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Abstract

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Regenerator vs. Simple-Relay with Optimum Transmit Power Control for Error Propagation

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***Abstract*— We study power dissipation in relay assisted wireless transmissions. Two types of assistance are considered: simple relaying and regenerative repeater. We find minimum transmit power levels to provision the same bit error rate (BER) in both cases. The simple-relay case considers power adjustment for error propagation at the intermediate relay node. Power consumption comparisons are made and results are discussed.**

***Index Terms*—Power control, error propagation, relay-assisted transmission.**

I. Introduction

In sensor networks and mobile ad-hoc networks, minimizing overall power consumption in the network, and accordingly increasing network lifetime is currently an active research area. Some of the approaches to achieve power efficiency are to decrease encoder and decoder complexities, to design low power circuitries, and to develop low signaling-cost routing protocols etc. There are also other techniques that try to exploit network topology to reduce power expenditure. Relay assisted transmission and power combining methods with diversity techniques [1]-[4] fall under the latter.

Assume a triplet as illustrated in Fig.1. In [1], [2] and [4], it is shown that when terminal A sends its data to B through C, the overall power consumption compared to the case when A transmits directly to B is reduced. This comparison is only valid provided that two scenarios provision the same bit error rate (BER) constraint, $\Pr_e^{AB}$, at destination B. In [1]-[4], this constraint is provisioned. However, their results do not take transmit power compensation due to error propagation into account at the relay node. This letter aims to fill this gap.

Intermediate node C may function in two different ways: a regenerator or a simple relay. In the regenerator mode (Fig.1-top), C receives data from A, at a power level that would satisfy $\Pr_e^{AB}$, corrects bits in errors and forwards the recovered data to B at a transmit power level that would again satisfy $\Pr_e^{AB}$ at receiver B. In a simple relay mode (Fig.1-bottom), C does not perform any data recovery (e.g., error correction etc.), but cooperates with A such that A adjusts its transmit power level to provision $\Pr_e^{AC}$ at receiver C; and C adjusts its transmit power level to provision $\Pr_e^{CB}$ at receiver B. Hence, assuming that the bit errors on these two paths are additive, the total BER is constrained to $\Pr_e^{AB}$.

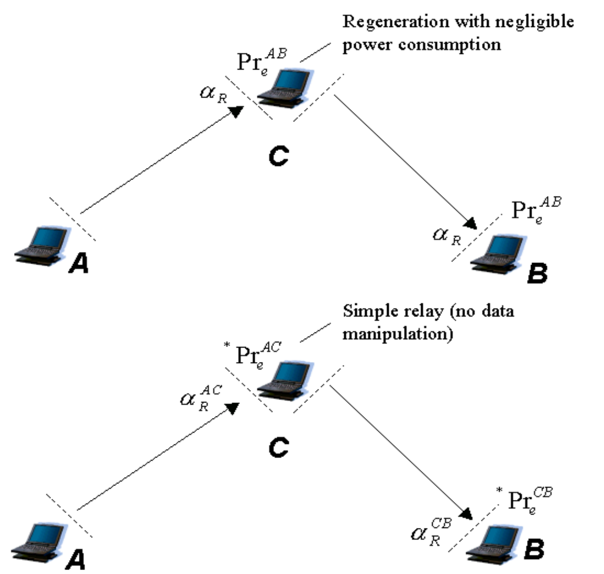

Fig. 1. Illustration of the received signal power levels and bit error rates at receiving terminals in wireless transmission over (top) the regenerator (bottom) simple relay intermediate node.

II. Radio Model

We denote by α_R, the required received signal power at rate R such that the BER is $\Pr_e^{AB}$. We use a simple radio model analogous to the ones in [2] and [3] to model wireless transmission. We assume that the power consumption of a communications node, P_t (in watts), consists of power consumed by the tx/rx radio circuitry, P_{circ}, which is fixed; the power consumed by the tx amplifier power, $P_{amp}(R)$, which is adjusted to achieve a desired BER at rate R; and power consumed to receive the bits at the receiver end, $P_{rec}(R)$. Also, different levels of clutter on propagation paths are modeled as *lognormal shadowing*. The shadowing effect between nodes *i* and *j*, x_{ij} (in dB), is expressed in the power equations as $10^{x_{ij}/10}$ (in watts).

$$P_t = P_{circ} + P_{amp}(R) + P_{rec}(R) \quad (1)$$

P_{amp} can be further expanded as $P_{amp}^{ij} = \alpha_R 10^{x_{ij}/10} d_{ij}^{\gamma}$, where d_{ij} is the distance between node pairs i and j, and γ is the path loss exponent. Antenna gain is assumed to be unity.

III. Power Analysis

In this section, we quantify power dissipation in a communications triplet with an intermediate node that is used as a regenerator or a simple-relay (amplifier). These two cases are compared to the power consumption in direct transmission.

A. Direct Transmission

Let P_{amp}^{ij} denote the amplifier power at transmitter i for streaming data to receiver j. When data are directly transmitted from A to B, the total power expended in the system, $P_t^{'}$, is

$$\begin{aligned} P_t^{'} &= P_{circ}^{A} + P_{amp}^{AB}(R) + P_{rec}(R) + P_{circ}^{B} \\ &= (2P_c + P_{rec}(R)) + \alpha_R 10^{x_{AB}/10} d_{AB}^{\gamma} \end{aligned} \quad (2)$$

In (2), we have made an assumption without loss of generality that $P_{circ}^{A} = P_{circ}^{B} = P_{circ}^{C} = P_c$.

B. Intermediate Regenerator Node

Assume that A transmits data to B using C as an intermediate regenerator node. C would consume power while both receiving data from A and forwarding it to B. Let $P_t^{''}$ denote the power dissipation in this triplet.

$$\begin{aligned} P_t^{''} &= P_{circ}^{A} + P_{amp}^{AC}(R) + P_{circ}^{C} + 2P_{rec}(R) + P_{amp}^{CB}(R) + P_{circ}^{B} \\ &= 3P_c + 2P_{rec}(R) + \alpha_R (10^{x_{AC}/10} d_{AC}^{\gamma} + 10^{x_{CB}/10} d_{CB}^{\gamma}) \end{aligned} \quad (3)$$

The satisfactory condition to have power savings with comparison to the direct transmission is $\Delta P_1 = P_t^{''} - P_t^{'} < 0$.

$$\begin{aligned} \Delta P_1 &= \alpha_R (-10^{x_{AB}/10} d_{AB}^{\gamma} + 10^{x_{AC}/10} d_{AC}^{\gamma} \\ &+ 10^{x_{CB}/10} d_{CB}^{\gamma}) + P_c + P_{rec}(R) \end{aligned} \quad (4)$$

After rearranging (4), the test condition for power savings is given in (5).

$$10^{x_{AC}/10} d_{AC}^{\gamma} + 10^{x_{CB}/10} d_{CB}^{\gamma} < 10^{x_{AB}/10} d_{AB}^{\gamma} - \frac{P_c + P_{rec}(R)}{\alpha_R} \quad (5)$$

C. Intermediate Simple Relay Node

In this scenario, C does not perform any recovery on the data received from A. Let α_R^{AC} and α_R^{CB} denote the required received signal power levels to provision $\Pr_e^{AC}$ from A to C and $\Pr_e^{CB}$ from C to B. Then, the overall power dissipation, $P_t^{'''}$, is

$$\begin{aligned} P_t^{'''} &= P_{circ}^{A} + P_{amp}^{AC}(R) + P_{circ}^{C} + 2P_{rec}(R) + P_{amp}^{CB}(R) + P_{circ}^{B} \\ &= 3P_c + 2P_{rec}(R) + \alpha_R^{AC} 10^{x_{AC}/10} d_{AC}^{\gamma} + \alpha_R^{CB} 10^{x_{CB}/10} d_{CB}^{\gamma} \end{aligned} \quad (6)$$

The difference in power expenditure between simple relaying and direct transmission is $\Delta P_2 = P_t^{'''} - P_t^{'}$.

$$\begin{aligned} \Delta P_2 &= \alpha_R^{AC} 10^{x_{AC}/10} d_{AC}^{\gamma} + \alpha_R^{CB} 10^{x_{CB}/10} d_{CB}^{\gamma} \\ &- \alpha_R 10^{x_{AB}/10} d_{AB}^{\gamma} - P_c + P_{rec}(R) \end{aligned} \quad (7)$$

To prove power efficiency of simple relaying compared to direct transmission $\Delta P_2 < 0$ must be satisfied. Eq.7 leads to test condition (8), which is slightly different from (5).

$$\begin{aligned} &\frac{\alpha_R^{AC}}{\alpha_R} 10^{x_{AC}/10} d_{AC}^{\gamma} + \frac{\alpha_R^{CB}}{\alpha_R} 10^{x_{CB}/10} d_{CB}^{\gamma} \\ &< 10^{x_{AB}/10} d_{AB}^{\gamma} - \frac{P_c + P_{rec}(R)}{\alpha_R} \end{aligned} \quad (8)$$

Note that the right sides of (5) and (8) are the same. In order to be able to compare power consumption in the intermediate simple relay scenario to either the direct transmission or to the intermediate regenerator scenario, α_R^{CB} and α_R^{AC} must be optimally selected such that the left side of (8) is minimized, under the constraint that $\Pr_e^{AC} + \Pr_e^{CB} = \Pr_e^{AB}$. Assuming a M-ary Quadrature amplitude modulation (QAM) scheme in direct transmission from A to B, $\Pr_e^{AB}$ can be approximated as in (9) [5].

$$\Pr_e^{AB} \cong \frac{4(1 - 1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R \log_2 M}{R(M-1)N_0}}\right) \quad (9)$$

where N_0 is the power density of additive white Gaussian noise (AWGN). In a similar way, we can formulate $\Pr_e^{AC}$ and $\Pr_e^{CB}$ as in (10) and (11) respectively.

$$\begin{aligned} \Pr_e^{AC} &\cong \frac{4(1 - 1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R^{AC} \log_2 M}{R(M-1)N_0}}\right) \\ &= (4/\beta) Q\left(\sqrt{\alpha_R^{AC}/\alpha_R}\, Q^{-1}\left(0.25\beta \Pr_e^{AB}\right)\right) \end{aligned} \quad (10)$$

$$\begin{aligned} \Pr_e^{CB} &\cong \frac{4(1 - 1/\sqrt{M})}{\log_2 M} Q\left(\sqrt{\frac{3\alpha_R^{CB} \log_2 M}{R(M-1)N_0}}\right) \\ &= (4/\beta) Q\left(\sqrt{\alpha_R^{CB}/\alpha_R}\, Q^{-1}\left(0.25\beta \Pr_e^{AB}\right)\right) \end{aligned} \quad (11)$$

where $\beta = (\log_2 M)/(1 - 1/\sqrt{M})$.

The goal is to find optimum error rates ${}^{*}\Pr_e^{AC}$ and ${}^{*}\Pr_e^{CB}$ that would minimize objective function (12), and then to compute optimum received signal power levels ${}^{*}\alpha_R^{AC}$ and ${}^{*}\alpha_R^{CB}$.

$$\arg\min_{\Pr_e^{AC}} \left(\left(\frac{Q^{-1}(0.25\beta \Pr_e^{AC})}{Q^{-1}(0.25\beta \Pr_e^{AB})}\right)^2 10^{x_{AC}/10} d_{AC}^{\gamma} + \left(\frac{Q^{-1}(0.25\beta \Pr_e^{CB})}{Q^{-1}(0.25\beta \Pr_e^{AB})}\right)^2 10^{x_{CB}/10} d_{CB}^{\gamma} \right) \quad (12)$$

After rearranging the derivative of (12) in terms of $\Pr_e^{AC}$, we get (13). It is then straightforward to find ${}^{*}\Pr_e^{AC}$.

$$\ln\left(\frac{d_{AC}^{\gamma}10^{x_{AC}/10}}{d_{CB}^{\gamma}10^{x_{CB}/10}}\right)=\ln\left(Q^{-1}\left(0.25\beta(\Pr_e^{AB}-\Pr_e^{AC})\right)\right)$$
$$+\left(erf^{-1}\left(1-0.5\beta(\Pr_e^{AB}-\Pr_e^{AC}\right)\right)^2-\left(erf^{-1}\left(1-0.5\beta\Pr_e^{AC}\right)\right)^2$$
$$-\ln\left(Q^{-1}\left(0.25\beta\Pr_e^{AC}\right)\right) \quad (13)$$

Note that $^{*}\Pr_e^{CB}=\Pr_e^{AB}-{}^{*}\Pr_e^{AC}$. Finally, $^{*}\alpha_R^{AC}$ and $^{*}\alpha_R^{CB}$ are derived from (10) and (11), and are used in (6) to compute the power dissipation in the simple relay triplet.

IV. RESULTS

Assume a grid area of 100x100 meters. Let us denote the Cartesian coordinates of nodes *A, B* and *C* as (0,0), (100,0) and (i,j) respectively, where i,j∈[1,99]. In order to compare relative power savings between the regenerator and the simple-relay, we define a new metric η in (14). It is the average ratio of the power expenditure in the relay mode to the power expenditure in the regenerator mode. This average is taken only over the set of coordinates where relaying has power savings, that is $P_t^{'''}<P_t^{'}$. The I(*x*) is the indicator function defined such that it returns 1 if *x* is TRUE, and 0 otherwise.

$$\eta=\frac{\sum_{i=1}^{99}\sum_{j=1}^{99}(P_t^{'''}(i,j)/P_t^{''}(i,j))\mathrm{I}(P_t^{'''}(i,j)<P^{'})}{\sum_{i=1}^{99}\sum_{j=1}^{99}\mathrm{I}(P_t^{'''}(i,j)<P^{'})} \quad (14)$$

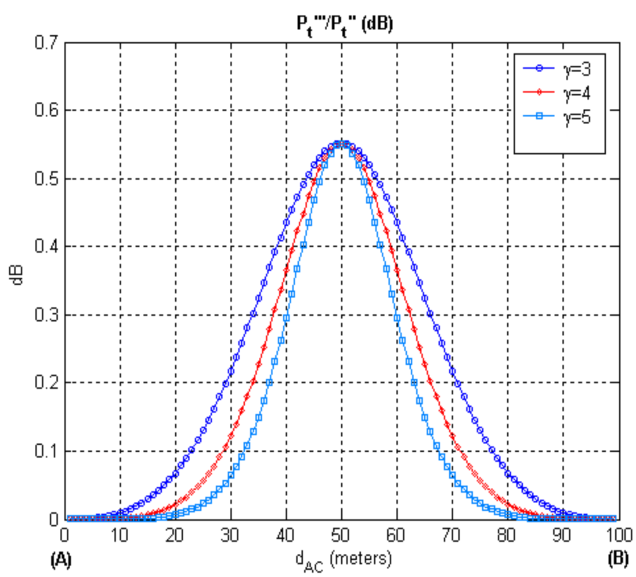

Fig. 2. Power savings of regenerating with respect to simple relaying at intermediate node positions over the direct path between A and B separated with 100m. Note: $\Pr_e^{AB}=10^{-3}$, $x_{AB},x_{CB},x_{AC}=0$, M=64, and $(3P_c+2P_{rec}(R))/\alpha_R=0.2$.

Figure 2 plots the numerator in (14) for only intermediate node positions on the direct path between A and B. The results show that regenerating may expend up to 0.55dB less power than simple relaying, as the intermediate node gets closer to the midpoint of A-B separation, intuitively which proves to be the optimum point to achieve the highest power savings in 1-hop assisted wireless transmission.

TABLE I

AVERAGE POWER SAVING DIFFERENCE BETWEEN SIMPLE-RELAYING AND REGENERATING (in dB). Note: M=64

γ		3	4	5
η	$\Pr_e^{AB} = 10^{-4}$	0.22	0.19	0.17
	$\Pr_e^{AB} = 10^{-3}$	0.31	0.27	0.23

On the other hand, the numerical results given in Table-1 for η prove that for lower $\Pr_e^{AB}$ and higher γ, the performance margin between the simple relay and the regenerator gets quite narrow. Noting that results in Fig.2 and Table-1 ignore power consumption due to regeneration process at the intermediate node, simple relaying may be preferred to regenerating. For instance, assume that the relay node is located on the direct path at 70 meters from A and 30 meters from B. In Fig.2, $P_t^{'''} < P_t^{''}$ corresponds to 0.12 (dB) for γ=4 at that position. This tells us that if the power dissipation for regeneration process exceeds 0.12 (dB), simple relaying is favored.

V. CONCLUSIONS

We quantify power savings performances of simple relaying and regenerating at an intermediate node in relay-assisted wireless transmission. Error propagation is taken into consideration for optimizing transmit-powers.

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