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A Decision-Dependent Chance-Constrained Planning Model for Distribution Networks Under Extreme Weather Events

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***Abstract*—Extreme weather events have posed tremendous challenges to the operation of distribution networks. In this paper, we propose a decision-dependent chance-constrained model for the optimal planning of diesel generators, renewable distributed generations (RDGs), energy storage systems, and switches under contingency. A promising moment-based ambiguity set that incorporates the information of decision variables is employed to depict the uncertainty arising from RDGs. By leveraging effective approximation methods such as the Bonferroni approximation method to handle the considered joint chance constraints, the proposed model is transformed into a tractable mixed-integer second-order conic programming problem, which means it can easily be implemented. Numerical experiments are put forward on the IEEE 33-bus test system to validate the effectiveness of the developed approach.**

Keywords—Chance constraint, contingency, distributionally robust optimization, distribution network, renewable energy.

I. INTRODUCTION

In recent decades, the ever-increasing frequency of extreme weather events such as hurricanes [1] has significantly influenced the economic and environmental benefits of modern power systems. The blackouts caused by these events will result in great difficulties for system operations. Hence, grid resilience is becoming a vital factor to protect against extreme weather events. Particularly, since most power outages are prone to happen in distribution networks, more investments need to be conducted at the distribution level to enhance the resilience.

To achieve this goal, one of the appealing approaches is to plan the integration of electric power infrastructures, such as renewable distributed generations (RDGs), energy storage systems (ESSs), and switches. In general, installing them efficiently will give rise to quite a few inspiring advantages. For example, RDGs can maximize the penetration of clean renewable energy and provide many services (e.g., reactive power support) to the grid [2]. However, there also exist a host of challenges. The biggest one is how to satisfactorily internalize the uncertainty originating from RDGs, as the energy generated by RDGs is naturally random. Stochastic optimization (SO) and robust optimization (RO) are thus extensively adopted in the existing literature to accommodate the uncertainty [3], [4]. Although these two methods can render moderate dispatch plans under uncertainty, SO is either over-optimistic or computationally challenging, while RO tends to deliver over-conservative strategies. In this regard, distributionally robust optimization (DRO), which acts as a complementary approach between SO and RO, is explored recently [5], [6]. DRO can bridge the gaps of SO and RO since it can embed the distributional information of uncertainties and guarantee the distributional robustness via the constructed ambiguity set. Moment-based and metric-based DRO are two of the most popular paradigms. On the other hand, chance-constrained programming is also a widely leveraged method to handle the uncertainty, which requires that security constraints will be satisfied with a predefined confidence level [7]. Currently, coupled with DRO, distributionally robust chance constrained (DR-CC) models are also investigated. This methodology requires that the chance constraints (CCs) will hold for all candidate distributions within the ambiguity set [8]. However, many works only focus on addressing the DR individual CCs [6]–[8]. Compared to individual CCs, the joint chance constraint [9] can enforce multiple constraints to be satisfied simultaneously, and thus provide a higher reliability to the system. Needless to say, joint CCs are often NP-hard and more challenging than individual CCs. Therefore, tractable reformulations are urgently needed to solve this issue.

In this work, to improve the resilience of distribution systems, a planning model that aims to design various electric power infrastructures is developed while utilizing the DR joint CC method. A novel moment-based ambiguity set with the information of decision variables (i.e., decision-dependent) is harnessed to deal with the renewable forecast uncertainty. This exploited ambiguity set can describe the uncertainty more accurately. By applying convex approximations, the proposed model is cast as a mixed-integer second-order conic programming (SOCP) problem. Case studies on the IEEE 33-bus test system illustrate the effectiveness of the proposed method.

This paper contains four other sections. Section II presents the model formulation. Section III describes the solution approach to the model. Section IV reports the case studies, and Section V concludes the paper.

II. MODEL FORMULATION

In this part, the planning model for a distribution network is first presented, which consists of normal conditions with renewable forecast uncertainty and blackout conditions

The work of A. Zhou was done while he was working at MERL.

induced by extreme weather events. Then the moment-based ambiguity set is introduced.

A. The Planning Model

The objective function of the planning model for distribution network infrastructures is described as:

$$\min_{\boldsymbol{x}}\{C_1 + C_2 + C_3 + C_4 + C_5 + C_6 + C_7\}, \tag{1a}$$

where $\boldsymbol{x}$ is the decision vector. C_1 means the set-up cost and the size-based maintenance cost for RDGs. C_2 is the cost of the power purchased from the main grid under the normal condition. C_3 includes the set-up cost, the power generation cost, and the emission cost of dispatchable diesel generators. C_4 denotes the set-up cost and the degradation cost for ESSs. C_5 represents the set-up cost and the switching cost for the specific switches. C_6 is the load shedding cost. C_7 denotes the expected adjustment cost regarding the uncertainty internalized by dispatchable diesel generators. The explicit expressions of them are given as:

$$C_1 = \sum_{k=1}^{K_r}\sum_{i=1}^{N}(c_{ki}^0 + c_k^1 Tr_k)z_{ki}, \tag{1b}$$

$$C_2 = \sum_{s\in S}\sum_{t\in T_n}\left(c_{sp}p_s^t + c_{sq}q_s^t\right), \tag{1c}$$

$$C_3 = \sum_{i=1}^{N} c_{di}^0 z_{di} + \sum_{t\in T}\sum_{i\in N_c}\left(c_i^f p_{ic}^t + c_i^e c^{ef} p_{ic}^t\right), \tag{1d}$$

$$C_4 = \sum_{i=1}^{N} c_{ei}^0 z_{ei} + \sum_{t\in T}\sum_{i\in N_e} c_{ess}\left(\eta_i^{ch} p_{ich}^t + p_{idch}^t/\eta_i^{dch}\right)\Delta t, \tag{1e}$$

$$C_5 = \sum_{(i,j)\in B_g} c_{sij}^0 z_{sij} + \sum_{t\in T}\sum_{(i,j)\in B_g}\left(c_{dis}m_{ij}^t + c_{con}n_{ij}^t\right), \tag{1f}$$

$$C_6 = \sum_{t\in T}\sum_{l\in L} c_l \Delta p_l^t, \tag{1g}$$

$$C_7 = \sup_{\mathbb{P}\in\mathcal{D}_t} \mathbb{E}_{\mathbb{P}}\left\{\sum_{t\in T}\sum_{i\in N_c} c_i^{ac}\beta_{i,t}\boldsymbol{s}_t\right\}. \tag{1h}$$

In (1b), K_r is the total number of RDGs to be installed. N is the total number of buses. T is the total number of time intervals in the planning horizon. c_{ki}^0 denotes the setup cost of placing the kth RDG at bus i. c_k^1 is the size-based maintenance cost of the kth RDG. r_k means the given capacity of the kth RDG. z_{ki} is the binary indicator if kth RDG is located at bus i. In (1c), T_n is the set of time intervals under normal conditions (i.e., without extreme weather). S denotes the set of substations. c_{sp} and c_{sq} are the costs of the active and reactive power purchased from the substation s, respectively. p_s^t and q_s^t indicate the active and reactive power purchased from the substation s at time t. In (1d), c_{di}^0 is the setup cost of dispatchable diesel generators at bus i. z_{di} is the binary indicator if a diesel generator is located at bus i. N_c is the set of buses with dispatchable diesel units. c_i^f and c_i^e are the fuel and emission cost coefficients of diesel generators, respectively. c^{ef} is the emission cost coefficient of diesel generators. p_{ic}^t is the active power generated by diesel generators. In (1e), c_{ei}^0 is the setup cost of ESSs at bus i. z_{ei} is the binary indicator if an ESS is located at bus i. N_e is the set of buses with ESSs. c_{ess} is the degradation cost coefficient. η_i^{ch} and η_i^{dch} are the charging and discharging efficiencies of ESS i. p_{ich}^t and p_{idch}^t are the charging and discharging power of ESS i at time t. Δt is the time step. In (1f), B_g is the set of given branches that switches can be installed. c_{sij}^0 is the setup cost of switches at branch (i,j). z_{sij} is the binary indicator if a switch is installed at branch (i,j). c_{dis} and c_{con} are the costs of disconnecting and connecting a switch. m_{ij}^t and n_{ij}^t are auxiliary binary variables that denote disconnecting and connecting the switch at branch (i,j). In (1g), L is the set of loads. c_l is the cost of load shedding. Δp_l^t is the interrupted load at time t. In (1h), $\mathbb{P}$ is a probability distribution. $\mathcal{D}_t$ means an ambiguity set. c_i^{ac} is the adjustment cost. $\beta_{i,t}$ is the participation factor. $\boldsymbol{s}_t$ is the sum of renewable power forecasting error at time t, which represents the corresponding uncertainty of RDGs.

The constraints of the planning model are then expressed by:

1). *Output constraints of diesel generators:*

$$z_{di}p_{ic}^{t,\min} \le p_{ic}^t \le z_{di}p_{ic}^{t,\max}, i\in N_c, t\in T, \tag{2a}$$

$$z_{di}q_{ic}^{t,\min} \le q_{ic}^t \le z_{di}q_{ic}^{t,\max}, i\in N_c, t\in T, \tag{2b}$$

$$\inf_{\mathbb{P}\in\mathcal{D}_t} \mathbb{P}\begin{pmatrix} p_{ic}^t + \beta_{i,t}\boldsymbol{s}_t \le z_{di}p_{ic}^{t,\max} \\ p_{ic}^t - \beta_{i,t}\boldsymbol{s}_t \ge z_{di}p_{ic}^{t,\min}\end{pmatrix} \ge 1-\epsilon_{ic}, i\in N_c, t\in T, \tag{2c}$$

where $p_{ic}^{t,\min}, p_{ic}^{t,\max}$ and $q_{ic}^{t,\min}, q_{ic}^{t,\max}$ denote the active and reactive power minimal and maximal limits of diesel generators, respectively. q_{ic}^t is the reactive power generated by diesel generators. ϵ_{ic} is the risk parameter. (2a) and (2b) are the constraints of active and reactive power limits of diesel generators. (2c) is the DR joint chance constraint regarding the maximal and minimal active power limitations of diesel generators, which leverages the affine control policy [5] to tackle the renewable uncertainty $\boldsymbol{s}_t$.

2). *Constraints of participation factors:*

$$\sum_{i\in N_c}\beta_{i,t} = 1, \beta_{i,t} \ge 0, t\in T, \tag{3}$$

where (3) restricts the values of participation factors of diesel generators and ensures that the variation of $\boldsymbol{s}_t$ is fully compensated.

3). *Constraints of ESSs:*

$$0 \le p_{ich}^t \le \alpha_{i,t}^c p_{ich}^{t,max}, i\in N_e, t\in T, \tag{4a}$$

$$0 \le p_{idch}^t \le \alpha_{i,t}^d p_{idch}^{t,max}, i\in N_e, t\in T, \tag{4b}$$

$$\alpha_{i,t}^c + \alpha_{i,t}^d \le 1, i\in N_e, t\in T, \tag{4c}$$

$$\alpha_{i,t}^c \le z_{ei}, i\in N_e, t\in T, \tag{4d}$$

$$\alpha_{i,t}^d \le z_{ei}, i\in N_e, t\in T, \tag{4e}$$

$$SOC_{i,t+1} = SOC_{i,t} + \left(\eta_i^{ch}p_{ich}^t - p_{idch}^t/\eta_i^{dch}\right)\Delta t, \quad i\in N_e, t\in T, \tag{4f}$$

$$SOC_{i,t}^{\min} \le SOC_{i,t} \le SOC_{i,t}^{\max}, i\in N_e, t\in T, \tag{4g}$$

where $p_{ich}^{t,max}$ and $p_{idch}^{t,max}$ mean the charging and discharging power limits of ESSs. $\alpha_{i,t}^c$ and $\alpha_{i,t}^d$ are binary variables that denote the charging and discharging states of ESSs. $SOC_{i,t}^{\min}$ and $SOC_{i,t}^{\max}$ indicate the energy storage limits of ESSs. $SOC_{i,t}$ is the energy storage of ESS i. (4a) and (4b) show the charging and discharging power limits of ESSs, respectively. (4c) implies that the charging and discharging cannot happen at the same time. (4d) and (4e) mean the relationship between $\alpha_{i,t}^c$, $\alpha_{i,t}^d$, and z_{ei}. (4f) depicts the dynamics of the energy of ESS i. (4g) imposes the minimal and maximal capacity limits of ESS i.

4). *Constraints of switches:*

$$o_{ij}^{t-1} - o_{ij}^t \le m_{ij}^t, (i,j)\in B_g, t\in T, \tag{5a}$$

$$o_{ij}^t - o_{ij}^{t-1} \le n_{ij}^t, (i,j)\in B_g, t\in T, \tag{5b}$$

$$\sum_{t\in T}\sum_{(i,j)\in B_g}\left(m_{ij}^t + n_{ij}^t\right) \le s_a, \tag{5c}$$

$$o_{ij}^t \ge 1 - z_{sij}, (i,j)\in B_g, t\in T, \tag{5d}$$

where o_{ij}^t is the status variable for switch (i, j). s_a is the maximal number of switching times. (5a) and (5b) present the relationship between switch action variables and switch status variables. (5c) defines the maximal number of switch operations over T. (5d) means the relationship between the switch status variable and setup binary variable.

5). *Constraints of the substation:*

$$p_s^t = q_s^t = 0,\ s \in S, t \notin T_n, \tag{6}$$

where (6) means that the substation cannot provide the active and reactive power due to the blackout time.

6). *Power balance constraints at each bus:*

$$\sum_{f:(f,i)\in B} p_{fi}^t + \sum_{s=i,s\in S} p_s^t + \sum_{g=i,g\in G} z_{ki} p_g^t + \sum_{i,i\in N_c} p_{ic}^t + \sum_{i,i\in N_e} p_{idch}^t = \sum_{j:(i,j)\in B} p_{ij}^t + \sum_{l=i,l\in L} (p_l^t - \Delta p_l^t) + \sum_{i,i\in N_e} p_{ich}^t,$$
$$i \in N, t \in T, \tag{7a}$$

$$\sum_{f:(f,i)\in B} q_{ki}^t + \sum_{s=i,s\in S} q_s^t + \sum_{g=i,g\in G} z_{ki} q_g^t + \sum_{i,i\in N_c} q_{ic}^t = \sum_{j:(i,j)\in B} q_{ij}^t + \sum_{l=i,l\in L} (q_l^t - \Delta q_l^t),$$
$$i \in N, t \in T, \tag{7b}$$

where B and G are the set of branches and RDGs. p_{ij}^t and q_{ij}^t are the active and reactive power flows of branch (i,j). p_g^t and q_g^t are the active and reactive power outputs of RDGs. p_l^t and q_l^t are the active and reactive loads. Δq_l^t is the interrupted reactive power loads.

7). *Constraints for blackout and normal cases:*

$$\Delta p_l^t = 0, l \in L, t \in T_n, \tag{8a}$$

$$\textstyle\sum_{t\notin T_n} o_l^t \geq T_{bl}, l \in L_c, \tag{8b}$$

$$o_l^{t-1} \geq o_l^t, l \in L_c, t \notin T_n, \tag{8c}$$

$$0 \leq \Delta p_l^t \leq (1 - o_l^t)\Delta p_l^t, l \in L_c, t \notin T_n, \tag{8d}$$

where o_l^t is a binary variable that ensures the full load at blackout time. L_c is the set of critical loads. T_{bl} is the minimal number of time intervals for full loads. (8a) means that there exist no interrupted loads under normal conditions. (8b) and (8c) denote that the minimal number of time intervals without load shedding for critical loads should be met. (8d) guarantees that the critical loads are full loads during T_{bl}.

8). *Thermal capacity constraints of branches:*

$$\left(p_{ij}^t\right)^2 + \left(q_{ij}^t\right)^2 \leq o_{ij}^t \left(s_{ij}^{\max}\right)^2, (i,j) \in B, t \in T, \tag{9}$$

where $s_{ij}^{\max}$ is the apparent power capacity of branch (i,j). (9) denotes the limitation of the power flow at branch (i,j).

9). *Voltage constraints:*

$$\left|v_i^{\min}\right|^2 \leq |v_i^t|^2 \leq |v_i^{\max}|^2, i \in N, t \in T, \tag{10a}$$

$$|v_i^t|^2 - \left|v_j^t\right|^2 \leq \left(1 - o_{ij}^t\right)M + 2R_{ij}p_{ij}^t + 2X_{ij}q_{ij}^t,$$
$$i \in N, t \in T, \tag{10b}$$

$$|v_i^t|^2 - \left|v_j^t\right|^2 \geq \left(o_{ij}^t - 1\right)M + 2R_{ij}p_{ij}^t + 2X_{ij}q_{ij}^t,$$
$$i \in N, t \in T, \tag{10c}$$

where $|v_i^t|^2$ is the squared voltage magnitude at bus i. $\left|v_i^{\min}\right|^2$ and $|v_i^{\max}|^2$ are the squared voltage limitations. M is a very large value. R_{ij} and X_{ij} are the resistance and reactance of branch (i,j). (10a) restricts the boundaries of voltage magnitude at each bus. (10b) and (10c) are based on the DistFlow model.

B. Construction of the Ambiguity Set

The design of a well-defined ambiguity set is crucial to capture the stochasticity and variability of uncertainties and tackle the proposed model. Generally speaking, the probability information of a random vector $\boldsymbol{\omega}$ cannot be accurately known and oftentimes only a series of observed samples $\{\widehat{\boldsymbol{\omega}}^1, \widehat{\boldsymbol{\omega}}^2, \cdots, \widehat{\boldsymbol{\omega}}^K\}$ with a support $\boldsymbol{S}$ are accessible. In this case, we can obtain the sample mean as $\widehat{\boldsymbol{\mu}} = \frac{1}{K}\sum_{i=1}^K \widehat{\boldsymbol{\omega}}^i$ and the sample covariance as $\widehat{\boldsymbol{\Sigma}} = \frac{1}{K}\sum_{i=1}^K (\widehat{\boldsymbol{\omega}}^i - \widehat{\boldsymbol{\mu}})(\widehat{\boldsymbol{\omega}}^i - \widehat{\boldsymbol{\mu}})^{\mathrm{T}}$.

Hence, we construct the following moment-based ambiguity set:

$$\mathcal{D} = \left\{\boldsymbol{\omega} \in \boldsymbol{S}: \mathbb{E}_{\mathbb{P}}[\boldsymbol{\omega}] = \widehat{\boldsymbol{\mu}}, \mathbb{E}_{\mathbb{P}}[\boldsymbol{\omega}\boldsymbol{\omega}^{\mathrm{T}}] = \widehat{\boldsymbol{\Sigma}}\right\}, \tag{11}$$

which means that it contains all the distributions satisfying the given moment constraints.

Besides, there are many forms of the support $\boldsymbol{S}$, e.g., a hyper-box, a polytope, or an ellipsoid. Here, we assume that the support $\boldsymbol{S}$ is a hyper-box and dependent on the decision vector $\boldsymbol{x}$. Then the radius $r(\boldsymbol{x})$ of the support can be calculated by the following expression [10]:

$$r(\boldsymbol{x}) = \tfrac{1}{2}\textstyle\sum_{i=1}^n |\boldsymbol{x}_i| \left(\overline{s_i} - \underline{s_i}\right), \tag{12}$$

where $\overline{s_i}$ and $\underline{s_i}$ are the upper and lower bounds in all dimensions in $\boldsymbol{S}$. n is the dimension.

As a result, (11) and (12) make up the decision-dependent moment-based ambiguity set.

III. Solution Methodology

In this part, we investigate the solution method for our planning model.

A. Reformulation of Objective Function

Since the empirical mean is known, the worst-case expected generation cost C_7 is equal to:

$$C_7 = \sup_{\mathbb{P}\in\mathcal{D}_t} \mathbb{E}_{\mathbb{P}} \left\{\textstyle\sum_{t\in T}\sum_{i\in N_c} c_i^{ac}\beta_{i,t}\boldsymbol{s}_t\right\} = \textstyle\sum_{t\in T}\sum_{i\in N_c} c_i^{ac}\beta_{i,t}\widehat{\boldsymbol{\mu}}_t, \tag{13}$$

where $\widehat{\boldsymbol{\mu}}_t$ is the sample mean of $\boldsymbol{s}_t$.

B. Reformulation of DR Joint CCs

For ease of exposition, we first re-express DR joint CC (2c) as a compact form:

$$\inf_{\mathbb{P}\in\mathcal{D}} \mathbb{P}\left\{a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\omega} \leq b_i(\boldsymbol{x}), i = 1,2,\cdots,I\right\} \geq 1 - \epsilon, \tag{14}$$

where $\boldsymbol{\omega} \in \mathbb{R}^N$ denotes the random vector. I is the number of DR individual CCs. $a_i(\boldsymbol{x}) \in \mathbb{R}^N$ and $b_i(\boldsymbol{x}) \in \mathbb{R}$ are both affine in $\boldsymbol{x}$.

Firstly, by leveraging the classical Bonferroni approximation method [11] to deal with the DR joint chance constraint (14), (14) can be approximatively transformed into the following problem:

$$\inf_{\mathbb{P}\in\mathcal{D}} \mathbb{P}\{a_i(\boldsymbol{x})^{\mathrm{T}}\boldsymbol{\omega} \le b_i(\boldsymbol{x})\} \ge 1-\epsilon_i, i = 1,2,\cdots,I, \tag{15a}$$

$$\textstyle\sum_{i=1}^{I}\epsilon_i \le \epsilon, \epsilon_i \ge 0, i = 1,2,\cdots,I, \tag{15b}$$

where ϵ_i indicates the risk parameter for DR individual chance constraint i. To obtain a tractable reformulation of (14), as recommended in [11], we set $\epsilon_i = \epsilon/I$.

Besides, under the ambiguity set (11), the DR individual chance constraint (15a) admits a deterministic SCOP problem, which is revealed in the following proposition:

Proposition: Assuming that the support of the random vector $\boldsymbol{\omega}$ is a hyper-box, then (15a) can be transferred approximately via:

$$\hat{\boldsymbol{\mu}}^{\mathrm{T}}a(\boldsymbol{x}) + \phi_{\mathcal{K}}\, r(\boldsymbol{x}) + \pi_{\mathcal{K}}\sqrt{\frac{1-\epsilon_i}{\epsilon_i}}\,\|\boldsymbol{y}\|_2 \le b(\boldsymbol{x}), \tag{16a}$$

$$\sqrt{a(\boldsymbol{x})^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}a(\boldsymbol{x})} \le y_1, \setminus \tag{16b}$$

$$\sqrt{2\phi_{\mathcal{K}}\, r(\boldsymbol{x})} \le y_2, \tag{16c}$$

where $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in\mathbb{R}^2$ is a vector of auxiliary variables, and (ϕ_K, π_K) are positive scalars that depend on the number of samples K and ϵ_i:

$$\phi_{\mathcal{K}} = \mathcal{K}^{\left(\frac{1}{p}-\frac{1}{2}\right)}, \tag{16d}$$

$$\pi_{\mathcal{K}} = \left(1-\frac{4}{\epsilon_i}\exp^{-\left(\mathcal{K}^{\frac{1}{p}}-2\right)^2/2}\right)^{-\frac{1}{2}}, \tag{16e}$$

while

$$p > 2, \mathcal{K} > \left(2+\sqrt{1ln(4/\epsilon_i)}\right)^p. \tag{16f}$$

By using (16) to reformulate the joint chance constraint (2c), (2c) is finally reduced to an SOCP problem that can readily be implemented.

C. Reformulation of (8d)

Since (8d) is a bilinear constraint, the big-M method is utilized here to linearize these constraints. Then (8d) can be replaced by:

$$0 \le \Delta p_l^t \le \gamma, \tag{17a}$$

$$-Mo_l^t \le \Delta p_l^t - \gamma \le Mo_l^t, \tag{17b}$$

$$-M(1-o_l^t) \le \gamma \le M(1-o_l^t), \tag{17c}$$

where γ is a new auxiliary variable. If $o_l^t = 0$, then $\Delta p_l^t = \gamma$, and if $o_l^t = 1$, then $\gamma = 0$.

D. Circular Constraint Linearization Method for (9)

In this study, two square constraints are exploited to approximate the circular constraint (9), which provides a sufficient level of precision for practical applications. The two square constraints invoked in this paper are cast as:

$$-o_{ij}^t s_{sij}^{max} \le p_{ij}^t \le o_{ij}^t s_{sij}^{max}, (i,j) \in B, t \in T, \tag{18a}$$

$$-o_{ij}^t s_{sij}^{max} \le q_{ij}^t \le o_{ij}^t s_{sij}^{max}, (i,j) \in B, t \in T, \tag{18b}$$

$$-\sqrt{2}o_{ij}^t s_{sij}^{max} \le p_{ij}^t + q_{ij}^t \le \sqrt{2}o_{ij}^t s_{sij}^{max}, (i,j) \in B, t \in T, \tag{18c}$$

$$-\sqrt{2}o_{ij}^t s_{sij}^{max} \le p_{ij}^t - q_{ij}^t \le \sqrt{2}o_{ij}^t s_{sij}^{max}, (i,j) \in B, t \in T, \tag{18d}$$

To sum up, by leveraging (13) to tackle the worst-case cost function (1h), using (16) to tackle the DR joint CC (2c), and applying (17) and (18) to handle (8d) and (9), the proposed planning model is reduced to a tractable mixed integer SOCP problem, which can be solved by CPLEX solver.

IV. Numerical Simulation

In this section, a modified IEEE 33-bus test system is introduced to validate the effectiveness of our proposed model. The structure of this system is depicted in Fig. 1. Two diesel generators are placed at buses 15 and 21, two ESSs are placed at buses 7 and 29, and two wind farms are placed at buses 17 and 31, respectively. A critical load is placed at bus 19, while three switches are placed at branches (2, 22), (5, 6), and (27, 28), respectively. The parameters of diesel generators, ESSs, wind farms, and loads can be seen in [2] and [12]. The parameters of switches can be seen in [6]. Other system parameters can be found in [2]. The parameters ϵ_{ic} and T_{bl} are set to be 0.1 and 10, respectively. The whole time horizon T = 48h, while the black out time is from t=25 to t =34. The parameters p = 5 and K = 4000.

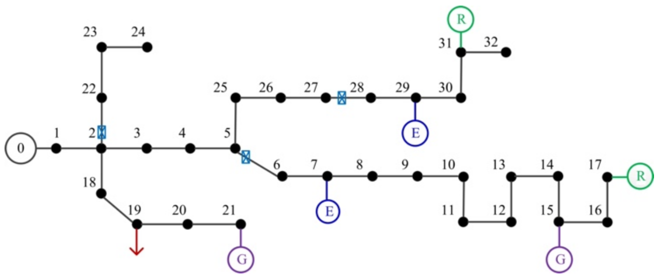

Fig. 1. Structure of the IEEE 33-bus test system.

Considering that the renewable forecast uncertainty $\boldsymbol{s}_t$ depends on the installed statues of wind farms, a RDG statues enumeration method is used to determine the optimal solution of the planning model. We first assume the installed statues for each candidate RDG, then get a corresponding estimation for $\boldsymbol{s}_t$, after that the planning model is solved by using the estimated $\boldsymbol{s}_t$. The final optimal solution is determined as the least cost solution among all solutions generated by using different combination of RDG installation statuses.

A. Set-Up Performance

In this subsection, the set-up performance of diesel generators, ESSs, wind farms, and switches is presented. The results are shown in Table I. In Table I, "1" means that the facility will be installed, and "0" otherwise.

TABLE I. Set-Up Results of the Proposed Model

Set-up Cost of One RDG	**Generators**	**ESSs**	**RDGs**	**Switches**
0.4×10^4($)	1,1	1,1	0,1	1,1,1
0.4×10^5($)	1,1	1,1	0,0	1,1,1
Set-up Cost of One ESS	**Generators**	**ESSs**	**RDGs**	**Switches**
0.1×10^4($)	1,1	1,1	0,1	1,1,1
0.4×10^5($)	1,1	0,0	0,1	1,1,1

As shown in Table I, it can be seen that the set-up decision will be affected by its cost. For example, when increasing the set-up cost of ESSs, the set-up performance of these two ESSs will be totally different. This is reasonable since there exists a trade-off between the set-up cost and operational cost. Once the set-up cost is high, installing the related facility will not be economical.

B. Comparison with Other Methods

To further assess the proposed method (denoted as M1), two other methods are applied here for comparisons, they are:

M2: Gaussian-based joint CC planning model with given mean ($\hat{\boldsymbol{\mu}}$) and covariance ($\hat{\boldsymbol{\Sigma}}$). In M2, it presumes that the uncertainty follows the Gaussian distribution. The parameter $\epsilon_i = \epsilon/\mathrm{I}$.

M3: Moment-based joint CC planning model with given mean ($\hat{\boldsymbol{\mu}}$) and covariance ($\hat{\boldsymbol{\Sigma}}$). In M3, the parameter $\epsilon_i = \epsilon/\mathrm{I}$.

It should be noted that M2 and M3 can also reformulate the individual chance constraint (15a) as SOCP problems [8]. The cost results and the lowest reliability results regarding the security constraint (2c) with 10^6 samples of the three methods are reported in Table II. As observed from Table II, M2 renders the lowest total cost among the three methods, since the particular Gaussian distribution is employed to capture the uncertainties, which is often aggressive. Besides, M1 gives a higher cost than M3 as the constructed decision-dependent ambiguity set in M1 describes the uncertainty more accurately. As for the reliability results, it can be seen that M1 and M3 can satisfy the reliability requirement (i.e., 90%), whereas M2 cannot. And M1 has the highest reliability level, validating the good performance of the proposed method.

TABLE II. COMPARISON WITH OTHER METHODS

Method	Total Cost ($)	Lowest Reliability (%)
M1	1.0841×10^6	97.8
M2	8.0360×10^5	87.6
M3	8.0410×10^5	94.3

V. CONCLUSION

This paper presents a chance-constrained planning model for distribution networks under contingency. To engage with uncertainty arising from RDGs, a decision-dependent moment- based ambiguity set is designed. Various effective approximation methods are then leveraged to reformulate the proposed model as a tractable mixed-integer second-order conic programming problem. Simulation results show the effectiveness of the developed approach. Our future work will capitalize on other ambiguity sets to accommodate the uncertainty.

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