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### Abstract

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# EMI Mitigation Using a Learning-based Frequency Modulation Carrier in PWM Inverters

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**Abstract**—Pulse-width-modulation (PWM) inverters generate electromagnetic interference (EMI) due to large  $dv/dt$  and  $di/dt$  of switching operations. When a periodic carrier is used to generate the PWM control signal, the corresponding EMI spectrum contains strong harmonic components of the carrier frequency. Research has shown that the carrier harmonic spectrum can be mitigated by modulating the carrier frequency. In this paper, we proposed a learning-based method to design the carrier frequency modulation of the inverter by solving an optimization problem, given EMI spectra of the PWM inverter under different carriers as learning data. As a result, a desired EMI spectrum envelop can be achieved according to the EMC regulation. Synthesized EMI results show that our learning-based method generalize the frequency modulation carrier design for EMI mitigation. Experiments provide promising results on EMI reduction.

**Index Terms**—EMI mitigation, PWM inverters, Carrier, Frequency modulation

## I. INTRODUCTION

Pulse-Width-Modulation (PWM) inverters are widely used in modern motor drive systems for their high efficiency and wide range of adjustable frequency output. However, the large  $dv/dt$  and  $di/dt$  of their switching devices during operations generate serious electromagnetic interference (EMI) [1], which could violate EMC regulations, interfere with other nearby equipment, or even cause malfunction of the system itself [2]. With the development of high-voltage, high-switching-frequency, and high-power-density semiconductor devices such as wide bandgap devices (WBG), there is an increasing need and interest in mitigating the EMI of power electronic devices to meet the rapid growing market in electric vehicles, electric aircraft, and renewable energy [3].

To reduce the EMI level in power electronic equipment, various methods have been studied and validated in the power electronics society, including soft switching [4], EMI filter (both passive and active) [5], and circuit layout design [6], *etc.* All these techniques aim to reduce the time-domain magnitude of EMI. Consequently, the overall EMI spectral level is also reduced.

In recent years, frequency modulation (FM)-based methods have attracted a lot of attention [7] in mitigating EMI spectral level of PWM inverters. It is well known that in power electronics the PWM signal is generated by comparing a reference signal of a desired driving frequency with a periodic carrier signal such as a sinusoidal or triangular signal. Further study shows that the EMI spectrum of PWM inverters contains strong harmonic components of the carrier frequency, which contribute the majority of frequency components that exceed EMC regulations. Therefore, it is helpful to modulate the frequency of the carrier signal such that its harmonic energy can be spread out in a relatively large frequency range. Although the overall EMI energy is not necessarily reduced by using the FM carrier signal, it has been demonstrated effective to mitigate harmonic peaks in the EMI frequency spectrum to meet the requirement of EMC regulations.

Following this frequency modulation idea, a lot of frequency modulation schemes have been proposed to mitigate the EMI level of PWM inverters. For instance, a random FM scheme [8]–[10] is proposed to dither the switching time randomly such that carrier harmonic energy can be spread out in a corresponding frequency range, resulting in a reduced EMI spectral level. However, due to the nature of randomness, random carrier FM works well in a statistical sense but not a deterministic way. It is also difficult to implement with simple hardware. Therefore, a deterministic frequency modulation scheme is desired in practice. Recently, a sinusoidal frequency modulation scheme is proposed, where the frequency of carrier signal changes within a range, following a sinusoidal wave pattern [11]. A more general frequency modulation method can be found in [12] where an adaptive frequency modulation method is proposed to reduce carrier harmonic EMI received by the victim with consideration of EMI propagation characteristics.

To further investigate and to generalize FM carrier signal design, in this paper we propose a learning-based method to design a customized FM carrier signal for the PWM control signal such that a desired EMI spectrum envelop can be achieved. Given EMI spectral data of a PWM inverter or the

same type of PWM inverters operating at different periodic carrier signals of various frequency respectively, we formulate the carrier design problem as a constrained optimization problem, where the objective function is formed according to the desired EMI spectrum, and the constraint is placed on the weights of different frequency components such that the total EMI energy is unchanged. Once the weights are determined, we design a FM carrier signal for the inverter with the carrier's frequency modulation time proportional to the corresponding weights. Under the assumption of the additivity property, the expected EMI spectrum is represented as a linear combination of the EMI spectra under different periodic carrier signals with weights corresponding to various frequency. Compared to other heuristic frequency modulated carrier design, our method is more flexible and can be used to customize carrier design for different devices. Simulation and experimental results show that a desired spectrum envelop can be achieved using our designed FM carrier.

This paper is organized as follows. In Section II, we summarize existing carriers and their corresponding EMI characteristics. In Section III we present our learning-based carrier design for desired EMI spectrum. Simulation and experimental results are shown in Section IV to demonstrate our proposed method. Conclusion is drawn in Section V.

## II. PWM CARRIERS AND EMI

In PWM inverters, the PWM control signal is generated by comparing a sinusoidal reference signal of a desired frequency with a high frequency carrier signal such as a sinusoidal or triangular wave to generate a train of rectangular pulses with different widths to control the operation of switching devices. Let  $f_r$  be the reference signal frequency, which is also the desired frequency of the inverter output, and  $f_c$  be the carrier frequency with  $f_c \gg f_r$ . In the following part, we analyze different carrier signals from the EMI point of view, ignoring other side effects such as total harmonic distortion, heat loss, and vibration, *etc.*

### A. Periodic carrier

In conventional PWM inverters, high frequency periodic carriers, such as sinusoidal, triangular, and saw-tooth signals, are used to generate PWM signals. For simplicity, we consider the sinusoidal carrier signal as an example. Given  $f_c$ , the sinusoidal carrier signal can be represented by

$$c_S(t) = A \sin[\phi_c(t)] = A \sin(2\pi f_c t + \phi_0), \quad (1)$$

where  $A$  is the amplitude,  $\phi_c(t) = 2\pi f_c t + \phi_0$  is the phase, and  $\phi_0$  is the initial phase of the sinusoidal carrier signal, respectively. Further Fourier analysis of the PWM inverter output shows that the output spectrum includes abundant harmonic frequency components of order  $(nf_c \pm kf_r)$ , where  $n$  and  $k$  are non-negative integers [13]. In Fig. 1 we plot an exemplar spectrum of PWM signal with  $f_r = 200\text{Hz}$  and  $f_c = 20\text{kHz}$ . We observe that the EMI spectrum contains strong harmonics of the carrier frequency, especially the first order harmonic.

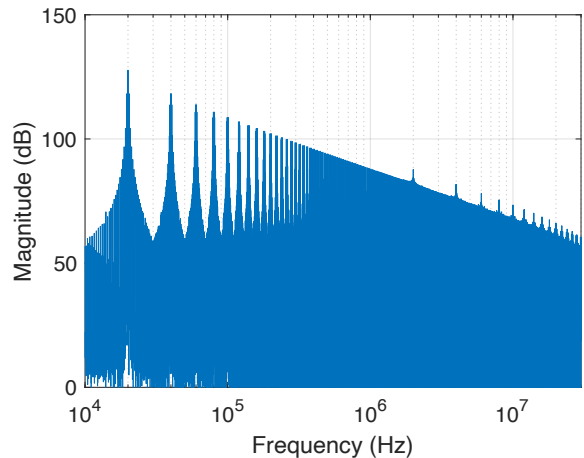


Fig. 1. EMI of a PWM inverter using a periodic carrier signal

### B. Random FM carrier

Random carrier frequency modulation (RCFM) has been proved to be an effective way to spread out the harmonic energy in the frequency domain and consequently to lower the EMI level [9]. Let  $t_{f_i}$  be the sweep duration of frequency  $f_i \in [f_{min}, f_{max}]$  and  $T$  be the total EMI measurement time according to EMC regulations. Then we have

$$t_{f_i} = v_i T, \quad s.t. \quad v_i \geq 0, \quad \sum_i^N v_i = 1, \quad (2)$$

where  $v_i$  is a fraction randomly drawn from  $[0, 1]$ . Because of the randomness, RCFM only provides statistically improved performance, not in a deterministic way. Since the carrier frequency is randomly modulated, the PWM inverter may perform differently when a different random modulation is employed.

### C. Linear FM carrier

Linear frequency modulation (LFM) equally distributes the sweep time in the frequency range,

$$t_{f_i} = T/N, \quad \text{for } i = 1, \dots, N. \quad (3)$$

Let the time-domain LFM carrier signal be

$$c_{LS}(t) = A \sin\left\{2\pi\left[\frac{f_{max} - f_{min}}{4}u(t) + \frac{f_{max} + 3f_{min}}{4}\right]t' + \phi_0\right\}, \quad (4)$$

where  $f_{min}$  and  $f_{max}$  are the minimal and maximal sweep frequency, respectively,  $u(t)$  is a saw-tooth signal of period  $T$  with magnitude in the range of  $[-1, 1]$ , which can be written as [12]

$$u(t) = 2\left[\frac{t}{T} - \left\lfloor \frac{t}{T} \right\rfloor\right] - 1, \quad (5)$$

where  $\lfloor \cdot \rfloor$  is the floor function that outputs the greatest integer less than the input real number; and

$$t' = t - \left\lfloor \frac{t}{T} \right\rfloor T = \frac{u(t) + 1}{2} T. \quad (6)$$

In general, given a fixed total EMI energy, the wider the sweep frequency range, the lower the EMI level. However, in practice the frequency range is restricted by various constraints. For example, the lowest carrier frequency  $f_{min}$  is typically constrained by the total harmonic distortion (THD) of the output voltage. If the carrier frequency is too low, the THD of output may be unacceptably high. The highest carrier frequency  $f_{max}$  is generally limited by the switching speed of devices or by other constraints such as switching loss.

#### D. Adaptive FM carrier

Above carrier designs aim to mitigate the EMI spectral level of the PWM signal, a.k.a. the EMI source. In practice, the EMI spectrum received by the victim is different from the source spectrum due to propagation. The adaptive carrier signal design incorporates the EMI source and the EMI propagation together, and pre-distorts the EMI source spectrum to compensate the influence of propagation such that the EMI spectrum received by the victim is well mitigated to a low level.

For adaptive FM carrier, given a discrete uniform frequency sampling [ $f_{min} = f_1 < f_2 < \dots < f_N = f_{max}$ ], the sweep duration  $t_{f_i}$  of frequency  $f_i$  is adaptively determined and inversely proportional to the corresponding EMI spectrum  $M(f_i)$  when a linear FM carrier is used. The sweep duration of frequency  $f_i$  can be expressed as [12]

$$t_{f_i} = t_{i+\frac{1}{2}} - t_{i-\frac{1}{2}} = \frac{\frac{1}{M(f_i)}}{\sum_{j=1}^N \frac{1}{M(f_j)}} T, \quad i = 1, \dots, N-1. \quad (7)$$

The frequency is then modulated adaptively according to

$$f_{NM}(t_{i+\frac{1}{2}}) = \frac{f_i + f_{i+1}}{2}, \quad (8)$$

where the time  $t_{i+\frac{1}{2}}$  is determined by (7) with  $t_{-\frac{1}{2}} \triangleq 0$ .

In the adaptive FM carrier, the sweep time depends on the magnitude of the first order harmonics, *i.e.*,  $f_1, f_2, \dots, f_N$ , not higher order harmonics. Therefore, it is desirable to design a FM carrier such that the whole conducted EMI spectrum can be mitigated.

### III. LEARNING-BASED PWM CARRIER DESIGN

From Section II, we learn that the performance of EMI mitigation using different FM carriers relies on the frequency modulation scheme.

The sweep time of each carrier frequency determines the contribution of the corresponding EMI spectrum to the overall EMI spectrum. The longer sweep time of a frequency, the larger fractional weight on the EMI spectrum of the same frequency carrier.

Since different devices exhibit different EMI propagation property, it is therefore desirable to optimize the FM carrier such that the overall EMI spectrum can meet EMC regulations with a large margin.

While the linear FM carrier and the adaptive FM carrier can achieve relatively low EMI spectrum level, they can not mitigate high order harmonics effectively. This is because high-order harmonics may be contributed by multiple fundamental

frequency. For example, 120kHz can be the 5<sup>th</sup> order harmonic of 24kHz, or the 3<sup>th</sup> order harmonic of 40kHz. It is difficult to decouple the impacts of different fundamental frequency. To generalize the frequency modulation carrier method, we propose a learning-based method for carrier design.

Let  $\mathbf{A} \in \mathcal{R}^{M \times N}$  be a non-negative matrix of EMI spectra as learning data, where  $A_i$  is the  $i^{\text{th}}$  column representing the EMI frequency spectrum magnitude when a periodic carrier of frequency  $f_i$  is employed to generate the PWM signal. Note that  $\mathbf{A}$  may be different from device to device. Without loss of generality, the sweep duration of frequency  $f_i$  can be expressed as a fraction of the total sweep period  $T$  as

$$t_{f_i} = t_{i+\frac{1}{2}} - t_{i-\frac{1}{2}} = w_i T, \quad (9)$$

where  $0 \leq w_i \leq 1$  is a scalar weight.

Assuming that the overall EMI spectrum is a superposition of EMI spectra using different single frequency carriers, the expected EMI with the designed FM carrier can be expressed as

$$E_{\text{FM}} = \mathbf{A}\mathbf{w}, \quad (10)$$

where  $\mathbf{w} = [w_1, w_2, \dots, w_N]^T \in \mathcal{R}^N$  represents the weight vector of all discrete sweep frequency.

Consider an EMC regulation where the maximum EMI level is constrained by a certain level. In such case, the weights of different frequency can be determined by solving an optimization problem

$$\min_{\mathbf{w}} \|\mathbf{A}\mathbf{w}\|_{\infty}, \text{ s.t. } w_i \geq 0, \quad \sum_{i=1}^N w_i = 1. \quad (11)$$

Once we get the weight  $\mathbf{w}$ , it is straightforward to get the time-varying modulation frequency with respect to time, *i.e.*

$$f_i = f(t_i) \quad \text{with} \quad t_i = \sum_{n=1}^{i-1} w_n T. \quad (12)$$

The learning-based carrier's signal can be computed as

$$c_{L_n}(t) = A \sin[\phi_{L_n}(t)], \quad (13)$$

where the phase

$$\phi_{L_n}(t_i) = 2\pi \sum_{n=1}^i f_n \cdot t_{f_n}. \quad (14)$$

In some situations, EMC standards regulate different EMI levels according to different frequency bands. To generalize the EMC regulation, we use  $\mathbf{A}\mathbf{w} \leq \mathbf{e}$ , where  $\mathbf{e}$  is a vector of frequency spectrum representing the up limit of the EMI level. Equivalently we have  $\mathbf{A}[\mathbf{e}]^{-1}\mathbf{w} \leq \mathbf{1}$ , where  $[\mathbf{e}]^{-1}$  is a diagonal matrix with the diagonal element being the inverse of the corresponding element of  $\mathbf{e}$ . The objective function is then modified by according to EMC regulations as

$$\min_{\mathbf{w}} \|\mathbf{A}[\mathbf{e}]^{-1}\mathbf{w}\|_{\infty}, \text{ s.t. } w_i \geq 0, \quad \sum_{i=1}^N w_i = 1. \quad (15)$$

It is straight forward to solve the optimization problem with disciplined convex programming such as CVX [14]. We omit

the details of solution to save space. As a result, the final EMI spectrum is optimized according to different EMC regulations.

#### IV. RESULTS AND COMPARISONS

##### A. Experimental setup

We carry out experiments on a Headspring HGCB-4A-401200 SiC inverter to test EMI spectra of different carriers, with a picture of the experimental setup shown in Fig. 2. A DC power supply made by Myway Plus APL-II (not shown) provides DC voltage to the inverter, and the AC output of the inverter drives a 18kW three-phase motor. PWM gate signals are generated by a controller corresponding to different carrier signals to control switches of the inverter. A Rohde&Schwarz (ESR7) EMI receiver (no shown) is used to measure conducted EMI in the range of [10kHz, 30MHz] generated by the inverter via a LISN (Model NNHV8123-200 made by Schwarzbeck) connected between the DC power supply and the converter.

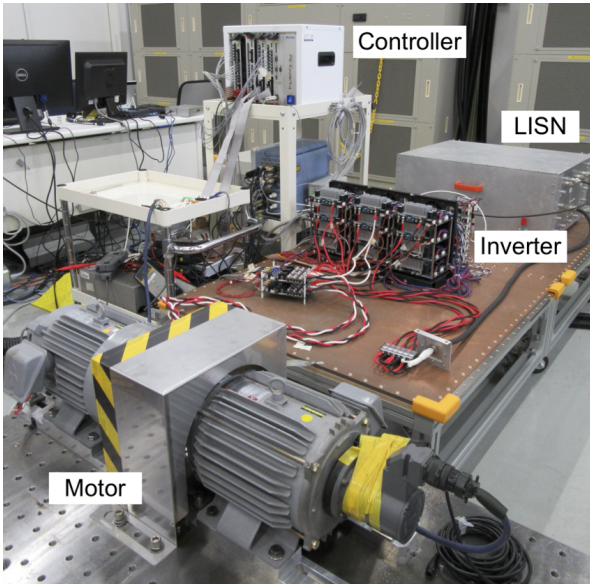


Fig. 2. Experiment setup

##### B. Synthesized EMI using experimental data

We first measure EMI levels using different periodic carriers. Fig. 3 shows the EMI spectral data map of A using periodic triangular carriers of different frequency from 50kHz to 150kHz, with step size of 1kHz. Each column represents the EMI spectrum using the corresponding frequency carrier. We can observe that the EMI spectrum contains strong harmonics of the carrier frequency, represented by straight lines of different slopes.

Based on the EMI data of different periodic carrier signals, we synthesize EMI using different FM carrier signals mentioned in Sec. II. Fig. 4 shows different FM functions of carrier signals designed by different methods. The corresponding EMI spectra of these carrier signals, synthesized by using experimental data are shown in Fig. 5.

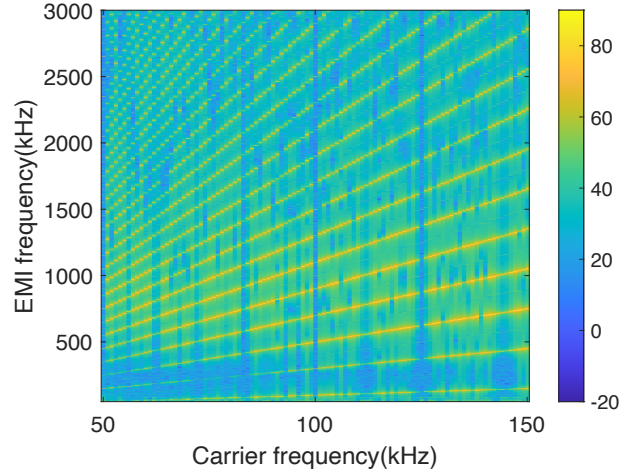


Fig. 3. Experimental data (color bar in dB scale) of different periodic carrier signals used for learning FM functions

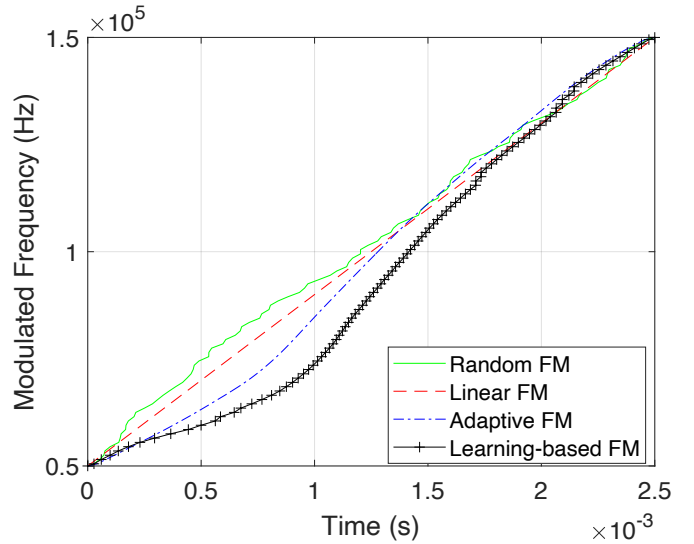


Fig. 4. Different FM functions of carrier signals.

Fig. 5(a)-(d) show the EMI spectrum using a random FM carrier, a linear FM carrier, an adaptive FM carrier, and a learning-based carrier using the proposed method, respectively. For all FM carriers, the sweep frequency range of [50, 150]kHz is considered. We observe that random carrier frequency modulation significantly mitigates the EMI level by about 10dB compared to that using a periodic triangular carrier of 100kHz overall. If a linear frequency modulation carrier is employed, the first order harmonic and conducted EMI is reduced further by about 2.5dB. When an adaptive FM carrier based on the first order harmonic is adopted, the first order harmonic is reduced by another 2.3dB. However, the maximum conducted EMI level in [200kHz, 300MHz] is increased at the same time. For our learning-based carrier (#1) that aims to reduce first order harmonics, the first order harmonic level is mitigated

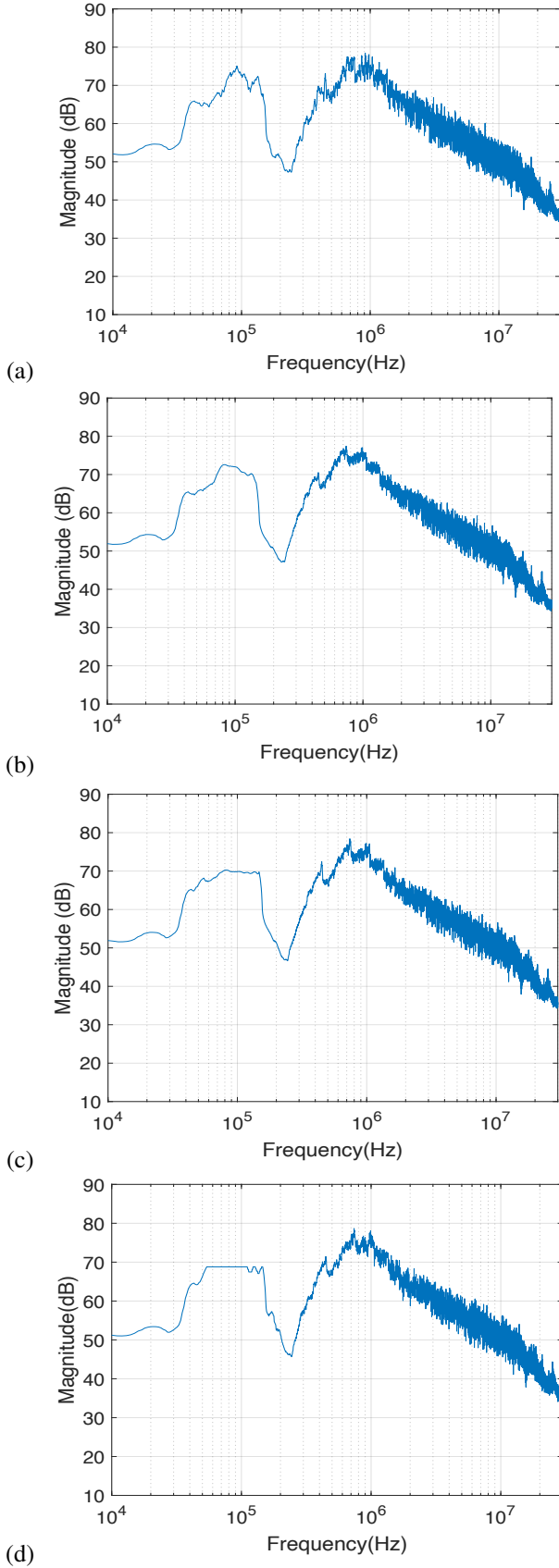


Fig. 5. EMI of a PWM inverter using (a) random, (b) linear, (c) adaptive, and (d) learning-based FM carrier signals.

to a minimum level of 68.78dB, the lowest among those of different carriers.

EMC standards regulate EMI levels of electronic products. However, EMC regulations vary for different types of products and different countries or area regions. Our learning-based carrier provides flexibility of EMI mitigation to some extent. If we aim to mitigate the whole conducted EMI level using a learning-based carrier(#2), the overall level of synthesized EMI is reduced to 76.8dB, which is also the lowest among those of different FM carriers, as shown in Fig. 6 (a). As another example, Fig. 6(b) presents a result of minimizing frequency-range-dependent EMI levels according to EMC regulations, where the red dashed line represents EMC regulation (e.g. EN IEC61000-6-3), and the green line represents the maximum level of specific frequency ranges [10kHz, 1MHz] and [1MHz, 30MHz] respectively. We observe that through learning-based carrier design, the EMI spectrum envelop can be reshaped to achieve about 6dB margin according to the EMC regulations.

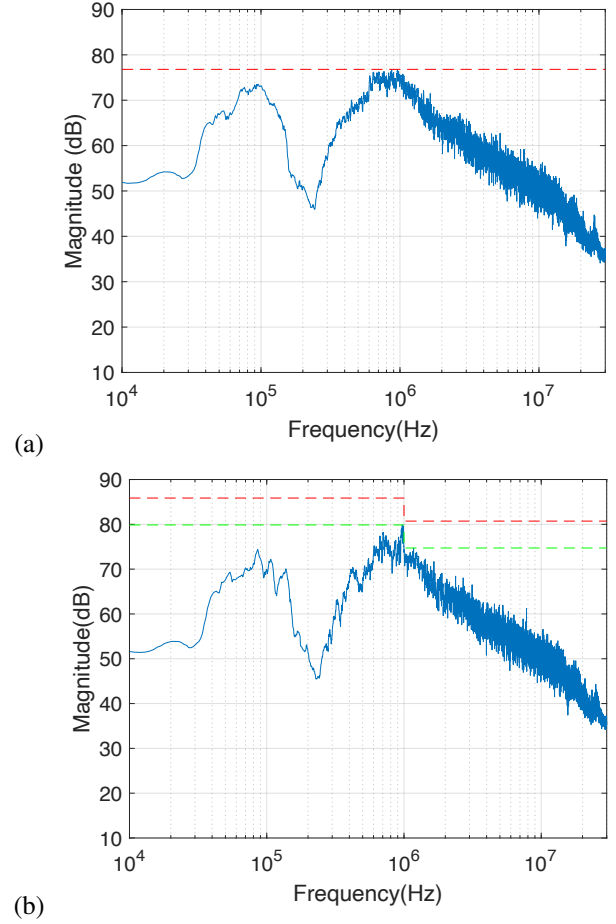


Fig. 6. Reshape EMI spectrum using learning-based FM carriers, (a) minimize the maximum EMI level and (b) minimize according to frequency-range dependent EMI levels according to EMC regulations.

EMI levels at different frequency ranges are summarized in Table I, where the frequency range [50kHz, 150kHz] corresponds to the first order harmonics, and [10kHz-30MHz]



corresponds to the conducted EMI.

TABLE I  
COMPARISON OF EMI LEVELS AT DIFFERENT FREQUENCY RANGES USING DIFFERENT CARRIERS.

| Carriers     | 50kHz-150kHz | 10kHz-30MHz  |
|--------------|--------------|--------------|
| Triangle     | 85.27        | 86.63        |
| Random FM    | 75.07        | 78.42        |
| Linear FM    | 72.59        | 77.43        |
| Adapative FM | 70.26        | 78.42        |
| Learned FM#1 | <b>68.78</b> | 78.67        |
| Learned FM#2 | 73.49        | <b>76.80</b> |

### C. Comparison of measured EMI using different carriers

We carry out experiments on the same PWM inverters with the FM carrier signal generated by the controller in Fig. 2 corresponding to different FM carriers. Fig. 7 shows experimental EMI spectra using different FM carrier signals. The EMI level is mitigated about 15-20dB when a linear FM carrier is used. When an adaptive FM carrier is adopted, the first order harmonic received by the victim in the frequency range [50kHz, 150kHz] is flattened, resulting a lower EMI level. The results agree with the synthesized results very well in EMI mitigation in the first order harmonic range. Further experiments to be carried out for validate our learning-based method.

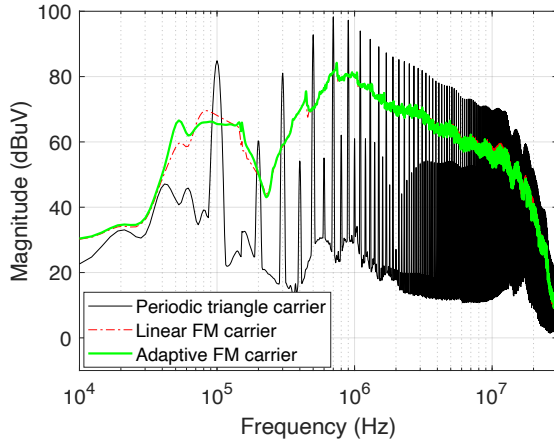


Fig. 7. EMI using different FM carrier signals

## V. CONCLUSION

We proposed a learning-based method to design FM carrier signal for PWM inverters to mitigate EMI levels by solving a constrained optimization problem given EMI spectra of different carriers as learning data. Synthesized results using measured EMI spectra show that our method is capable of mitigating the EMI level at different frequency ranges and even forming to a desired spectral envelop according to EMC regulations. Compared to other FM carrier methods, our method generalizes the FM carrier design for PWM inverters. The method can be used to customize the EMI spectral envelop for PWM inverters to meet different EMC regulations.

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