analysis, we assume a guard interval between information symbols that is equal to the length of the channel impulse response (e.g. [18]), which avoids inter-symbol interference (ISI).

III. Linear MMSE Receiver

We first consider a linear receiver for user 1 that combines all the samples from the received signal optimally, according to the MMSE criterion [12].

Let $\mathbf{r}$ be an $N\times1$ vector denoting the distinct samples $r_{l,j}$

for $(l,j) \in \mathcal{L} \times \mathcal{F}_i$:

$$\mathbf{r} = \left[r_{l_1, j_1^{(1)}} \cdots r_{l_1, j_{m_1}^{(1)}} \cdots r_{l_M, j_1^{(M)}} \cdots r_{l_M, j_{m_M}^{(M)}} \right]^T, \quad (5)$$

where $\sum_{i=1}^{M} m_i = N$ denotes the total number of samples, with $N \leq MN_f$.

Using (3), $\mathbf{r}$ can be expressed as

$$\mathbf{r} = \mathbf{SAb}_i + \mathbf{n}, \quad (6)$$

where $\mathbf{A}$ and $\mathbf{b}_i$ are as in (3) and $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I})$. $\mathbf{S}$ is a signature matrix, which has $\mathbf{s}_{l,j}^T$ of (4) for $(l,j) \in \mathcal{C}$ as its rows, where $\mathcal{C} = \{(l_1, j_1^{(1)}), \ldots, (l_1, j_{m_1}^{(1)}), \ldots, (l_M, j_1^{(M)}), \ldots, (l_M, j_{m_M}^{(M)})\}$.

After some manipulation, the received samples in (6) can be expressed as the summation of the signal and the total noise terms [14]:

$$\mathbf{r} = b_i^{(1)} \boldsymbol{\beta} + \mathbf{w}, \quad (7)$$

where $\boldsymbol{\beta}$ is the signal part[3], and

$$\mathbf{w} = \mathbf{S}^{(\mathrm{MAI})} \mathbf{Ab}_i + \mathbf{n} \quad (8)$$

is the total noise term including the MAI and the background noise, with $\mathbf{S}^{(\mathrm{MAI})}$ denoting the MAI part of the signature matrix in (6).

A linear receiver combines the elements of $\mathbf{r}$ and obtains a decision statistic as follows:

$$y = \boldsymbol{\theta}^T \mathbf{r}, \quad (9)$$

where $\boldsymbol{\theta}$ is the weighting vector.

The MMSE weights that maximize the SINR of the received signal in (7) can be obtained as [19]

$$\boldsymbol{\theta}_{\mathrm{MMSE}} = \left(\boldsymbol{\beta}\boldsymbol{\beta}^T + \mathbf{R_w} \right)^{-1} \boldsymbol{\beta} = c\, \mathbf{R_w}^{-1} \boldsymbol{\beta}, \quad (10)$$

where $\mathbf{R_w} = \mathrm{E}\{\mathbf{ww}^T\}$ is the correlation matrix and $c = (1 + \mathrm{SINR})^{-1}$, with $\mathrm{SINR} = \boldsymbol{\beta}^T \mathbf{R_w}^{-1} \boldsymbol{\beta}$.

Assuming equiprobable information symbols, the correlation matrix can be expressed as

$$\mathbf{R_w} = \mathbf{S}^{(\mathrm{MAI})} \mathbf{A}^2 \left(\mathbf{S}^{(\mathrm{MAI})} \right)^T + \sigma_n^2 \mathbf{I}, \quad (11)$$

and the linear MMSE receiver is given by

$$\hat{b}_i^{(1)} = \mathrm{sign}\left\{ \mathbf{r}^T \mathbf{R_w}^{-1} \boldsymbol{\beta} \right\}. \quad (12)$$

Note that this receiver requires the inversion of an $N \times N$ matrix ($N \leq MN_f$). Therefore, the computational complexity can be quite high when the number of frames and/or the number of Rake fingers are large. For that reason, we present various suboptimal algorithms with lower computational complexity in the following sections.

IV. Optimal Frame Combining (OFC)

Since the combination of all the samples by the MMSE criterion requires intense computations, the optimality in the multipath or repetition diversity domain can be sacrificed for low complexity solutions. In an OFC receiver, the MPCs in each frame are combined according to the maximal ratio combining (MRC) criterion, which is suboptimal, and then those combined components in different frames are combined according to the optimal linear MMSE criterion (Figure 2). That is, the decision variable is given by

$$y = \sum_{j=iN_f}^{(i+1)N_f - 1} \hat{\theta}_j \sum_{l \in \mathcal{L}} \alpha_l^{(1)} r_{l,j}, \quad (13)$$

where $\hat{\theta}_{iN_f}, \ldots, \hat{\theta}_{(i+1)N_f-1}$ are the weighting factors for the ith bit.

From (3)-(8), MMSE weights can be obtained for $\hat{\boldsymbol{\theta}}_i = [\hat{\theta}_{iN_f} \cdots \hat{\theta}_{(i+1)N_f-1}]^T$ [12]. In other words, the samples combined by MRC in different frames can be combined by MMSE in the second step to obtain the decision variable. Since the MMSE combining in the second step combines N_f combined samples, the OFC algorithm requires the inversion of an $N_f \times N_f$ matrix. The reduction in complexity compared to the optimal linear MMSE receiver of the previous section is due to the suboptimal combination of MPCs in different frames.

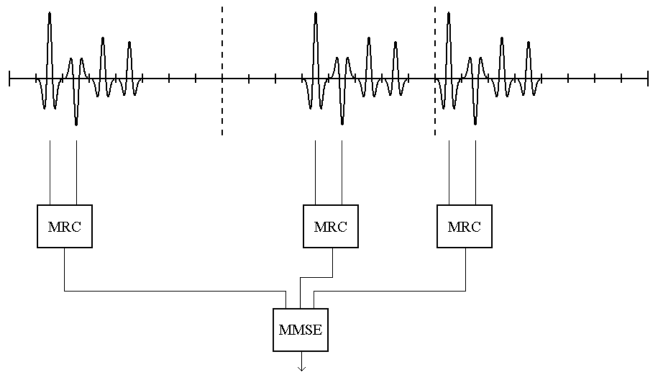

Fig. 2. The OFC receiver, which combines samples from different MPCs in each frame by MRC and employs MMSE combining in the second step.

V. Optimal Multipath Combining (OMC)

Similar to the OFC receiver, we can also consider a receiver that combines different MPCs according to the optimal linear MMSE criterion, while employing equal gain combining (EGC) for contributions from different frames (Figure 3). In this case, the decision variable is given by

$$y = \sum_{l \in \mathcal{L}} \tilde{\theta}_l \sum_{j=iN_f}^{(i+1)N_f - 1} r_{l,j}, \quad (14)$$

where $\tilde{\boldsymbol{\theta}} = [\tilde{\theta}_{l_1} \cdots \tilde{\theta}_{l_M}]^T$ is the MMSE weighting vector [12].

The OMC receiver needs to invert an $M \times M$ correlation matrix since it combines M combined samples by the MMSE algorithm. The reduction in complexity compared to the optimal linear receiver in Section III is the result of suboptimal combination of the contributions from different frames.

[3] Please refer to [14] for the detailed expressions.

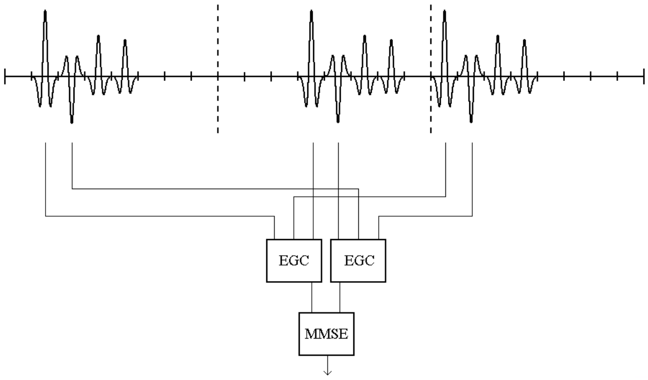

Fig. 3. The OMC receiver, which combines samples from different frames for each MPC by EGC and employs MMSE combining in the second step.

We can show that the OFC and OMC receivers are equivalent for a single user system in which the pulses in a frame never collide with any pulse in another frame. In other words, in the absence of IFI and MAI, both systems have the same SINR values:

$$\mathrm{SINR}_{\mathrm{OFC}} = \mathrm{SINR}_{\mathrm{OMC}} = \frac{E_1}{\sigma_n^2}\sum_{l\in\mathcal{L}}\alpha_l^2. \tag{15}$$

VI. Two-Step MMSE Combining

In this section, we consider another two-step combining algorithm, which is more generic in that it does not limit the sample-grouping operation to only frame or multipath components.

We first divide the received samples $\mathbf{r}$ into N_1 groups:

$$\mathbf{r}_j = b^{(1)}\boldsymbol{\beta}_j + \mathbf{w}_j, \tag{16}$$

for $j = 1,\ldots,N_1$. Then, we combine the samples in each group by the MMSE criterion using the following weighting vectors:

$$\boldsymbol{\theta}_j = \left(\boldsymbol{\beta}_j\boldsymbol{\beta}_j^T + \mathbf{R}_{\mathbf{w}_j}\right)^{-1}\boldsymbol{\beta}_j = c_j\mathbf{R}_{\mathbf{w}_j}^{-1}\boldsymbol{\beta}_j, \tag{17}$$

where

$$c_j = \left(1 + \boldsymbol{\beta}_j^T\mathbf{R}_{\mathbf{w}_j}^{-1}\boldsymbol{\beta}_j\right)^{-1} \tag{18}$$

and $\mathbf{R}_{\mathbf{w}_j} = \mathrm{E}\{\mathbf{w}_j\mathbf{w}_j^T\}$. We then obtain N_1 combined samples: $\left\{\boldsymbol{\theta}_j^T\mathbf{r}_j\right\}_{j=1}^{N_1}$. Note that the MMSE combining in each group ignores the information about the other groups, which causes a loss in optimality. However, this is the main source of complexity reduction.

Let $\hat{\mathbf{r}}$ denote the set of combined samples:

$$\hat{\mathbf{r}} = \begin{bmatrix}\boldsymbol{\theta}_1^T\mathbf{r}_1\\ \vdots \\ \boldsymbol{\theta}_{N_1}^T\mathbf{r}_{N_1}\end{bmatrix}, \tag{19}$$

which can be expressed as

$$\hat{\mathbf{r}} = b^{(1)}\hat{\boldsymbol{\beta}} + \hat{\mathbf{w}}, \tag{20}$$

where $\hat{\boldsymbol{\beta}} = [\boldsymbol{\theta}_1^T\boldsymbol{\beta}_1 \cdots \boldsymbol{\theta}_{N_1}^T\boldsymbol{\beta}_{N_1}]^T$ and $\hat{\mathbf{w}} = [\boldsymbol{\theta}_1^T\mathbf{w}_1 \cdots \boldsymbol{\theta}_{N_1}^T\mathbf{w}_{N_1}]^T$. Then, the bit estimate is obtained as

$$\hat{b}^{(1)} = \mathrm{sgn}\{\boldsymbol{\gamma}^T\hat{\mathbf{r}}\}, \tag{21}$$

where $\boldsymbol{\gamma}$ is the MMSE weighting vector for the samples in $\hat{\mathbf{r}}$, which is given by

$$\boldsymbol{\gamma} = \left(\hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}^T + \mathbf{R}_{\hat{\mathbf{w}}}\right)^{-1}\hat{\boldsymbol{\beta}} = \hat{c}\,\mathbf{R}_{\hat{\mathbf{w}}}^{-1}\hat{\boldsymbol{\beta}}, \tag{22}$$

with $\mathbf{R}_{\hat{\mathbf{w}}} = \mathrm{E}\{\hat{\mathbf{w}}\hat{\mathbf{w}}^T\}$.

Note that unlike the OFC and OMC receivers, the two-step MMSE scheme uses MMSE combining in both steps of the algorithm. In addition, the samples in the first step of the algorithm does not have to come from the same frame or the same MPC.

It can be shown that the complexity of the two-step MMSE algorithm is $\mathcal{O}(N^{1.8})$ compared to $\mathcal{O}(N^3)$ of the optimal MMSE scheme in Section III [14].

A. Suboptimality

The two-step MMSE algorithm is suboptimal in general, since, in the first step, each group ignores the information about the other groups. However, under the following condition, the two-step algorithm is the same as the optimal linear MMSE algorithm in Section III:

Proposition [14]: *In order for the two-step MMSE algorithm to be the optimal linear scheme for any given channel gains, it is necessary and sufficient that the noise samples in* $\mathbf{w}_1,\ldots,\mathbf{w}_{N_1}$ *of (16) are mutually uncorrelated.*

B. Grouping Received Samples

As the proposition states, the two-step MMSE scheme is optimal when the correlation matrix $\mathbf{R}_{\mathbf{w}}$ has a block diagonal structure. However, in many situations, $\mathbf{R}_{\mathbf{w}}$ does not have this structure. In such cases, it is reasonable to group highly correlated samples into the same group in order to obtain a "near block diagonal" structure. For that purpose, the following low-complexity grouping algorithm can be used for a given correlation matrix $\mathbf{R}_{\mathbf{w}}$:

1) $\mathcal{S} = \{1,\ldots,N\}$
2) for $i = 1 : N_1 - 1$
3) Choose a random sample s from $\mathcal{S}$
4) $\mathcal{S} = \mathcal{S} - \{s\}$
5) $\tilde{\mathcal{S}}_i = \{s\}$
6) for $j = 1 : \hat{N}_i - 1$
7) $\tilde{l} = \arg\max_{l\in\mathcal{S}} \sum_{k\in\tilde{\mathcal{S}}_i} |\rho_{lk}|$
8) $\tilde{\mathcal{S}}_i = \tilde{\mathcal{S}}_i \cup \{\tilde{l}\}$
9) $\mathcal{S} = \mathcal{S} - \{\tilde{l}\}$
10) $\tilde{\mathcal{S}}_{N_1} = \mathcal{S}$

where $\hat{N}_i$ denotes the number of samples in group i, for $i = 1,\ldots,N_1$. In the 7th step, the correlation ρ_{lk} is given by

$$\rho_{lk} = \frac{[\mathbf{R}_{\mathbf{w}}]_{lk}}{\sqrt{[\mathbf{R}_{\mathbf{w}}]_{ll}[\mathbf{R}_{\mathbf{w}}]_{kk}}}, \tag{23}$$

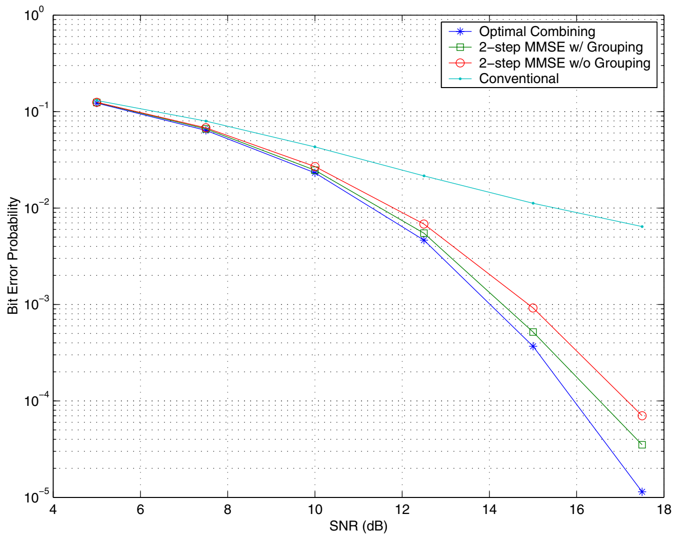

Fig. 4. BEP versus SNR for the optimal, conventional and two-step algorithms in a 5-user IR-UWB system over the channel $\boldsymbol{\alpha}^{(k)} = [-0.4019\ 0.5403\ 0.1069 - 0.0479\ 0.0608\ 0.0005]\ \forall k$, where $N_c = 10$, $N_f = 8$, $\mathcal{L} = \{1, 2, 3, 4\}$ and $E_k = 1\ \forall k$.

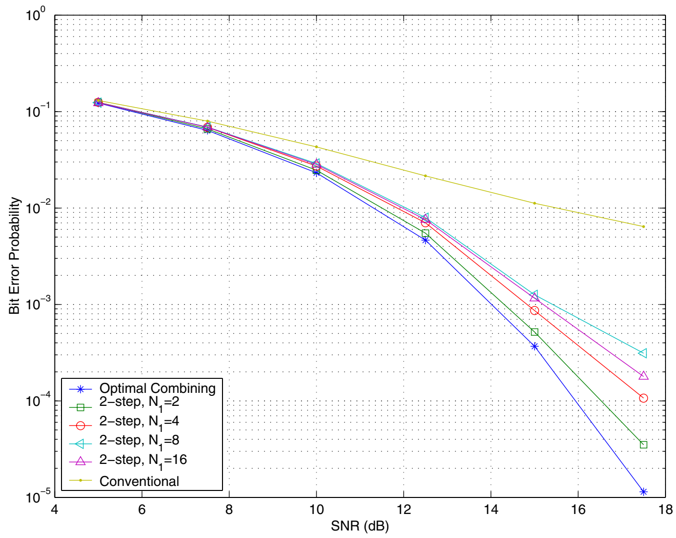

Fig. 5. BEP versus SNR for the optimal, conventional and two-step algorithms, where effects of N_1 are investigated. The same parameters are used as in Figure 4.

which is used as a measure to determine the level of correlation between any two samples.

Note that the algorithm starts with a random sample for each group, and then chooses the most correlated samples from the available index set $\mathcal{S}$ to form a group of highly correlated samples. At the end, the algorithm outputs the sets of indices $\tilde{\mathcal{S}}_1, \ldots, \tilde{\mathcal{S}}_{N_1}$, which determine the received sample groups to be combined at the first step of the two-step MMSE algorithm.

VII. Simulation Results

In this section, we consider the downlink of an IR-UWB system with 5 users ($K = 5$), where $E_k = 1\ \forall k$. The number of chips per frame, N_c, is equal to 10 and the discrete channel impulse response is given by $\boldsymbol{\alpha}^{(k)} = [-0.4019\ 0.5403\ 0.1069\ -\ 0.0479\ 0.0608\ 0.0005]\ \forall k$ [20]. The TH sequences and polarity codes of the users are chosen from uniform distributions, and the results are averaged over different realizations. For the two-step MMSE algorithm, the numbers of samples in the groups are chosen to be the same.

In the first scenario, $N_1 = 2$, the number of frames per symbol, N_f, is equal to 8 and the first four multipaths are sampled at the receiver; that is, $\mathcal{L} = \{1, 2, 3, 4\}$. Figure 4 shows the bit error probability (BEP) for different SNR values for the optimal MMSE, the conventional, and the two-step MMSE (with and without grouping) receivers. It is observed from the plot that the two-step MMSE receiver has a performance close to the optimal MMSE receiver, and the conventional receiver, which combines the MPCs by MRC, performs poorly. Also the effects of grouping on the performance of the two-step MMSE is observed.

Next, effects of N_1, the number of groups, on the performance of the two-step MMSE algorithm (with grouping) are investigated in Figure 5, for which the same parameters as in the previous scenario are used. Note that the optimal MMSE can be thought of a special case of the two-step MMSE algorithm for $N_1 = 1$. As the number of groups are increased from $N_1 = 1$, the algorithm gets further from optimal due to the fact that the MMSE combining in each group ignores the information about the others. However, as N_1 gets close to N, which is 32 in this case, the algorithm starts to perform better since the MMSE combining in the second step becomes more effective ($N_1 = 16$ performs better than $N_1 = 8$). In fact, for $N_1 = N$, the two-step MMSE scheme reduces to the optimal MMSE since there occurs no combining in the first step since each group consists of a single sample in that case; hence, the second step becomes the optimal MMSE.

Finally, the performance of the two-step MMSE receiver and that of the OMC and OFC receivers are compared for $N_f = N_1 = 5$ and $\mathcal{L} = \{1, 2, 3, 4, 5\}$. It is observed from Figure 6 that the two-step MMSE algorithm performs better than the OMC and OFC algorithms since the MMSE criterion is employed in both steps of the two-step MMSE algorithm whereas the OMC and OFC receivers employ EGC and MRC, respectively, in their first steps.

VIII. Conclusions and Extensions

In this paper, we have considered optimal and suboptimal linear receivers for IR-UWB systems. The optimal linear MMSE receiver combines the samples from different frame and multipath components optimally in one step. However, it can require the inversion of a large matrix for the calculation of the optimal weights. Therefore, we have studied suboptimal receivers that combine available samples in two steps. The OFC and OMC receivers employ MRC and EGC, respectively, in their first steps in order to reduce computational complexity. However, the performance degradation can be significant in some cases. The two-step MMSE combining applies MMSE

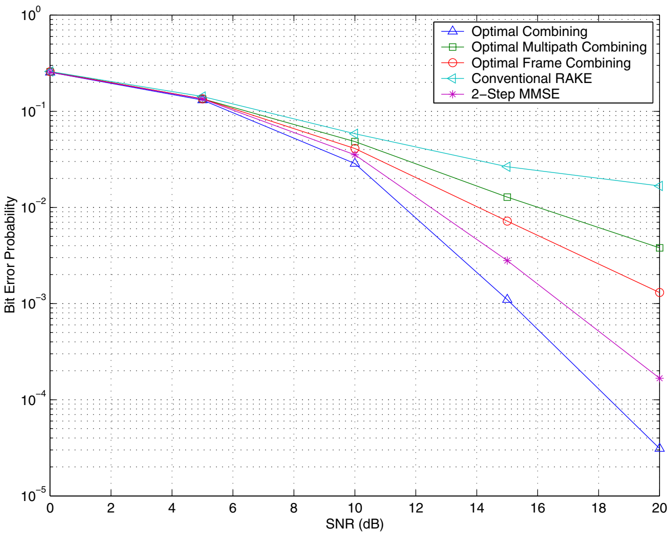

Fig. 6. BEP versus SNR for the optimal, conventional, OMC, OFC, and two-step MMSE receivers. $N_f = N_1 = 5$, $\mathcal{L} = \{1, 2, 3, 4, 5\}$, and all the other parameters are the same as in Figure 5.

combining in both steps and also provides a trade-off between complexity and performance by allowing different group numbers and sizes. In addition, the two-step MMSE approach can be extended to a multi-step approach in a straightforward manner. By increasing the number of steps, the complexity can be further reduced, although the obtained solution may be further from the optimal solution.

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