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# OPTIMUM COMBINING FOR TIME HOPPING IMPULSE RADIO UWB RAKE RECEIVERS

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**Abstract**—In this paper, we present results on the application of reduced-rank adaptive filtering techniques to the problem of interference suppression in ultra-wideband (UWB) communications. It is shown that reduced-rank optimum combining (OC) methods, in particular the eigencanceler (EC), are effective in suppressing interference modeled as 802.11a signals. Simulation results are presented to show that the EC requires a shorter data record than minimum mean square error (MMSE) Rake receivers.

## I. INTRODUCTION

Impulse radio ultra-wideband (UWB) is being considered for a variety of applications including as a possible physical layer for emerging wireless personal area networks (WPANs). The ability to resolve multipath is one of the most attractive features of UWB. Numerous investigations have confirmed that the impulse radio UWB channel can be resolved into a significant number of multipath components (for example [1]). A Rake receiver can be employed to exploit the multipath diversity [2][3]. The Rake receiver, using maximum ratio combining (MRC), is optimum only when the disturbance to the desired signal is sourced by additive white Gaussian noise (AWGN). WPANs, including those with a UWB physical layer, will be typically required to operate in proximity to other wireless networks, for example, the proliferating local area networks (LANs). In the presence of narrowband interference emitted by LANs, a UWB receiver with a conventional Rake combiner will exhibit an error floor dependent on the signal-to-interference-plus-noise ratio (SINR). A more suitable diversity scheme to employ in this case is *optimum combining* (OC), whereby the received signals are weighted and combined to maximize the output SINR [4].

The minimum mean square error (MMSE) Rake is a possible implementation of OC [5][6]. The MMSE scheme is optimal (in the sense that it achieves the maximum likelihood solution for Gaussian interference plus noise) if the correlation of the received signal (aggregate of transmitted signal, interference, and noise) is known. When this correlation matrix has to be estimated, the MMSE solution is affected by measurement noise and is not optimal any more. An eigenanalysis based OC scheme, referred to as *eigencanceler* (EC), has been suggested for various applications, among them suppression of narrowband interference in direct-sequence

spread spectrum [7]. The EC exploits the inherent low-rank property of the narrowband interference correlation matrix. It is designed as a weight vector orthogonal to the *interference subspace*. The interference subspace is defined as the signal space spanned by the eigenvectors associated with the dominant eigenvalues. For the method to be effective, the dominant eigenvalues need to be contributed mainly by the interference. This is the case for low SINR. The EC is motivated by the observation that the correlation matrix of the received signal consists of a limited number of large eigenvalues contributed by the narrowband interference, and a large number of small and almost equal eigenvalues contributed by the desired signal and AWGN. A tap-weight vector orthogonal to the interference eigenvectors effectively cancels the interference, leaving most of the data untouched. The EC is computed from relatively few, stable eigenvectors spanning the interference subspace. Thus, even with a short data record, it can obtain a high degree of interference cancellation. In this paper, we study the application of the EC to impulse radio UWB systems over time dispersive channels.

This paper is organized as follows. In Section II, the impulse radio UWB system model is described. Section III presents the two approaches, MMSE and EC, to optimum combining for the UWB Rake receiver. Simulation results are provided in Section IV to compare the performance of the MMSE and EC according to bit error rate, output power of the interference-plus-noise, SINR improvement, and normalized weight variance. In Section V, we conclude this paper and propose further investigations.

## II. SYSTEM MODEL

Consider a binary bit stream  $\{d_k\} \in \{\pm 1\}$  transmitted over a multipath channel. Each data bit is represented by a sequence of  $N_p$  time-delayed pulses. The basic pulse is chosen to meet the limits imposed by the FCC emissions mask [8]. An example of such pulse is given in [9]. Denote  $p(t)$  the basic pulse as seen at the receiver. This pulse shape may be different from the transmitted one due to the effect of the receiver antenna (typically modeled as a high-pass filter or a derivative operation). Then the basic waveform representing a data bit as seen

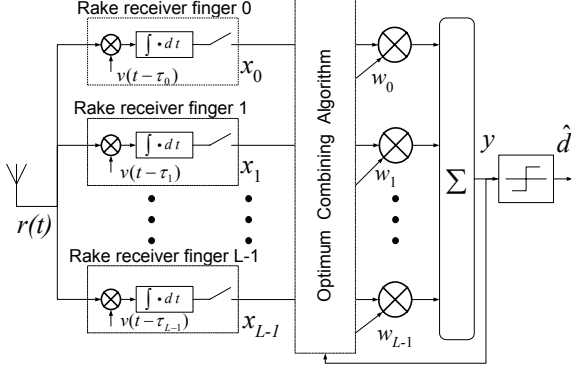


Fig. 1. A Rake receiver with optimum combining to cancel narrow-band interference for impulse radio UWB systems.

at the receiver is given by

$$q(t) = \sum_{j=1}^{N_p} p(t - jT_f - c_jT_c), \quad (1)$$

where without loss of generality  $p(t)$  is scaled such that  $\int_{-\infty}^{+\infty} q^2(t)dt = 1$ . The average pulse interval is  $T_f$ , and  $T_c$  provides an additional time shift to the  $j$ th pulse due to the time-hopping sequence  $c_j$ . Polarity reversals can eliminate the spectral lines and reduce the peak-to-average ratio [10][11]. Two types of modulation, biphasic and binary pulse-position modulation (PPM), are considered. Hence, the following model is used for the received signal at time epoch  $k$ ,

$$r_k(t) = \begin{cases} d_k \sqrt{E_b} q(t - kT_s) * h(t) & \text{biphase} \\ +i(t) + n(t), \\ \sqrt{E_b} q(t - kT_s - d_k \tau_p) * h(t) & \text{binary PPM} \\ +i(t) + n(t), \end{cases} \quad (2)$$

where the subscript  $k$  represents the bit index,  $E_b$  is the energy per bit,  $T_s = T_f N_p$  is the symbol duration, and  $\tau_p$  is a PPM shift that ensures orthogonality between the two symbols of the modulation. The dispersive channel response,  $h(t)$ , is modeled by a tapped-delay-line

$$h(t) = \sum_{l=0}^{\tilde{L}-1} h_l \delta(t - \tau_l), \quad (3)$$

where  $\tilde{L}$  is the total number of the channel taps,  $h_l$  and  $\tau_l$  are the channel tap amplitude and delay, respectively. Denote  $i(t)$  the narrowband interference and residual inter-symbol interference (ISI). The noise  $n(t)$  is white, Gaussian with zero-mean and two-sided power spectral density of  $N_0/2$ . Consistent with the baseband model assumed, all quantities are real-valued.

An OC Rake receiver is composed of  $L$  correlators followed by a linear combiner as shown in Fig. 1. The Rake receiver samples the received signals at the symbol

rate and correlates them with suitably delayed references [12]. The correlator is operated with perfect knowledge of the channel amplitudes and delays. The reference  $v(t)$ , is given by [6]

$$v(t) = \begin{cases} q(t), & \text{biphase} \\ q(t - \tau_p) - q(t + \tau_p), & \text{binary PPM.} \end{cases} \quad (4)$$

The received signal at the output of the correlator corresponding to the  $l$ th finger of the Rake receiver and the reference delayed by  $\tau_l$ , is

$$\begin{aligned} x_{l,k} &= \int_{-\infty}^{+\infty} r_k(t) v(t - kT_s - \tau_l) dt \\ &= d_k \sqrt{E_b} h_l + i_{l,k} + n_{l,k}, \quad 0 \leq l \leq L-1 \end{aligned} \quad (5)$$

where  $i_{l,k}$  and  $n_{l,k}$  are the interference and noise at the output of the correlators, respectively. The signal model in vector notation is

$$\mathbf{x}_k = d_k \sqrt{E_b} \mathbf{h} + \mathbf{i}_k + \mathbf{n}_k, \quad (6)$$

where  $\mathbf{x}_k = [x_{0,k}, \dots, x_{L-1,k}]^T$ ,  $\mathbf{h} = [h_0, \dots, h_{L-1}]^T$ ,  $\mathbf{i}_k = [i_{0,k}, \dots, i_{L-1,k}]^T$ ,  $\mathbf{n}_k = [n_{0,k}, \dots, n_{L-1,k}]^T$ , and the superscript  $T$  denotes vector transposition. A bit decision is made at the output of the combiner,  $\hat{d}_k = \text{sgn}(\mathbf{w}^T \mathbf{x}_k)$ , where  $\mathbf{w} = [w_0, \dots, w_{L-1}]^T$  is the combiner weight vector.

### III. OPTIMUM COMBINING FOR RAKE RECEIVERS

#### A. MMSE Rake Receiver

The MMSE filter parameters are varied such that the mean square error between the desired and the actual output is minimized. The SINR is maximized when the optimal weight vector of the MMSE combiner is adopted [13]. The optimal weight vector satisfies

$$\tilde{\mathbf{w}}_0 = \arg \min_{\mathbf{w}} E \left[ \|d_k - \mathbf{w}^T \mathbf{x}_k\|^2 \right]. \quad (7)$$

The solution to the MMSE combiner is

$$\tilde{\mathbf{w}}_0 = \alpha \mathbf{M}^{-1} \mathbf{h}, \quad (8)$$

where  $\alpha$  is a scaling constant, and  $\mathbf{M}$  is the correlation matrix of the interference-plus-noise,  $\mathbf{M} = E [\mathbf{i}_k \mathbf{i}_k^T + \mathbf{n}_k \mathbf{n}_k^T]$ . In practice, the matrix  $\mathbf{M}$  has to be estimated from a block of training symbols. The maximum likelihood estimate is given by the sample covariance matrix  $\hat{\mathbf{M}} = (1/K) \sum_{k=0}^{K-1} \mathbf{x}_k \mathbf{x}_k^T$ , where  $K$  is the block size. Since training is an overhead function that consumes resources, it is of interest to develop techniques that can work with short training sets. It is well known that the number of vector samples required to estimate an  $L \times L$  correlation matrix within 3 dB of its true value is  $2L$  [14]. For a dispersive channel resulting in a large number of non-zero taps  $L$ , the number of samples required to train MMSE might be prohibitive.

We propose a reduced-rank optimum combiner based on an eigenanalysis approach, which is more robust to errors caused by short training sets.

### B. EC Rake Receiver

By eigen-decomposition, the correlation matrix of the interference-plus-noise is expressed as

$$\mathbf{M} = \mathbf{Q}_I \mathbf{\Lambda}_I \mathbf{Q}_I^T + \mathbf{Q}_v \mathbf{\Lambda}_v \mathbf{Q}_v^T, \quad (9)$$

where the columns of  $\mathbf{Q}_I$  and  $\mathbf{Q}_v$  consist of, respectively, the interference eigenvectors and the noise eigenvectors. The matrices  $\mathbf{\Lambda}_I$  and  $\mathbf{\Lambda}_v$  are diagonal and contain the interference and noise eigenvalues, respectively. For an interference with bandwidth considerably smaller than the signal bandwidth, the eigenanalysis of the interference-plus-noise matrix reveals a few large eigenvalues and a large number of small eigenvalues. The eigenvectors associated with large eigenvalues span the interference subspace. Since the interference subspace is orthogonal to the noise subspace, a tap-weight vector residing in the noise subspace will effectively cancel the interference, leaving most of the information data untouched. The tap-weight of the EC is designed to minimize the norm of the weight vector while maintaining linear and eigenvector constraints given by

$$\min \mathbf{w}^T \mathbf{w} \text{ subject to } \mathbf{w}^T \mathbf{h} = g \text{ and } \mathbf{Q}_I^T \mathbf{w} = \mathbf{0}, \quad (10)$$

where  $g$  is a constant. The solution then follows [15]

$$\tilde{\mathbf{w}}_e = \beta (\mathbf{I} - \mathbf{Q}_I \mathbf{Q}_I^T) \mathbf{h}, \quad (11)$$

where  $\beta$  is a fixed scalar. It shows the weight vector of the EC is constructed from the stable eigenvectors of the largest eigenvalues of the received signal. It is unaffected by fluctuations in the noise eigenvalues. This leads to a high degree of interference cancellation shown next.

## IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of MMSE and EC-Rake receivers with respect to bit error rate, residual interference-plus-noise power, SINR improvement, and normalized variance of the weight vectors.

The channel is simulated using the IEEE 802.15.3a channel model [16]. This channel is passed through a lowpass filter with 1 GHz bandwidth, corresponding to the assumed UWB system bandwidth. Biphasic modulation is used with the data rate of 40 Mbps over the dispersive 802.15.3a line-of-sight (LOS) channel. With this data rate, ISI can be ignored and about 98% energy can be collected within one bit duration.

The transmitted UWB signal power is restricted by the FCC allowing emissions with a power spectral density of -41.3 dBm/MHz. Due to the very low transmission power, even the large processing gain of the UWB

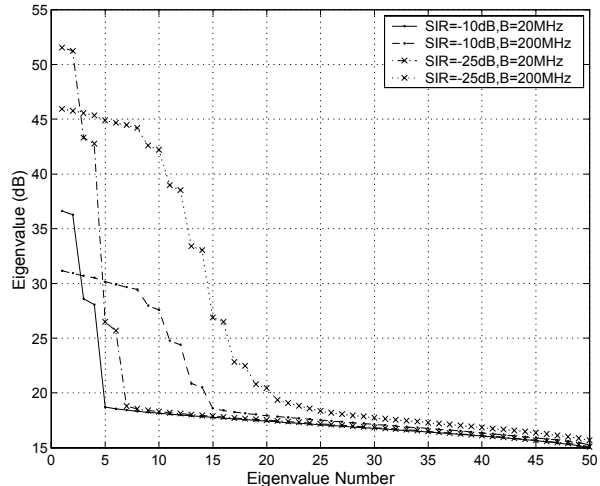


Fig. 2. Eigenvalues of the correlation matrix over the IEEE 802.15.3a LOS channel in the presence of narrowband interference.

system is not sufficient to suppress high levels of interference [17]. The narrowband interference is modeled as a bandpass Gaussian signal representing the IEEE 802.11a interference with bandwidth  $B = 20$  MHz [5]. The transmitted power of the interference is assumed 15 dBm. Assume a scenario where the UWB receiver is placed 1 m from the UWB transmitter, and 2 m from the interfering transmitter. A simple link budget calculation shows that the average signal-to-interference ratio (SIR) is approximately -10 dB in free-space. In the simulation, the SIR is assumed -10 dB, and the signal-to-noise ratio per bit ( $E_b/N_0$ ) is 10 dB.

In Fig. 2, we plotted the eigenvalues of the interference-plus-noise correlation matrix for a 50-tap receiver. It is noted then that for a 1 GHz system, and an interference with SIR = -10 dB and bandwidth 20 MHz, the number of dominant eigenvalues is 4 out of a total of 50. The interference subspace used to determine the tap weights of the EC is constructed from the 4 eigenvectors associated with these eigenvalues.

Fig. 3 shows the bit error rate (BER) performance of selective-combining (SC) Rake parameterized by the number of branches combined,  $L$ , for a biphasic system. The combining is carried out utilizing MRC. It is observed that the interference causes a high error floor. This motivates seeking optimum combining techniques.

The performance of SC-Rake with MMSE and EC receivers is compared in Fig. 4. The channel's  $L = 40$  largest energy taps are selected for processing. At BER =  $10^{-4}$ , the EC with  $K = 30$  bit training has a 2.5 dB advantage over MMSE with 80 bit training, and about 5 dB gain compared with MMSE with 60 bit training.

Further insight into the effect of the training data

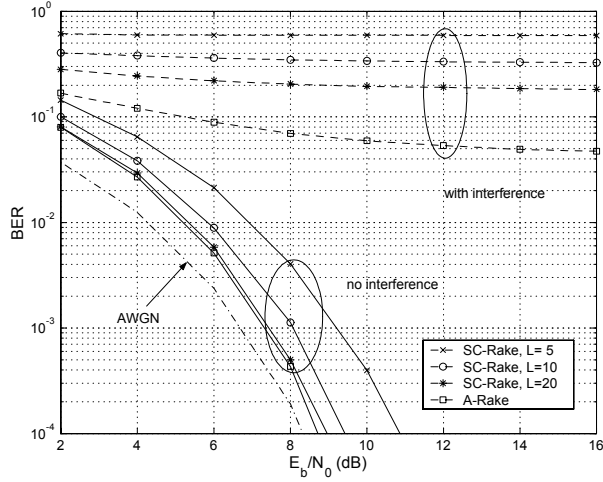


Fig. 3. BER performance of biphas UWB systems by MRC with SC-Rake over IEEE 802.15.3a LOS channel in the presence of 802.11a interference.

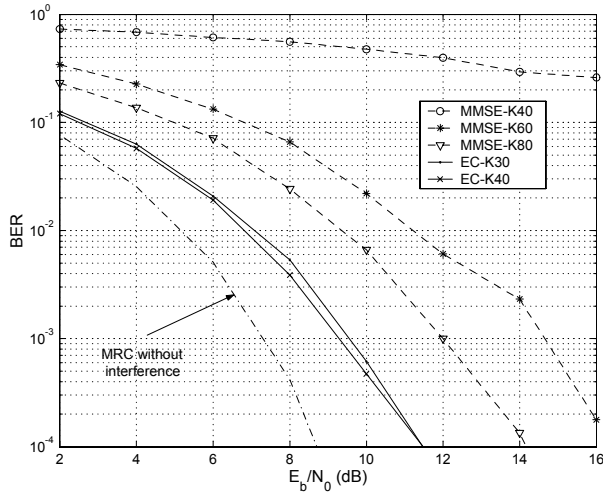


Fig. 4. BER for biphas UWB systems by MMSE and EC combiner with selected  $L = 40$  largest fingers over the 802.15.3a LOS channel in the presence of 802.11a interference.

set can be gained by measuring the residual interference power  $P_J = \tilde{\mathbf{w}}^T \mathbf{M} \tilde{\mathbf{w}}$ . In Fig. 5, we plotted the interference-plus-noise power at the output of the EC, MMSE combiners versus the size of the training data. An all-Rake (A-Rake) receiver was assumed. The figure indicates that with a limited size of data, the output interference-plus-noise power from the EC is much lower than that from the MMSE combiner.

The advantage of the EC can be also gleaned from the SINR improvement defined as the ratio of SINRs at the combiner output and input. From (6), the SINR per

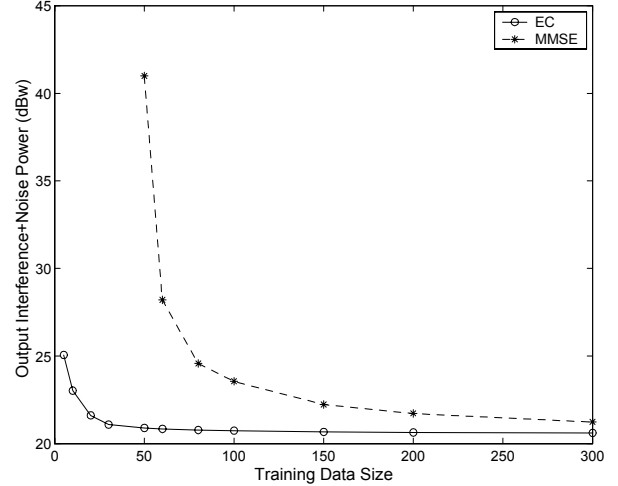


Fig. 5. Comparison of the output power of the interference-plus-noise as a function of training data size between the MMSE and EC Rake receiver. The  $L=50$  taps are used.

bit at the input to the combiner is

$$\text{SINR}_i = \frac{E_b}{I_0 + N_0}, \quad (12)$$

where  $I_0$  is the power spectral density of the interference. The output SINR is given by

$$\text{SINR}_o = \frac{E_b \|\mathbf{w}^T \mathbf{h}\|^2}{\mathbf{w}^T \mathbf{M} \mathbf{w}}. \quad (13)$$

The SINR improvement as a function of input SINR is plotted in Fig. 6 in the presence of a 802.11a interference. The receiver employed is A-Rake. With a small data size  $K$ , the EC achieves larger SINR improvement than the MMSE. For example, the SINR improvement of the EC with 30 bit training is 2 dB larger than that of MMSE with 100 bits training, and 13 dB than that with 50 bit training. Even with very short training of 10 bits, the EC still provides a larger SINR improvement than the MMSE with a training of 50.

The normalized variance of the weight vectors is a measure of the robustness of the combiners to the effects of noise. When the correlation matrix is estimated from a finite number of samples, the receiver noise causes perturbations in the values of the weight vectors. The normalized weight variance is defined as [15]

$$E \left[ \frac{\|\Delta \mathbf{w}\|^2}{\|\mathbf{w}\|^2} \right] = \frac{E \left[ \|\hat{\mathbf{w}} - \mathbf{w}\|^2 \right]}{\mathbf{w}^T \mathbf{w}}, \quad (14)$$

where  $\hat{\mathbf{w}} = \mathbf{w} + \Delta \mathbf{w}$  is the estimated weight vector,  $\mathbf{w}$  is the optimal weight vector computed with known signal statistics, and  $\Delta \mathbf{w}$  is the perturbation of the weight vector. Table I lists the normalized weight variances for the MMSE and EC combiners as a function of the

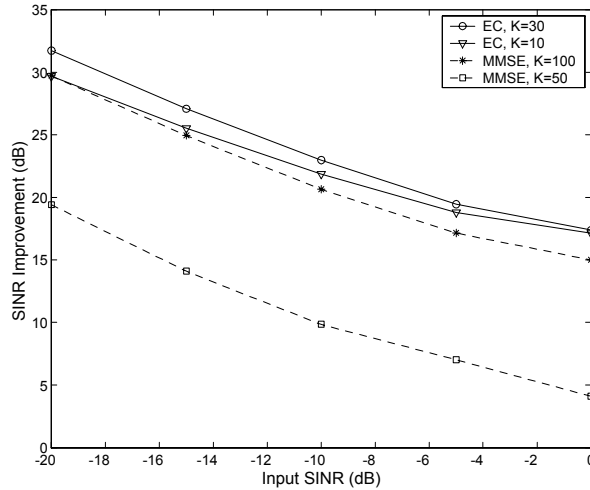


Fig. 6. SINR improvement as a function of input SINR in the presence of the 802.11a OFDM interference over the 802.15.3a LOS channel. The  $L=50$  taps are employed.

training data size  $K$ . It is observed that the EC has a much lower normalized weight variance than the MMSE, where the same SIR of -10 dB and SNR per bit of 10 dB are used as in other figures.

TABLE I  
NORMALIZED VARIANCE OF THE WEIGHT VECTOR

$K$	50	60	80	100	150
MMSE	3.9037	1.4250	1.1237	0.9749	0.8167
EC	0.0092	0.0074	0.0055	0.0043	0.0029

## V. CONCLUSION

In this paper, an eigenanalysis based optimum combining scheme, for impulse radio UWB Rake receiver, has been proposed. Simulation results have been presented to demonstrate that the performance of the proposed method is superior to the MMSE, when the correlation matrix is estimated from a limited amount of data. In this work, ideal channel information was assumed. In practice, the channel also has to be estimated from the training data. Future work will assess the effect of Rake combiners when channel estimation is taken into account.

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