Phase Unwrapping in Correlated Noise for FMCW Lidar Depth Estimation

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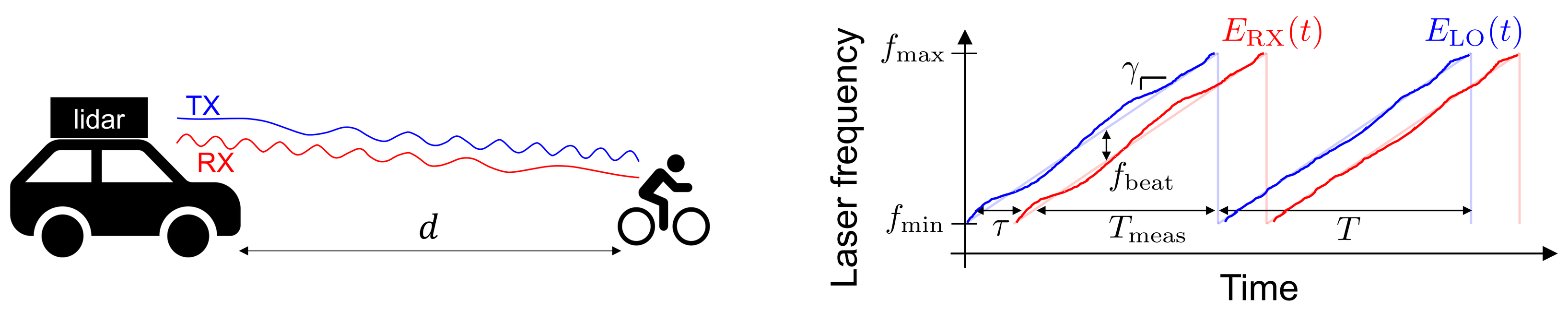
Motivation



- Object distance d causes a delay τ in received light RX
- Optically mixing RX and local oscillator LO creates an interference pattern with beat frequency f_{beat} proportional to τ
- Goal: estimate distance from f_{beat}
- Problem: phase noise causes deviation from f_{beat} that degrades performance of depth estimators

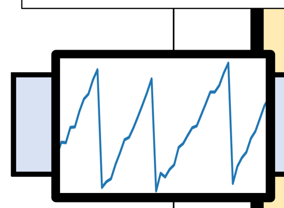
Measurement Model

- Transmitted signal: $E_{\text{LO}}(t) = \cos(\gamma t^2 + \omega_0 t + \phi_n(t))$
- Received signal: $E_{\text{RX}}(t) = \sqrt{R}\cos(\gamma(t-\tau)^2 + \omega_0(t-\tau) + \phi_n(t-\tau))$
- Interference signal intensity:

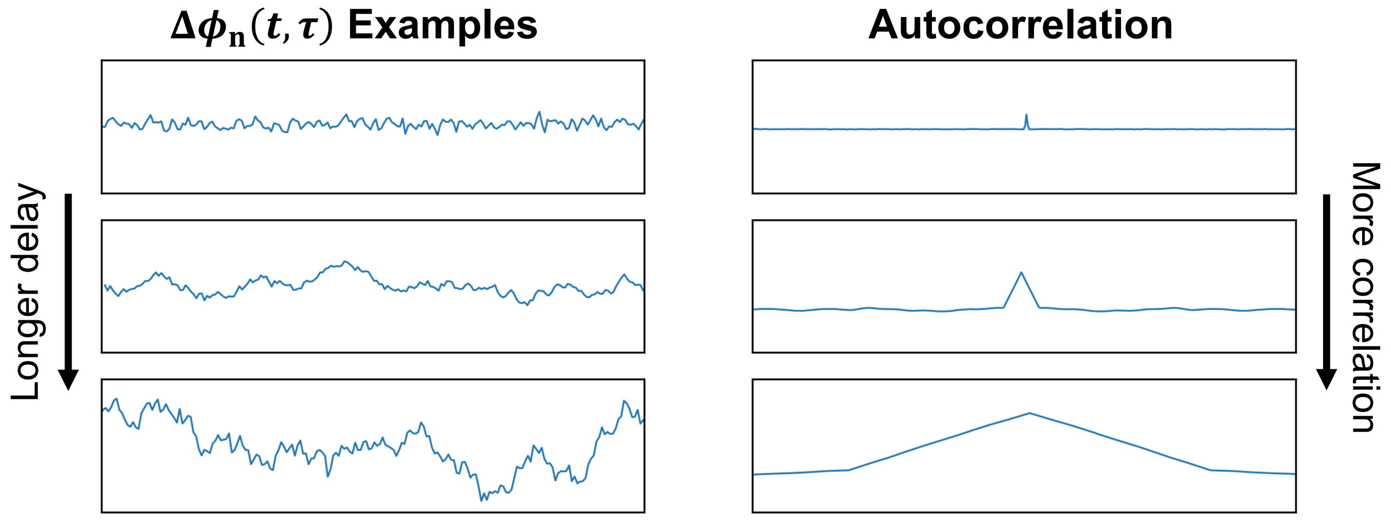
$$i(t) = \sqrt{R}\cos\left(\underbrace{\gamma\tau t}_{\text{Beat frequency } f} + \underbrace{\omega_0\tau - \frac{\gamma\tau^2}{2}}_{\text{Phase offset } \theta} + \underbrace{\Delta\phi_n(t,\tau)}_{\text{Observed phase noise}}\right) + \underbrace{w(t)}_{\text{AWGN}}$$

Observed phase noise: $\Delta\phi_n(t,\tau) = \phi_n(t) - \phi_n(t-\tau)$

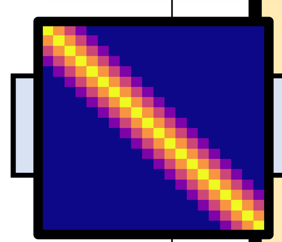
Wrapped Phase Extraction



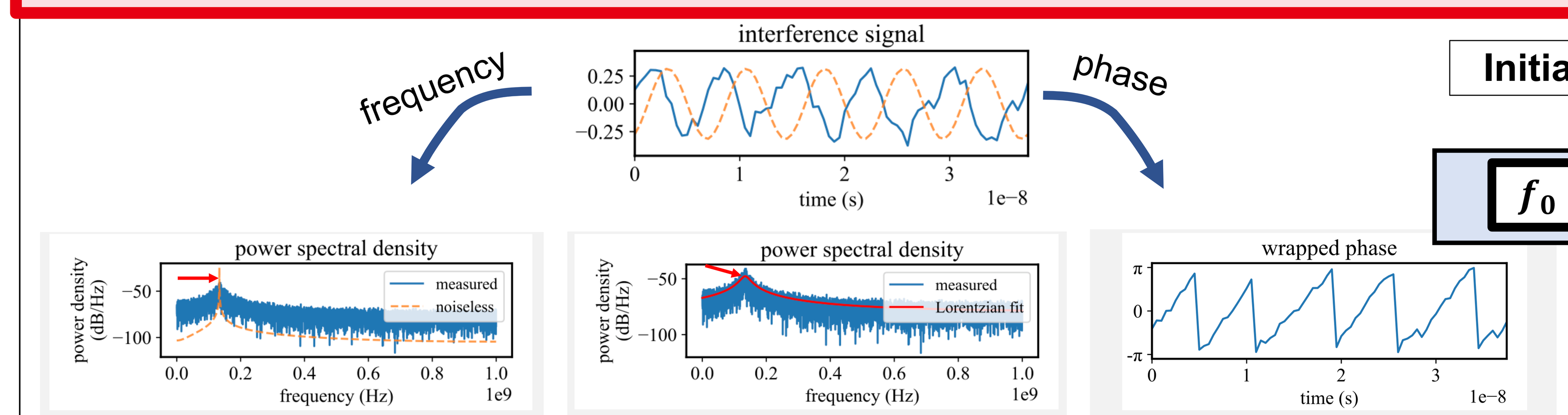
- Phase noise variance and correlation increases with delay τ



Phase Noise Statistics



Alternative Depth Estimation Methods



Periodogram maximization (Rife and Boorstyn 1974)
- ML estimator of frequency in AWGN
- Doesn't account for phase noise

Lorentzian fitting (Kim et al. 2018)
- PSD of interference signal approaches Lorentzian distribution
- Robust to phase noise
- Approximate noise model lacks precision at high SNR

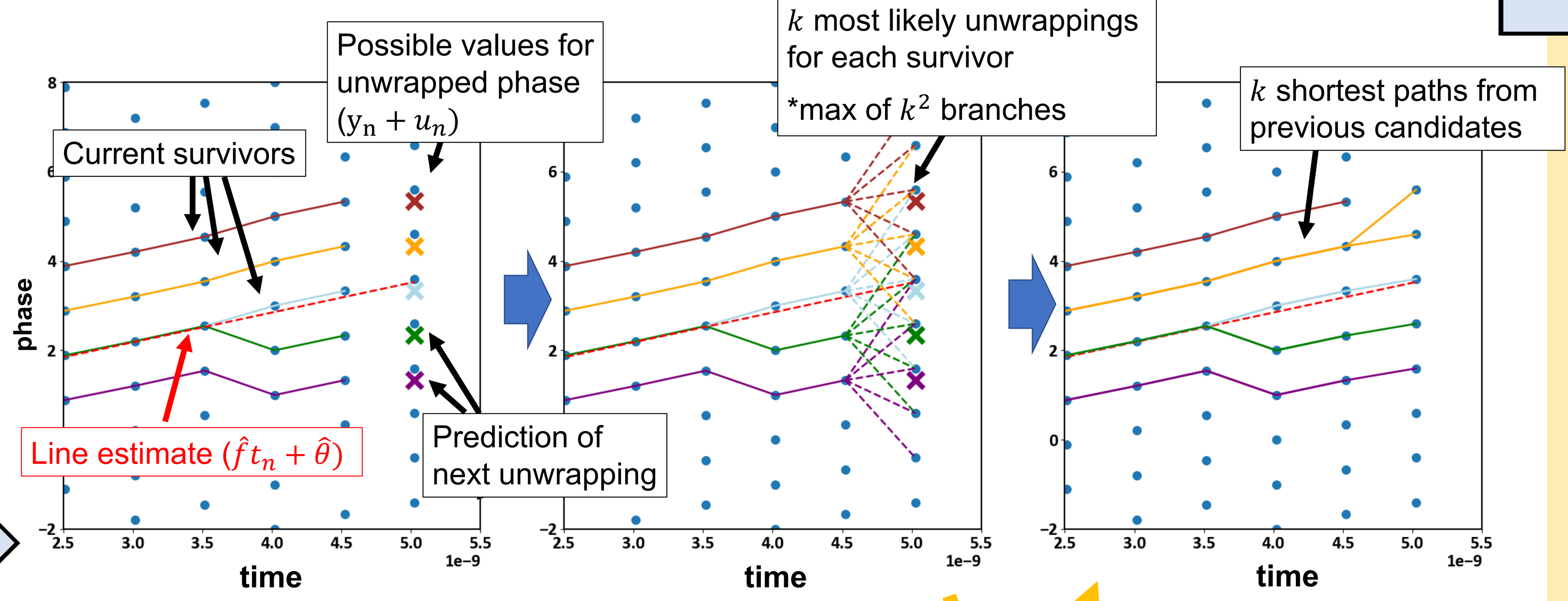
Phase unwrapping + linear regression (Tretter 1985)
- Unwrapped phase is affine function in additive Gaussian noise
- Requires accurate phase unwrapping

Proposed Depth Estimation Algorithm

- Our method estimates the beat frequency by alternating between phase unwrapping and linear regression
- Both steps account for the second-order statistics of the phase noise

Viterbi Phase Unwrapping

- Solve for phase unwrapping by predicting the phase noise for each sample
- Select the most likely unwrapping sequence among multiple possible trajectories



Find u_{n+1}'s that yield the k shortest path lengths

$$\underbrace{-\sum_{\ell=0}^{n}\log\left(p(y_\ell|u_\ell, \boldsymbol{y}_{\ell-C}^{\ell-1}, \boldsymbol{u}_{\ell-C}^{\ell-1}, \hat{f}, \hat{\theta})\right)}_{\text{length of survivor path}} \underbrace{- \log\left(p(y_{n+1}|u_{n+1}, \boldsymbol{y}_{n+1-C}^{n}, \boldsymbol{u}_{n+1-C}^{n}, \hat{f}, \hat{\theta})\right)}_{\text{length of new branch}}$$

LMMSE prediction of observed phase noise

$$\hat{\xi}_n^{\text{LMMSE}} = \boldsymbol{p}_C^T Q_C^{-1}(\boldsymbol{y}_{n-C}^{n-1} + \hat{\boldsymbol{u}}_{n-C}^{n-1} - \hat{f}\boldsymbol{t}_{n-C}^{n-1} - \hat{\theta})$$

Approximate log-likelihood of the unwrapped phase

$$-\log p(\boldsymbol{y}|\boldsymbol{u}, \hat{f}, \hat{\theta}) \approx \sum_{n=0}^{N-1}\frac{(y_n + u_n - \hat{f}t_n - \hat{\theta} - \hat{\xi}_n^{\text{LMMSE}})^2}{2\sigma_\epsilon^2} + \text{N}\cdot\log(\sigma_\epsilon^2)$$

Notation

- y_n: wrapped phase
- u_n: integer value added to unwrap y_n ($y_n + u_n$ = unwrapped phase)
- η_n: time-delay difference of Wiener Process ($\Delta\phi_n(t_n)$)
- ϵ_n: Additive Observed Phase Noise (phase noise from AWGN)
- ξ_n: Total phase noise ($\eta_n + \epsilon_n$)
- $(\hat{f}, \hat{\theta})$: frequency and phase offset estimate from GLS

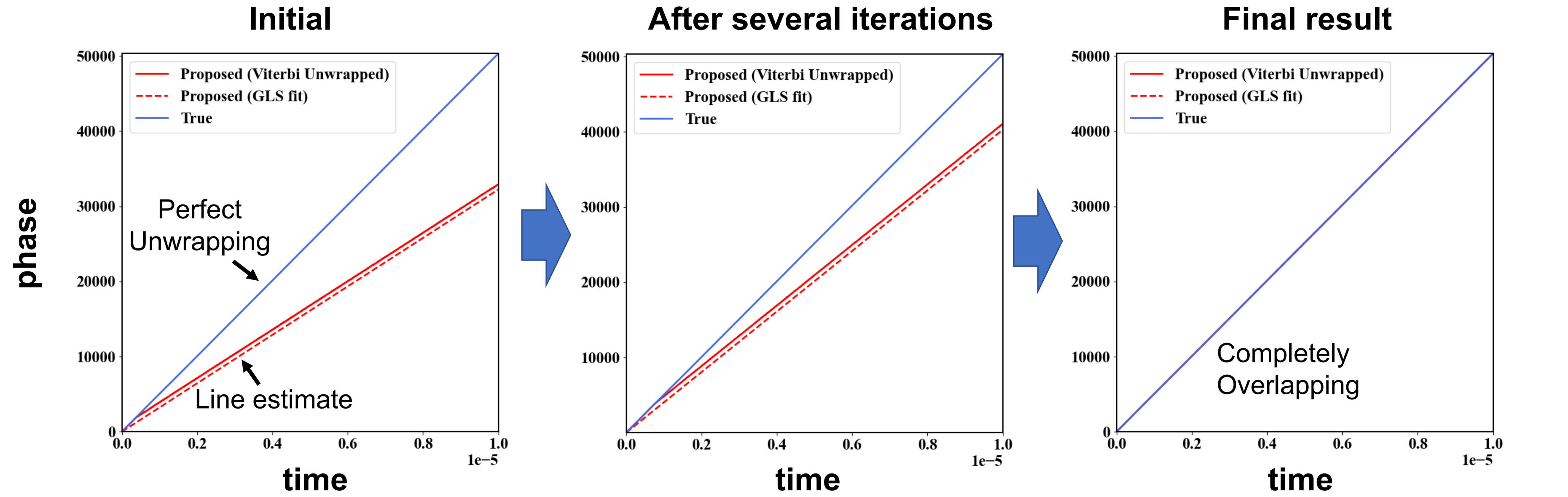
ALTERNATE

Initialization

$f_0 \leftarrow ?$

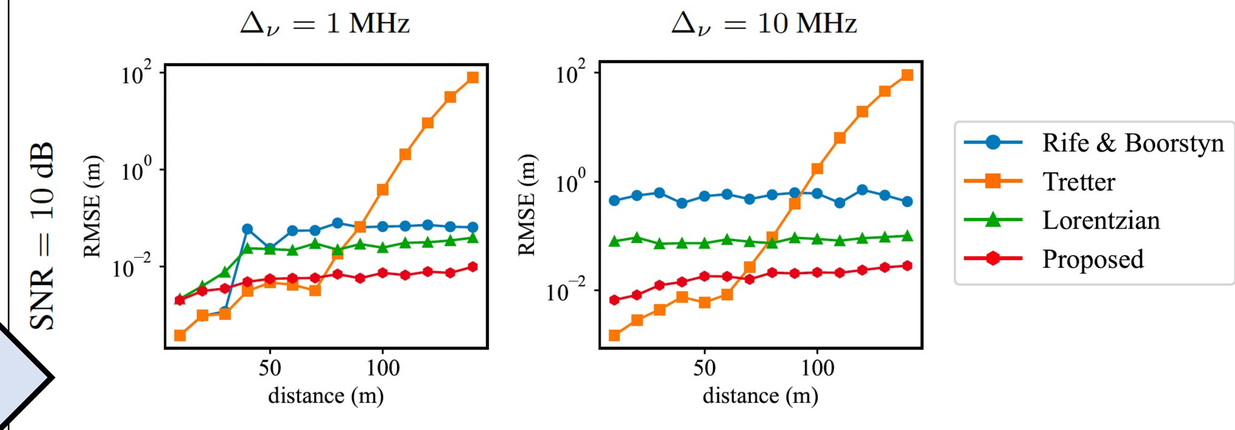
Frequency Estimate Update via Generalized Least Squares

- Update the line estimate by applying GLS on the unwrapped phase
- Our iterative approach often converges to the true unwrapped phase, even when the initial frequency estimate is far from the true value



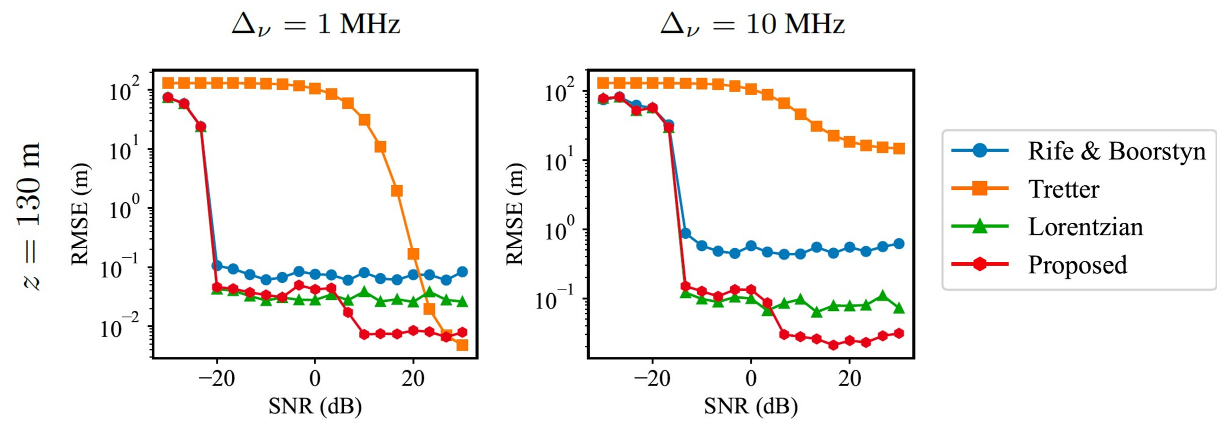
Numerical Results

Performance vs distance



Proposed approach is the most robust to phase noise (which increases with linewidth Δ_ν and distance d)

Performance vs SNR (additive noise)



Proposed approach is robust to low SNR and achieves best overall performance at high SNR (low additive noise)

Conclusion

1. Introduced a phase unwrapping-based algorithm specifically for accurate depth estimation from FMCW lidar measurements with significant phase noise and over long range
2. Demonstrated that our method can achieve the best accuracy at high SNR
3. Could enable use of cheaper swept-frequency lasers with significant phase noise for use in FMCW lidar, SS-OCT, coherent communications, etc.

References

1. D. Rife and R. Boorstyn, "Single tone parameter estimation from discrete-time observations," *IEEE Transactions on Information Theory*, vol. 20, no. 5, pp. 591–598, Sep. 1974.
2. T. Kim, P. Bhargava, and V. Stojanovic, "Optimal spectral estimation and system trade-off in long-distance frequency-modulated continuous-wave lidar," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, IEEE, Apr. 2018, pp. 1583–1587.
3. S. Tretter, "Estimating the frequency of a noisy sinusoid by linear regression," *IEEE Transactions on Information Theory*, vol. 31, no. 6, pp. 832–835, Nov. 1985.