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Abstract

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Time hopping and frequency hopping in ultrawideband systems

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***Abstract*— This paper analyzes frequency-hopping (FH) and time-hopping (TH) as multiple access format for ultrawideband communications. We apply the concept of "fourthegy", recently introduced by Subramanian and Hajek, to both TH and FH. We find that the design rules are different for FCC-compliant systems (where the power spectral density is limited) than for conventional systems (where the energy per bit is limited). We find that fourthegy, and thus possible information rate, is maximized by using a bandwidth that is as large allowed by the FCC. For TH systems, a low duty cycle should be used. For FH, a subdivision into as many bands as possible should be used, and the dwell time on each frequency should be at least as long as the delay spread of the channel.**

***Index Terms*— *Index Terms*— UWB, time hopping, frequency hopping**

I. INTRODUCTION

Ultrawide bandwidth (UWB) spread - spectrum (SS) multiple access techniques have recently received considerable attention for future commercial and military wireless communication systems [1], [2], [3], [4] . The report and order of the FCC (Federal Communications Commission) in the USA that allowed UWB communications systems in the 3.1-10.6 GHz range has intensified the interest especially from possible chip and equipment manufacturers. One possible application lies in Personal Area Networks (PAN), where high data rates are sent over a short distance. The FCC has imposed two restrictions on the use of the spectrum: a requirement that the transmission bandwidth is a minimum of 500MHz (though it is not completely clear over which time duration the instantaneous spectrum must fulfill that condition), and a restriction on the transmit power spectral density, namely -41.3dBm/MHz. However, the FCC imposes no specific modulation or multiple-access (MA) format as long as those restrictions are fulfilled.

This fact gives a great practical as well as theoretical value to the problem of finding a good modulation and MA scheme for ultrawideband communications. This topic is also a major factor in the deliberations of the IEEE 802.15.3a standardization committee, which has been established to develop an UWB system that can provide multiple piconets with 110Mbit/s at 10m distance, as well as higher data rates at smaller distances. Two candidate schemes are frequency-hopping (FH) and time-hopping (TH). Recent information-theoretic results [5], [6], [7], [8] allow interesting conclusions about good spreading schemes. However, those investigations do not consider the constraints put on the signalling schemes by the FCC regulations, nor do they cover quantitatively the cases of TH and FH. It is the purpose of this paper to generalize those results to FCC-compliant, TH and FH systems, and investigate the impact on system design.

The paper is organized the following way: in Section II, we set up the system model, and describe the boundary conditions imposed both by the FCC, the propagation environment, and by practical considerations. Next, information-theoretic considerations, especially based on the recently-introduced concept of fourthegy, are discussed both for TH and FH systems. A discussion of the implementation complexity, especially the required hardware effort, wraps up the paper.

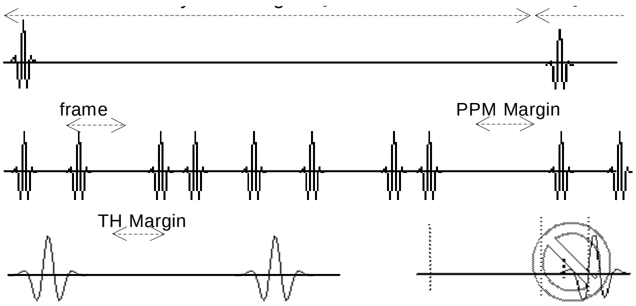

Fig. 1. Symbol structure 1 1 pulse/symbol 2 TH sequence of 8 pulses 3 Frame

II. SYSTEM MODEL

A. Time hopping

Figure 1 shows the structure of the TH signal under consideration. Each symbol is represented by a sequence of time-shifted pulses. A detailed description of the signalling format is given in [2]. The symbol duration is divided into N_f so-called "frames", and the system transmits one pulse (duration T_c) per frame. the location of the pulse within the frame is (pseudo-) random. For our analysis below, we will assume a pseudorandom signal that transmits on average with a duty cycle of N_f/N_p. We will also assume that the shape of the pulses is rectangular. While this does not fill the FCC spectral mask (see Sec. II.3) in an optimum way, it is a reasonable first approximation and allows a closed-form analysis.

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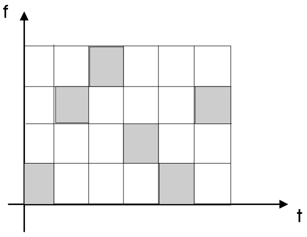

Fig. 2. Principle of frequency hopping. Shaded areas show active frequency bands.

B. Frequency hopping

Figure 2 shows the structure of a frequency-hopped signal. The total bandwidth B is divided into N_b bands of equal width; within each band, a signal with a chip duration T_c is used; the relation $B/N_b = 1/T_c$ holds . We analyze the situation of slow frequency hopping, so that the signal stays in the same frequency band for a duration of $N_h T_c$. The frequency-hopping pattern is pseudorandom, with each frequency having the same probability $1/N_b$ to occur, and is periodic with period $N_h N_c T_c$. This allows to use FH as multiple access scheme for unsynchronised users.

C. FCC rules

The considered system is operating with several restrictions or "boundary conditions", some of which are imposed by frequency regulators, some stem from the wireless channel, and some from restrictions on the implementation complexity.

The rules of the FCC require that

1) the bandwidth of the system must be larger than 500MHz. This implies that for FH systems, the width of the subband must be at least 500MHz.
2) the power spectral density must be smaller than -41.3dBm/MHz in the admissible band (3.1-10.6 GHz).
3) The peak transmit power must be less than the admissible average transmit power, plus $10 + 10\log(B/50MHz)$ dB.[1] This implies that for TH systems with large peak-to-average ratio, there is a penalty in terms of the mean transmitted power.
4) the measurement of the spectrum for testing conformance with the FCC rules can occur only for a finite duration, so that an "instantaneous spectrum" is analyzed. There is some ambiguity about the measurement duration for which conditions 1 and 2 must be fulfilled. In this paper, we will assume that this has to be $1\mu s$, (corresponding to the 1MHz video bandwidth of the spectrum analyzer mentioned in the report and order). [2] This implies that a FH system has to cycle through all possible subbands within this period.

III. INFORMATION-THEORETIC RESULTS

A. Modulation format

Recent information-theoretic work treated topics that are relevant both for finding both good modulation format and the multiple-access scheme. The work of Medard and Gallager [7] as well as Subramanian and Hajek [8] showed that signals need to be "peaky" in time or frequency; the term "flash signalling" was coined by Verdu [5], [6] for signals that have very large amplitude on a small support (in time or frequency), and no signal for the remainder. However, such schemes cannot be used in practice:

(i) as shown in [5], [6], the spectral efficiency of flash signalling approaches zero

(ii) the duration of a pulse for flash signalling in the time domain is lower-limited to about 100ps; shorter pulses would not comply with the FCC spectral mask

(iii) flash signalling in the time domain has a large peak-to-average ratio. The FCC regulations require a decrease in transmit power for this case.

(iv) the spectral width of the signal for flash signalling in the frequency domain is very narrow, and thus does not comply with the FCC rules.

For these reasons, QPSK (or BPSK for pure-baseband systems) remains the modulation format of largest practical interest.

B. Multiple access format

A key paper for the analysis of different signalling format is the work of Subramanian and Hajek [8]. They show that the mutual information in any wideband channel is upper-bounded by a so-called "fourthegy" of the signal arriving at the receiver. In other words, the mutual information between a transmit vector U and a received vector Y is upper bounded as

$$I(U;Y) \leq \frac{1}{2\sigma^2} E\{J_c(U)\}$$

where σ is the noise variance. For a WSSUS channel, this fourthegy can be computed as [8]

$$J_c(u) = \int_{\nu}\int_{\tau} |\chi(\nu,\tau)|^2 \psi_H(\nu,\tau) d\tau d\nu \tag{1}$$

where the channel response function $\psi_H(\nu,\tau)$ is given by

$$\psi_H(\nu,\tau) = \int_{f}\int_{t} S_H(f,t) S_H(f+\nu.t+\tau) dt df \tag{2}$$

where $S_H(f,t)$ is the *spreading function* of the channel [9]. The ambiguity function $\chi(\nu,\tau)$ of the signal $u(t)$ is defined as

$$\chi(\nu,\tau) = \int u(t+\tau/2) u^*(t-\tau/2) \exp(-j2\pi\nu t) dt \tag{3}$$

Ref. [8] derived several important conclusions from this formulation of the fourthegy. The first is that as the spreading bandwidth goes to infinity, the mutual information (per unit energy) that can be transmitted with "white-like" signals like DS-CDMA approaches ·zero. This conclusion agrees with the work

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[1] The full ruling of the FCC is more complicated. We use this rule here as an approximation.

[2] Other parts of the report and order mention that the impact of a victim receiver with 50MHz bandwidth should be the guideline for all interference assessment, while additional comments refer to $1ms$ averaging time for the spectrum analyzer.

of Medard and Gallager [7]. An intuitive explanation can be given by the fact that DS-CDMA requires coherent reception, which in turn necessitates channel estimation. The channel estimation becomes progressively more difficult as the spreading bandwidth increases.

However, UWB communications according to FCC standards differ in three important respects from the assumptions of the above-mentioned papers:

- the restriction lies not on the energy per bit, but rather on the power spectral density. Thus, as the bandwidth increases, the admissible energy per bit increases, which means that the mutual information does not go to zero as the spreading bandwidth increases. Consider the case of a block-frequency fading channel, where the fading in each of the N_b frequency bands is constant, and independent of the fading in the other bands. It has been pointed out in [8] that if the energy of the transmit signal is distributed evenly among the bands, the fourthegy per band scales like $1/N_b^2$, and the total fourthegy scales with $1/N_b$. However, following the FCC rules, the admissible energy of the transmit signal increases linearly with N_b, the fourthegy of the signal *increases* with N_b.
- the spreading bandwidth cannot be increased *ad infinitum*, but rather has a strict upper limit of 7.5GHz. For communications according to IEEE 802.15.3a requirements, the spreading factor is limited to about 10-100; in other words, less than for speech communications in W-CDMA.
- while DS-CDMA approaches have been proposed for UWB within the IEEE 802.15.3a standardization, TH and FH are more popular. It is thus of great interest to extend the analysis of [8] to those cases.

C. Time-hopping signals

Our approximations (see Sec. II.1) allow to treat a TH signal as a generalized CDMA signal

$$u(t) = \sum_{i=0}^{N-1} a_i p(t - iT_c) \tag{4}$$

where the coefficients a_i are independent random variables that take on the values $\pm\sqrt{N_p/N_f}$ (or $\pm\sqrt{N_p/N_f}, \pm j\sqrt{N_p/N_f}$) with probability N_f/N_p and 0 with probability $1 - N_f/N_p$. This normalization is chosen so that $E\{|a_i|^2\} = 1$ for a DS-CDMA system with constant modulus transmit signal, and any TH system has the same normalization. The admissible transmit power is proportional to $B = 1/T_c$, so that the squared magnitude of the ambiguity function of the signal is proportional (with proportionality constant K_{rd} to (compare also [8], Eq. (31))

$$E\{|\chi(\nu,\tau)|^2\} = \frac{K_{rd}}{T_c^2} P\left(\frac{N_p}{N_f}\right) \cdot \tag{5}$$

$$\Bigg[\Bigg(E\{|a_n|^2\}^2 \sum_{m=-N-1}^{N-1} (N - |m|)\exp(-j2\pi m\nu T_c) +$$

$$+N\left[E\{|a_n|^4\} - E\{|a_n|^2\}^2\right]\Big)|\chi_p(\nu,\tau)|^2 +$$

$$E\{|a_n|^2\}^2 \sum_{|m=-N-1, m\neq 0}^{N-1} (N - |m|)|\chi_p(\nu,\tau + mT_c)|^2\Bigg]$$

with $E\{|a_n|^2\}^2 = 1$ and $E\{|a_n|^4\} = N_p/N_f$, and $P(N_p/N_f)$ is the power penalty for high peak-to-average ratio. A first estimate for the fourthegy can be made by integrating squared magnitude of the ambiguity function over the support of the channel response function $\psi_H(\nu,\tau)$, i.e., the range $[-2f_d < \nu, 2f_d; -\tau_{max} < \tau < \tau_{max}]$. We also assume that the chip duration is shorter than the delay spread of the channel (an assumption always fulfilled in practice). Then we can make further upper boundings of the fourthegy per unit time (for a derivation see [10])

$$\begin{aligned}\frac{J_c(u)}{T} \quad &\le \quad K_{rd} P\left(\frac{N_p}{N_f}\right)\frac{1}{T_c} \\ &\left(1 + \frac{2T_{coh}}{\tau_{max}} + \frac{T_c}{\tau_{max}}(\frac{N_p}{N_f} - 1)\right)\end{aligned} \tag{6}$$

where T_{coh} is the coherence time of the channel. From 5, we see that the duty cycle in a TH system should be as low as possible (i.e., $\frac{N_p}{N_f}$ as large as possible. This also fits with the conclusions of [7], [8], [5] that signals should be as peaky as possible. However, there are two restrictions to this statement: (i) the existence of a power penalty for high (temporal) peak-to-average ratios means upper-bounds N_p/N_f to about 1500 in FCC-compliant systems that exploit the full admissible bandwidth (ii) there is a significant input on the total fourthegy only if T_c/τ_{max} is not too small. In typical indoor cases, this quantity is on the order of 100 or larger, so that the effect of this term (compared to T_{coh}/τ_{max}) is rather small. Note also that the fourthegy is proportional to 1/T_c (and not to T_c, as in [8]). This is due to the possible increase of the transmit power with used bandwidth.

D. Frequency-hopping signals

The squared magnitude of the ambiguity function of the signal is (for a derivation see [10])

$$E\{|\chi(\nu,\tau)|^2\} = K_{fh} B^2 \tag{7}$$

$$\Bigg(E\{|a_n|^2\}^2 \sum_{m=-N-1}^{N-1} (N - |m|)\exp(-j2\pi m\nu T_c) +$$

$$+N\left[E\{|a_n|^4\} - E\{|a_n|^2\}^2\right]\Big)|\chi_p(\nu,\tau)|^2 + K_{fh}B^2 \cdot$$

$$\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} E\{|a_n|^2\}E\{|a_m|^2\}|\chi_p(\nu + f_n - f_m, \tau + mT_c)|^2$$

where f_n and f_m are the frequencies for chips n and m. Further simplifications for that equation can be achieved by assuming that the transmission within each subband is done by constant-modulus signals $E\{|a_n|^2\} = 1$, and that there is no overlap between the subbands (and the subbands are significantly larger than the Doppler spreads), so that χ_p is zero unless $f_m = f_n$.

Using those simplifications, the squared ambiguity function can be approximated as

$$E\{|\chi(\nu,\tau)|^2\} \approx K_{fh}B^2 \tag{8}$$
$$\left(\sum_{m=-N-1}^{N-1}(N-|m|)\exp(-j2\pi m\nu T_c)|\chi_p(\nu,\tau)|^2+\right.$$
$$\left.\sum_{m=0}^{N-1}\Lambda(m)|\chi_p(\nu,\tau+mT_c)|^2\right)$$

where

$$\Lambda(m)=\begin{cases} N-|m| & \text{for } |n \bmod (N_h N_b)| < N_b/2 \\ 0 & \text{otherwise}\end{cases} \tag{9}$$

The fourthegy per unit time can be then upper-bounded as

$$\frac{J_c(u)}{T} \leq K_{fh}BN_h\left(\frac{2T_{cdh}}{\tau_{max}}+\frac{T_c}{\tau_{max}}\min(N_b,\frac{\tau_{max}}{T_c})\right) \tag{10}$$

This again allows important conclusions about design rules: the dwell time on each frequency should be at least as long as the delay spread, and the frequency band should be divided into as many bands as possible. This is again in line with the general rule that signals should be "peaky" (in the frequency domain, in this case). However, the FCC ruling gives a lower limit, namely 15, on the number of possible bands.

IV. IMPLEMENTATION COMPLEXITY

For a FH transceiver, there are two possible ways of implementation. One is to actually have parallel transceiver chains, each of them dedicated to a specific frequency range (note that the frequency band covered by one of the chains can be larger than the subbandwidth used by the frequency-hopping scheme). The advantages of this scheme are

- that narrowband components (amplifiers, antennas, etc.) can be used within each chain.
- multiuser detection can be used, because all information is available.

The alternative is to have a single RF front end, with receive antenna and amplifier covering the whole UWB frequency range. The local oscillator changes its center frequency in synchronization with the frequency hopper of the transmitter. The advantage is that at each time instant, the signal that has to be sampled and processed has only the bandwidth of B/N_b. This drastically reduces the costs and power consumption of the A/D conversion and digital processing. The drawbacks are that

- multiuser detection is not easily possible
- multipath components that arrive after the LO has changed to a different frequency, cannot be collected. Thus, if the dwell time N_bT_c is smaller or comparable to the maximum delay spread, an energy loss (which enters quadratically into the fourthegy) occurs. This puts an additional constraint on the dwell time.

For a TH receiver, we also have two different possible implementations. The first one is sampling the received signal with the Nyquist bandwidth (20Gsamples/s); the A/D conversion has to have a small resolution (1 or 2bits), and the energy consumption of such a high-rate ADC is usually larger. Furthermore, also the subsequent processing has to be done at the high speed; on the upside, this scheme allows an almost ideal demodulation including multiuser detection, as the full information about the received signal is present.

An alternative scheme [11] uses analog correlators, which consist of pulse generators, multipliers, and low-pass filters. With that scheme, sampling is only required at the symbol rate, not at the chip rate. For each received multipath component, we then need one pulse generator. For a dense multipath channel, τ_{max}/T_c Rake fingers are required. However, in a sparse channel like the IEEE 802.15.3a channel model, that number is much smaller. A discussion of the optimization of low-complexity Rake receivers can be found in [12].

V. SUMMARY AND CONCLUSION

We have analyzed and compared FH and TH, applying and modifying some recent information-theoretic results to FCC-compliant UWB communications. The "fourthegy" of the arriving signal is a vital characteristic. While a rough upper bound on the mutual information is given by the product of the squared energies of signal and channel, further information can be gathered from how well the channel and the signal are matched. Some optimum (in the limit) schemes, like FH with a large number of subbands, are not allowed by the FCC regulations. This paper gave a first insight into those topics. Further investigations on capacity in broadcast/multiple access channels, tightness of the bounds, effect of multiuser detection, and the effect of more refined channel models, will be presented in future work.

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