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Spectral Efficiency Analysis of Cellular Systems with Channel-Aware Schedulers

Jingxian Wu, [†,*], Neelesh B. Mehta, *Member, IEEE*[‡], Jin Zhang, *Senior Member, IEEE*[‡]

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I. INTRODUCTION

Cellular systems employ frequency reuse extensively to service a large number of users with a limited allocated spectrum. As cellular technology has evolved from second generation to third generation (3G) systems such as high speed downlink packet access (HSDPA) and High Data Rate (HDR), the spectrum utilization in terms of bits/sec/Hz, has improved significantly. This is due to the use of innovative techniques such as adaptive modulation and coding, incremental redundancy, and cross-layer schedulers that exploit multiuser diversity.

The presence of co-channel interference (CCI), which is a distinguishing characteristic of cellular systems, and the competition for radio resources among users make it very difficult to extrapolate the system-level performance from link-level results, obtained by studying individual transmitter and receiver pairs. Given the complexity of the system, most performance analyses, with a few exceptions, have been simulation studies [1]–[4], many of which simulate standard-specific models. Alouini and Goldsmith [5] derived analytical expressions for the area spectral efficiency (ASE), which measures the Shannon throughput, averaged over Rayleigh fading, shadowing, and user locations in an interference-limited cellular system. These results were extended to multiple antennas in [6], [7].

One key limitation of the analyses in [5]–[7] is the requirement that all the co-channel interferers be identically distributed, which is not the case in cellular layouts. As a result, the performance bounds based on best-case and worst-case interferer locations are quite loose for small reuse distances, at which 3G and next generation systems operate. Also, sectorization could not accurately modeled and quantified. Moreover, the results are applicable only to a Round-Robin scheduler, which allocates equal time to all users regardless of their channel conditions [4]. While [8] derived the throughput for different schedulers using numerical integration, the results are applicable only to a single-cell (no-CCI) system.

This paper presents exact closed-form expressions for the short-term Rayleigh fading-averaged spectral efficiency of cellular systems with channel-aware schedulers that operate with *non-identical* co-channel interferers and noise. We present results for the Max-SINR scheduler, which fully exploits the short-term fading variations but is unfair [8], and compare them with the Round-Robin scheduler. These results serve as upper and lower bounds for the spectral efficiency of proportional-fair schedulers [2], which trade-off system throughput for fairness. The effect of sectorization and limited modulation alphabets is also incorporated in the model. We show that small modulation alphabets, as is the case in today's 3G systems, can annul most of the multiuser diversity gains.

The rest of this paper is organized as follows. The cellular system model is introduced in Section II. Based on the statistical properties of post-detection signal to interference plus noise ratio (SINR) at the receiver, the theoretical spectral efficiencies of the Round-Robin scheduler and the Max-SINR scheduler are derived in Sections III and IV, respectively. Numerical examples are presented in Section V, followed by our conclusions in Section VI.

II. CELLULAR SYSTEM DOWNLINK MODEL

Consider a cellular system with N users per cell. On the downlink, each user has to cope with co-channel interference from M neighboring BSs. The received signal at the nth user can be modeled as:

$$r_n = h_{n0}x_{n0} + \sum_{m=1}^{M} h_{nm}x_{nm} + z_n, \tag{1}$$

where x_{n0} is the desired signal, x_{nm} is the mth interfering signal, and z_n is additive white Gaussian noise. The Rayleigh fading term h_{nm} represents the instantaneous channel state between the nth user and the mth BS, and is a zero-mean complex Gaussian random variable (RV) with variance α_{nm}, which depends on pathloss and shadowing. The channels are taken to be flat-fading. Furthermore, x_{nm} and h_{nm}, $0 \le m \le M$, are independent RVs.

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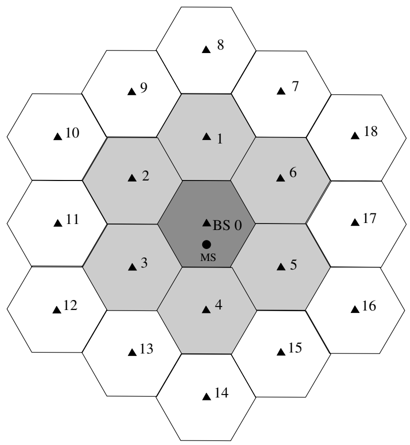

Fig. 1. Hexagonal layout of a 19-cell cellular system.

The number of interferers depends on the geometric layout of the cellular system and sectorization. For example, for the hexagonal layout shown in Fig. 1, when only first-tier interferers are considered, we have $M = 6$ without sectorization, $M = 2$ for 3 sectors per cell, and $M = 1$ for 6 sectors per cell [9]. When the second-tier interferers are also considered, the corresponding values are $M = 18$, $M = 7$, and $M = 4$.[1]

The instantaneous SINR, γ_n, at the receiver of the nth user is given by

$$\gamma_n = \frac{|h_{n0}|^2}{\sum_{m=1}^{M} |h_{nm}|^2 + 1/\rho}, \tag{2}$$

where ρ is the signal to noise ratio (SNR).

Spectral efficiency captures the highest data throughput per unit bandwidth achievable by the entire cellular system under the limitations imposed by the system model assumptions. We therefore use the Shannon capacity formula to measure throughput [5], as it is the maximum throughput the channel can reliably support. This also models the case where capacity-achieving error-free codes are used and the transmitter adapts its transmission rates on a continuous scale [1]. However, current systems have severe limitations on the maximum transmission rate. For example, 16-QAM, which is the highest modulation in HSDPA, cannot deliver a rate greater than 4 bits/symbol. The impact of a limited modulation alphabet is modeled by means of a cap, $C_{\max}$, on the achievable throughput per unit bandwidth as follows:

$$C(\gamma_n) = \begin{cases} \log_2(1+\gamma_n), & \gamma_n \le \gamma_T \\ C_{\max}, & \gamma_n > \gamma_T \end{cases}, \tag{3}$$

where $C_{\max} = \log_2(1+\gamma_T)$. When no constraint is placed on the alphabet size, we have $C_{\max} = \infty$.

The schedulers operate at a rate fast enough to adapt to the short-term Rayleigh fading variations; we do not average over shadowing, which varies much more slowly.

[1]These values of M arise when the interference from other sectors is neglected. This is justifiable because the antenna pattern attenuates adjacent sector interference by 20 dB or more.

III. Spectral Efficiency of the RR Scheduler

In a system with the RR scheduler, one user is scheduled at any time instant. Once a user is served by the BS, it is not served again until all the other users in the system have been served. The RR scheduler has the same average spectral efficiency as a random scheduler, which schedules all users with the same probability without taking into account the channel state. We state, without proof, the following simple Lemma on the average spectral efficiency of the RR scheduler.

Lemma *1:* The average spectral efficiency, C_{RR}, of a cellular system with N users and an RR scheduler is

$$C_{\text{RR}} = \frac{1}{N}\sum_{n=1}^{N} C_n, \tag{4}$$

where C_n is the average spectral efficiency of the nth user. ■

Thus, C_{RR} is simply the average of all the individual user's average spectral efficiencies. It is for this reason that the analysis in [1], [5] applies to the RR scheduler. To calculate C_{RR} and C_n, we now analyze the statistical properties of γ_n without requiring that the CCIs are identically distributed.

A. Statistical Properties of Post-Detection SINR

Lemma *2:* The probability density function (pdf) of the post-detection SINR of the nth user can be written as

$$f_{\gamma_n}(\gamma) = -\frac{\partial}{\partial\gamma}\left[\prod_{m=1}^{M}\left(1+\frac{\alpha_{nm}}{\alpha_{n0}}\gamma\right)^{-1}\exp\left(-\frac{\gamma}{\rho}\frac{1}{\alpha_{n0}}\right)\right]. \tag{5}$$

Proof: Let η denote the denominator in the SINR formula given in (2). Then, $\eta = \sum_{m=1}^{M}|h_{nm}|^2 + \frac{1}{\rho}$. As the numerator is an exponential RV with mean α_{n0}, the pdf of the SINR is

$$f_{\gamma_n}(\gamma) = \int_0^{+\infty} \frac{\eta}{\alpha_{n0}}\exp\left(-\frac{\gamma}{\alpha_{n0}}\eta\right) f_\eta(\eta)d\eta. \tag{6}$$

The form of (6) leads to an alternate and convenient representation of the pdf $f_{\gamma_n}(\gamma)$ as [6]:

$$f_{\gamma_n}(\gamma) = \frac{1}{\alpha_{n0}}\frac{\partial}{\partial s}M_\eta(s)\Big|_{s=-\gamma/\alpha_{n0}}, \tag{7}$$

where $M_\eta(s)$ is the moment generating function of η, and can be easily calculated even though η is the sum of non-identical (but independent) χ^2 RVs. $M_\eta(s)$ is given by

$$M_\eta(s) = \int_0^{+\infty} e^{\eta s} f(\eta)d\eta = \prod_{m=1}^{M}(1-\alpha_{nm}s)^{-1} e^{s/\rho}. \tag{8}$$

Combining (7) and (8) results in (5). ■

The differential form in (5) shall come in handy later on. It also leads to the following corollary about the cumulative distribution function (CDF) of γ_n.

Corollary *1:* For the nth user inside the multi-user cellular system defined by (1), the cumulative distribution function of the post-detection SINR, γ_n, is

$$F_{\gamma_n}(\gamma) = 1 - \prod_{m=1}^{M}\left(1+\frac{\alpha_{nm}}{\alpha_{n0}}\gamma\right)^{-1}\exp\left(-\frac{\gamma}{\rho}\frac{1}{\alpha_{n0}}\right). \tag{9}$$

■

B. Average Spectral Efficiency for a Single User

The instantaneous spectral efficiency, $C(\gamma_n)$, varies with time due to short-term Rayleigh fading in the channel. The average spectral efficiency of the nth user is then given by $C_n = \int_0^{+\infty} C(\gamma) f_{\gamma_n}(\gamma) d\gamma$. In the derivation below of the average spectral efficiency, C_n, for the nth user, we will need to partition the $(M+1)$ variables $\{\alpha_{nm}\}_{m=0}^{M}$ into L_n subsets, such that each subset contains all and only the variables with the same value. Let $m_n^{(l)}$ denote the cardinality of the lth subset ($M+1 = \sum_{l=1}^{L_n} m_n^{(l)}$), and $\alpha_n^{(l)}$ be the corresponding value. With this definition, C_n can be written in closed-form as

$$C_n = \log_2(e) \sum_{l=1}^{L_n} \sum_{i=1}^{m_n^{(l)}} \frac{\beta_i^{(l)}}{(i-1)!} \left[\frac{\alpha_{n0}}{\alpha_n^{(l)}}\right]^{m_n^{(l)}} \tilde{\gamma}_{n0}^{\,i-m_n^{(l)}} e^{\frac{1}{\tilde{\gamma}_n^{(l)}}} \times \left[\Gamma\left(i-m_n^{(l)}, \frac{1}{\tilde{\gamma}_n^{(l)}}\right) - \Gamma\left(i-m_n^{(l)}, \frac{1}{\tilde{\gamma}_n^{(l)}} + \frac{C_{\max}-1}{\tilde{\gamma}_{n0}}\right)\right], \tag{10}$$

where $\Gamma(k,x)$ is the incomplete Gamma function [10], L_n is the number of distinct values of the variables α_{nm}, $C_{\max}$ is the maximum rate allowed by the system, $\tilde{\gamma}_n^{(l)} = \rho\alpha_n^{(l)}$, $\tilde{\gamma}_{n0} = \rho\alpha_{n0}$, and the coefficient $\beta_i^{(l)}$ is given by

$$\beta_i^{(l)} = \frac{\partial^{i-1}}{\partial\gamma^{i-1}} \left[\left(1+\frac{\alpha_n^{(l)}}{\alpha_{n0}}\gamma\right)^{m_n^{(l)}} \prod_{m=0}^{M} \left(1+\frac{\alpha_{nm}}{\alpha_{n0}}\gamma\right)^{-1}\right]\Bigg|_{\gamma=-\frac{\alpha_{n0}}{\alpha_n^{(l)}}}. \tag{11}$$

The derivation is relegated to Appendix A.

The spectral efficiency is a function of the signal power, α_{n0}, and interference powers, $\{\alpha_{nm}\}_{m=1}^{M}$. These are in turn determined by the relative position between the user and the serving and interfering BSs. As mentioned, setting $C_{\max} = \infty$ removes the limitation on the modulation alphabet size.

In a practical setting, the variances, α_{nm}, are distinct due to different pathlosses and shadowing. C_n then simplifies to

$$C_n = \log_2(e) \sum_{m=0}^{M} \frac{\alpha_{n0}}{\alpha_{nm}} \left(\prod_{\substack{i=0 \\ i\neq m}}^{M} \frac{\alpha_{nm}}{\alpha_{nm}-\alpha_{ni}}\right) e^{\frac{1}{\tilde{\gamma}_{nm}}} \times \left[\Gamma\left(0, \frac{1}{\tilde{\gamma}_{nm}}\right) - \Gamma\left(0, \frac{1}{\tilde{\gamma}_{nm}} + \frac{C_{\max}-1}{\tilde{\gamma}_{n0}}\right)\right]. \tag{12}$$

The overall spectral efficiency of the Round-Robin scheduler is computed from the single user values using Lemma 1. The above result can be specialized to obtain the spectral-efficiency of a noise-limited system or an interference-limited system. Restricting all the CCIs to have the same power and neglecting noise results in the formulae derived in [5].

IV. Spectral Efficiency of Max-SINR Scheduler

While the RR scheduler ensures fairness among users, it does so at the expense of reduced overall system throughput. The Max-SINR scheduler, on the other hand, serves the mobile station with the highest SINR among all the users. It thus sacrifices fairness to maximize system spectral efficiency. Let $\gamma_{\max} = \max\{\gamma_1, \gamma_2, \cdots, \gamma_N\}$ denote the maximum SINR among all users at any instant. The average spectral efficiency of a system with the Max-SINR scheduler can be written as

$$C_{\text{MSINR}} = \int_0^{\gamma_T} \log_2(1+\gamma) f_{\gamma_{\max}}(\gamma) d\gamma + C_{\max}\left[1 - F_{\gamma_{\max}}(\gamma_T)\right]. \tag{13}$$

where $f_{\gamma_{\max}}(\gamma)$ and $F_{\gamma_{\max}}(\gamma)$ are the pdf and CDF of $\gamma_{\max}$, respectively.

A. Statistical Properties of Post-Detection SINR

We first derive the CDF of $\gamma_{\max}$ in the following Lemma.

Lemma *3:* The CDF of $\gamma_{\max}$ is given by

$$F_{\gamma_{\max}}(\gamma) = 1 + \sum_{n=1}^{N} (-1)^n \sum_{k=1}^{\binom{N}{n}} \times \exp(-\sigma_{nk}\gamma) \prod_{i\in\mathcal{S}_k(N,n)} \prod_{m=1}^{M} \left(1+\frac{\alpha_{im}}{\alpha_{i0}}\gamma\right)^{-1}, \tag{14}$$

where $\mathcal{S}_k(N,n)$ is an n-element subset of the index set $\{1,2,\cdots,N\}$, where the variable k indexes all of these subsets. As n elements can be chosen from N elements in $\binom{N}{n}$ ways, the number of such subsets is $\binom{N}{n}$. σ_{nk} is defined as $\sigma_{nk} = \sum_{i\in\mathcal{S}_k(N,n)} \frac{1}{\gamma_{i0}\alpha_{i0}}$.

Proof: The CDF of $\gamma_{\max}$ is given by $F_{\gamma_{\max}}(\gamma) = \prod_{n=1}^{N} F_{\gamma_n}(\gamma)$, where $F_{\gamma_n}(\gamma)$ is the CDF of the post-detection SINR of the nth user. Substituting the results of Corollary 1 into the expression for $F_{\gamma_{\max}}(\gamma)$ above results in (14). ■

B. Spectral Efficiency Analysis

With the formula for the CDF of $\gamma_{\max}$ at hand, we can now calculate the average spectral efficiency of the Max-SINR scheduler. To facilitate the analysis, we define the following function:

$$\Phi_{n,k}(\gamma) = \frac{1}{1+\gamma} \prod_{i\in\mathcal{S}_k(N,n)} \prod_{m=1}^{M} (1+\lambda_{im}\gamma)^{-1}, \tag{15}$$

with $\lambda_{im} = \alpha_{im}/\alpha_{i0}$. The product terms of the form $(1+\lambda_{im}\gamma)^{-1}$ in $\Phi_{n,k}(x)$ can be partitioned into $L(n,k)$ subsets, such that each subset contains all the terms with the same value of λ_{im}. Let $m_l(n,k)$ denote the number of terms in the lth subset – all the terms in the lth subset are of the form $(1+\lambda_l(n,k)\gamma)^{-1}$. With the above book-keeping notation in place, we have the following theorem about the average spectral efficiency of Max-SINR scheduler.

Theorem *1:* The average spectral efficiency expression for a cellular system with N users and the Max-SINR scheduler is given by

$$C_{\text{MSINR}} = \log_2(e) \sum_{n=0}^{N} (-1)^{n-1} \sum_{k=1}^{\binom{N}{n}} \sum_{l=1}^{L(n,k)} \lambda_l(n,k)^{-m_l(n,k)} \times \sum_{i=1}^{m_l(n,k)} \beta_i^{(l)}(n,k) \sigma_{nk}^{m_l(n,k)-i} \exp\left[\frac{\sigma_{nk}}{\lambda_l(n,k)}\right] \times \left\{ \begin{array}{l} \Gamma\left[i - m_l(n,k), \frac{\sigma_{nk}}{\lambda_l(n,k)}\right] \\ -\Gamma\left[i - m_l(n,k), \frac{\sigma_{nk}}{\lambda_l(n,k)} + \sigma_{nk}(C_{\max} - 1)\right] \end{array} \right\}. \quad (16)$$

Here, σ_{nk} is defined in Lemma 3 and the coefficients $\beta_i^{(l)}(n,k)$, for $i = 1, \cdots, m_l(n,k)$, are given by

$$\beta_i^{(l)}(n,k) = \frac{\partial^{i-1}}{\partial \gamma^{i-1}} \left[(1+\lambda_l(n,k)\gamma)^{m_l(n,k)} \Phi_{n,k}(\gamma)\right]\Bigg|_{\gamma = -\frac{1}{\lambda_l(n,k)}}. \quad (17)$$

Proof: The proof is in Appendix B. ■

The average spectral efficiency of a noise-limited system with the Max-SINR scheduler is obtained by setting $M = 0$.

V. Numerical Examples

We now compare the analytical results with simulations. A representative hexagonal cellular layout, shown in Fig. 1, with a reuse factor of 1 and up to two tiers of interfering BSs is used. The variances, α_{nm}, depend on the user location in the cell and BS transmission power, which is adjusted to achieve an SNR of μ dB at the corner of a cell of radius R. A pathloss exponent of 3.7 is assumed. While the analysis incorporates the effect of shadowing, it is not included in the numerical results for the sake of simplicity. The BSs are assumed to be always transmitting.

We first study the case where there is no limit on the modulation alphabet. Fig. 2 plots the Max-SINR spectral efficiency of a 10-user system without interferers, with only first-tier interferers, and with both first and second-tier interferers. All the users are placed at a distance of $R/2$ from the serving BS. It can be seen that CCI has a significant impact on system spectral efficiency. While the spectral efficiency of a noise-limited system increases linearly with μ (dB), it saturates for $\mu > 12$ dB in the presence of CCI. Not considering the second-tier interferers overestimates the spectral efficiency by 0.5 bits/sec/Hz when $\mu = 15$ dB. The simulation and analytical results agree well.

The spectral efficiencies of Max-SINR and RR schedulers are compared in Fig. 3 as a function of the number of users, N, in the system. As in Fig. 2, all the users are at a distance of $R/2$ from the BS, and $\mu = 10$ dB. As expected, while the spectral efficiency of RR scheduler is independent of N, that of the Max-SINR scheduler increases with N. Neglecting the second-tier interferers overestimates spectral efficiency by 0.25 bits/sec/Hz.

The system spectral efficiency with RR scheduler is depicted in Fig. 4 for different number of sectors. The users are assumed to be uniformly distributed in the center cell. For sectored cells, the base station antenna pattern, $A(\theta)$ in dB, is given by [11]

$$A(\theta) = -\min\left[12\left(\frac{\theta}{\theta_0}\right)^2, A_0\right], -180^{\text{o}} \leq \theta \leq 180^{\text{o}}, \quad (18)$$

where θ is the angle between direction of interest and the boresight of the antenna. For 3-sector cell, $\theta_0 = 70^{\text{o}}$, $A_0 = 20$ dB; for 6-sector cell, $\theta_0 = 35^{\text{o}}$, and $A_0 = 23$ dB. Unsectored cells use omni-directional antennas. It can be seen that sectorization benefits the system performance by reducing the number of co-channel interferers; the largest improvement occurs when the number of sectors increases from 1 to 3.

The spectral efficiencies for different modulation alphabet limits are shown in Fig. 5 as a function of the distance of the users from the BS. The number of users is taken to be 5, and, for convenience, all the users are placed at the same distance from the BS. It is interesting to observe that both the RR scheduler and Max-SINR scheduler have the same spectral efficiency $C_{\max}$, when the mobile users are close enough to the serving BS. The spectral efficiency decreases as the distance from the BS increases. When the users are close to cell edge, the constellation limit does not affect system performance, and the Max-SINR scheduler outperforms the RR scheduler.

VI. Conclusions

We derived closed-from expressions for the average spectral efficiency of cellular systems that employ the Max-SINR scheduler or the RR scheduler. The Max-SINR scheduler exploits the variations in the users' channels due to short-term Rayleigh fading. The results are sufficiently general to include an arbitrary number of identical/non-identical interferers as well as the effect of noise. The spectral efficiency turns out to be a function of the average received powers of the transmitted signal and interferers, which depend on the transmission power and geometric layout of the system. The analytical results were in excellent agreement with the simulation results. We observed that the using small constellations annuls a large fraction of the spectral efficiency gain obtainable from the channel-aware schedulers. The results derived for the Round-Robin and Max-SINR schedulers serve as lower and upper bounds for the performance of proportional-fair schedulers that trade-off between throughput and fairness.

Appendix A: Proof of (10)

From (3), the average spectral efficiency for the nth user can be shown to be

$$C_n = \log_2(e) \int_0^{\gamma_T} \frac{1}{1+\gamma} [1 - F_{\gamma_n}(\gamma)] d\gamma. \quad (19)$$

Using the expression for $F_{\gamma_n}(\gamma)$ from Corollary 1 in (19), we get

$$C_n = \log_2(e) \int_0^{\gamma_T} \exp\left(-\frac{\gamma}{\rho\alpha_{n0}}\right) \prod_{l=1}^{L_n} \left(1 + \frac{\alpha_n^{(l)}}{\alpha_{n0}}\gamma\right)^{-m_n^{(l)}} d\gamma. \quad (20)$$

Expanding the integrand in terms of partial fractions and simplifying leads to (10).

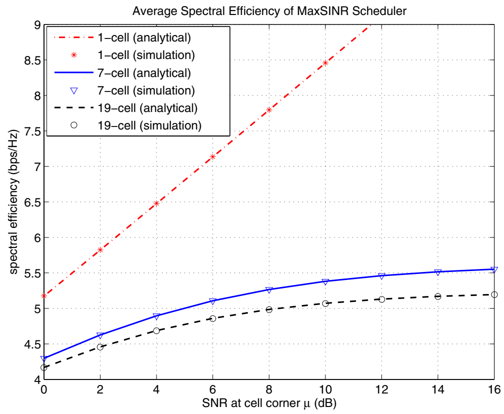

Fig. 2. Spectral efficiency of Max-SINR scheduler (10 users per cell and no sectorization).

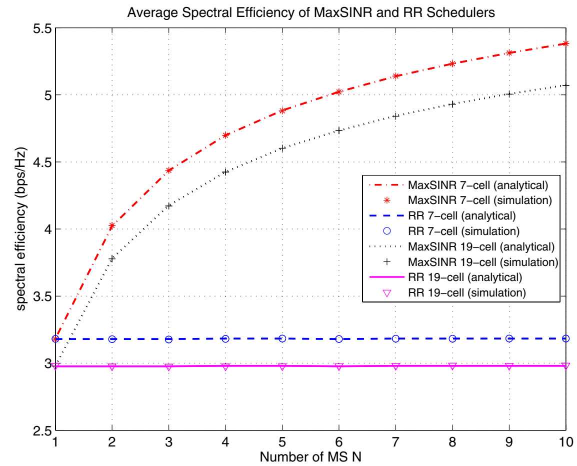

Fig. 3. Spectral efficiency Max-SINR and RR schedulers ($\mu = 10$ dB and no sectorization).

APPENDIX B: PROOF OF THEOREM 1

From (13) and integration by parts, C_{MSINR} can be written as

$$C_{\text{MSINR}} = \log_2(e) \int_0^{\gamma_T} \frac{1}{1+\gamma} \left[1 - F_{\gamma_{\max}}(\gamma)\right] d\gamma, \tag{21}$$

where $F_{\gamma_{\max}}(\gamma)$ is the CDF of $\gamma_{\max}$ defined in Lemma 3. From (14) and (15), the integrand in (21) can be written by

$$\frac{\left[1 - F_{\gamma_{\max}}(\gamma)\right]}{1+\gamma} = \sum_{n=1}^{N} (-1)^{n-1} \sum_{k=1}^{\binom{N}{n}} e^{-\sigma_{nk}\gamma} \Phi_{n,k}(\gamma). \tag{22}$$

We then perform a partial fraction expansion of $\Phi_{n,k}(\gamma)$ (defined in (15)), use it in (21), and simplify to get (16).

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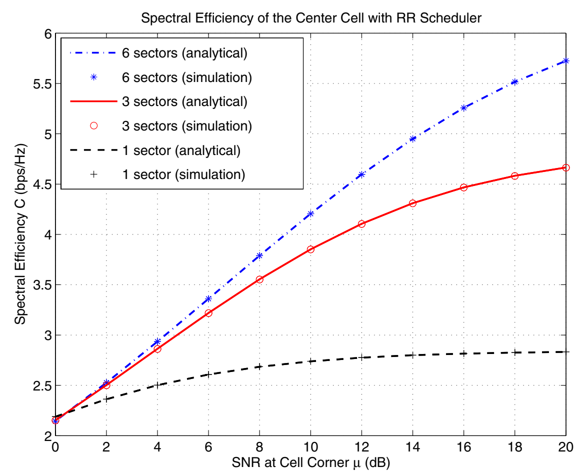

Fig. 4. Spectral efficiency with RR scheduler as a function of sectorization.

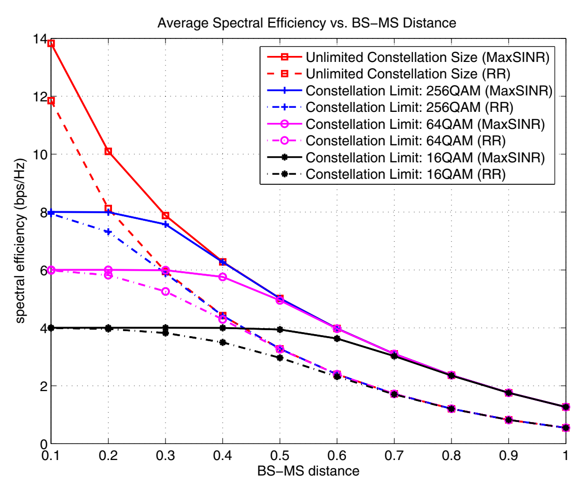

Fig. 5. Spectral efficiency with different modulation constellation size limits ($\mu = 15$ dB, 5 users per cell, and no sectorization).